THE GLUONS GREEN FUNCTION IN NLO

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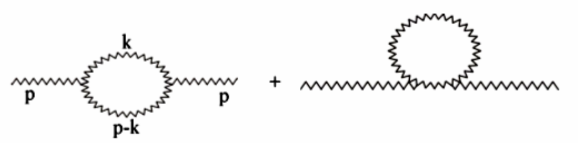
In given article the polarizing operator of $\chi_\mu A_\mu=0$ and $\partial_\mu A_\mu=0$ gauges is calculated. It was shown, that the polarizing operator corresponds to a cross condition for two conditions, only in gauge Landau. Results received in gauges $\chi_\mu A_\mu=0$ and $\partial_\mu A_\mu=0$ strongly differ from each other and in both cases the condition corresponds is not carried out.

The gluons Green function in Yang-Mills gauge theories in a general form can be represented as,

$$G^{ab}_{\mu\nu} = G^{(0)}_{\mu\nu}\delta_{ab} + \sum G^{(0)}_{\mu\mu'}\delta_{aa'}\Pi^{a'b'}_{\mu'\nu'}\delta_{bb'}G^{(0)}_{\nu\nu'} + \ldots \quad (1)$$

We shall limited by first two terms, since the second order is determined by the second terms. The sum before the second member means a summation of all diagrams of the second order. There are following two diagrams in our consideration (see fig.1). The ghosts loop, we do not consider, because limited by pure Yang - Mills theory. The calculation of the Green function



in the second order, leads us to calculated polarization operator, according to formula (1). The polarization operator corresponding to first diagram can be written as follows [1,2]:

$$\Pi^{ab}_{\mu\nu} = -\frac{ig^2}{(2\pi)^4}\int\frac{d^4k}{k^2(p-k)^2} f^{aa_1a_2} f^{bb_1b_2}\Gamma_{\mu\mu_1\mu_2}G^{(0)}_{\mu_1\mu_2}\Gamma_{\nu\nu_1\nu_2}G^{(0)}_{\nu_1\nu_2}\delta_{a_1b_1}\delta_{a_2b_2} \quad (2)$$

Here g is constant of interaction, $f^{aa_1a_2} f^{bb_1b_2}$ - are color indices, $G^{(0)}_{\mu_1\nu_1}, G^{(0)}_{\mu_2\nu_2}$ - the free gluons Green function, and $\Gamma_{\mu\mu_1\mu_2}, \Gamma_{\nu\nu_1