INVERSE CHAOS SYNCHRONIZATION IN THE MULTI-FEEDBACK IKEDA MODEL

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We investigate inverse synchronization between two uni-directionally coupled chaotic multi-feedback Ikeda systems and find both the existence and stability conditions for anticipating, lag, and complete synchronizations.

Keywords: Time-delayed systems, chaos synchronization, multi-feedback systems

1. INTRODUCTION

Recently chaos synchronization [1] in coupled systems have been extensively studied in the context of laser dynamics, electronic circuits, chemical and biological systems, etc. [2]. This phenomenon can be applicable in secure communication, optimization of nonlinear system performance, pattern recognition phenomena, species population control, etc., see e.g. [2] and references there in.

Finite signal transmission times, memory effects make systems with a single and multiple delays ubiquitous in nature and technology [3]. Dynamics of multi-feedback systems are representative examples of the multi-delay systems. Therefore, the study of synchronization phenomena in time-delayed systems is of high practical importance. Prominent examples of such dynamics can be found in biological and biomedical systems, laser physics, integrated communications [3]. In laser physics such a situation arises in lasers subject to two or more optical or electro-optical feedback. Second optical feedback could be useful to stabilize laser intensity [4]. Chaotic behavior of laser systems with two optical feedback mechanisms is studied in recent works [5]. Chaos synchronization between the uni-directionally coupled continuous multi-feedback systems is investigated in [6].

Recently in [7] we reported a type of synchronization: inverse anticipating synchronization, where a time-delayed chaotic system x drives another system y in such a way that a driven system anticipates the driver by synchronizing to its inverse future state: $x(t)=-y_{\tau}=-y(t-\tau)$ or equivalently $y(t)=-x(t+\tau)$ with $\tau>0$. In [7] we focused our attention on cases when a driving system contains a single delay time.

In this paper for the first time we investigate inverse synchronization between two uni-directionally coupled chaotic multi-feedback Ikeda systems and find both the existence and stability conditions for different synchronization regimes (retarded, complete, and anticipating).

2. SYNCHRONIZATION BETWEEN THE MULTI-FEEDBACK IKEDA SYSTEMS

Consider inverse synchronization between the multi-feedback Ikeda systems,

$$\frac{dx}{dt}=-\alpha x+m_1 \sin x_{\tau_1} + m_2 \sin x_{\tau_2} , \quad (1)$$

$$\frac{dy}{dt}=-\alpha+m_3 \sin y_{\tau_1}+m_4 \sin y_{\tau_2}+K \sin x_{\tau_3} \quad (2)$$

with positive $\alpha_{1,2}$ and $-m1,2,3,4$.

This investigation is of considerable practical importance, as the equations of the class B lasers with feedback (typical representatives of class B are solid-state, semiconductor, and low pressure CO2 lasers [8]) can be reduced to an equation of the Ikeda type [9].

The Ikeda model was introduced to describe the dynamics of an optical bi-stable resonator, plays an important role in electronics and physiological studies and is well-known for delay-induced chaotic behavior [10-11], see also e.g. [12]. Physically x is the phase lag of the electric field across the resonator; α is the relaxation coefficient for the driving x and driven y dynamical variables; $m_{1,2}$ and $m_{3,4}$ are the laser intensities injected into the driving and driven systems, respectively. $\tau_{1,2}$ are the feedback delay times in the coupled systems; τ_3 is the coupling delay time between systems x and y; K is the coupling rate between the driver x and the response system y.

We find that systems (1) and (2) can be synchronized on the synchronization manifold

$$y=-x_{\tau_3-\tau_1} \quad (3)$$

as the error signal $\Delta=-x_{\tau_3-\tau_1}+y$ for small Δ under the condition

$$m_1+K=m_3 ,\ m_2=m_4 \quad (4)$$

obey the following dynamics

$$\frac{d\Delta}{dt}=-\alpha\Delta+m_3\Delta_{\tau_1}\cos x_{\tau_3}+m_2\Delta_{\tau_2}\cos x_{\tau_2+\tau_3-\tau_1} \quad (5)$$

It is obvious that $\Delta=0$ is a solution of system (5). We notice that for $\tau_3>\tau_1$, $\tau_3=\tau_1$, and $\tau_3<\tau_1$ (3) is the inverse retarded, complete and anticipating synchronization manifold [12], respectively. To study the stability of the synchronization manifold $y=-x_{\tau_3-\tau_1}$ one can use the Krasovskii-Lyapunov functional approach. According to [3], the sufficient stability condition for the trivial solution $\Delta=0$ of time-delayed equation

$$\frac{d\Delta}{dt} = -r(t)\Delta + s_1(t)\Delta_{\tau_1} + s_2(t)\Delta_{\tau_2}$$

is: $r(t) > |s_1(t)| + |s_2(t)|$.

Thus, we obtain that the sufficient stability condition for the synchronization manifold $y = -x_{\tau_3-\tau_1}$ (3) can be written as:

$$\alpha > |m_3| + |m_2|. \quad (6)$$

As Eq.(5) is valid for small *Δ* stability condition (6) found above holds locally. Conditions (4) are the existence conditions for the synchronization manifold (3) between uni-directionally coupled multi-feedback systems (1) and (2). Analogously we find that $y = -x_{\tau_3-\tau_2}$ is the synchronization manifold between systems (1) and (2) with the existence $m_2+K=m_4$ and $m_1=m_3$ and stability conditions $\alpha > |m_3| + |m_4|$.

We notice that in the case of drivers with several feedback mechanisms synchronization manifold's stability condition requires larger value for the relaxation coefficient in comparison with the case of single feedback.

One can generalize the previous results to *n*- tuple feedback Ikeda systems. Applying the error dynamics approach to synchronization between the following Ikeda models

$$\frac{dx}{dt} = -\alpha x + m_{1x}\sin x_{\tau_1} + m_{2x}\sin x_{\tau_2} + \cdots + m_{nx}\sin x_{\tau_n} \quad (7)$$

$$\frac{dy}{dt} = -\alpha y + m_{1y}\sin y_{\tau_1} + m_{2y}\sin y_{\tau_2} + \cdots + m_{ny}\sin y_{\tau_n} + k\sin x_{\tau_k}, \quad (8)$$

we find that the existence and sufficient stability conditions e.g. for the synchronization manifold $y = -x_{\tau_k-\tau_1}$ are: $m_{1x} + k = m_{1y}$, $m_{nx} = m_{ny}$ and $\alpha > |m_{1y}| + |m_{2y}| + \cdots + |m_{ny}|$, respectively. For the synchronization manifold $y = -x_{\tau_k-\tau_2}$, $m_{2x} + k = m_{2y}$ and $m_{nx} = m_{ny}$ are the existence conditions, and $\alpha > |m_{1y}| + |m_{2y}| + \cdots + |m_{ny}|$ is the sufficient stability condition.

3. CONCLUSIONS

For the first *time* we have investigated inverse synchronization between two uni-directionally coupled chaotic multi-feedback Ikeda systems and find both the existence and stability conditions for inverse anticipating, lag, and complete synchronization regimes. We established that in general compared to the case of driver systems with a single feedback system additional feedback channels requires larger values for the relaxation coefficient.

Having in mind different application possibilities of chaos synchronization, synchronization in multi-feedback systems can provide more flexibility e.g. in obtaining different anticipating time scales, etc. and opportunities in practical applications.

It is well known that laser arrays hold great promise for space communication applications, which require compact sources with high optical intensities. The most efficient result can be achieved when the array elements are synchronized. Additional feedback channels could be useful to stabilize nonlinear system's output, e.g. laser intensity.

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ƏKS RABİTƏLƏRLİ İKEDA MODELİNDƏ İNVERSİON XAOC SİNXRONLAŞMASI

Bir istiqamətdə əlaqələndirilmiş bir neçə əks rabitəli xaotik İkeda modelinin inversion sinxronlaşması öyrənilib. Müxtəlif sinxronlaşma rejimləri üçün zəruri və stabillik şərtləri tapılıb.

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ИНВЕРСИОННАЯ ХАОТИЧЕСКАЯ СИНХРОНИЗАЦИЯ В МОДЕЛИ ИКЕДЫ С НЕСКОЛЬКИМИ ОБРАТНЫМИ СВЯЗЯМИ

На примере популярной модели Икеды инверсионная хаотическая синхронизация анализирована в системах с несколькими обратными связями. Найдены условия существования и стабильности различных синхронизационных режимов.

Received:23.03.2005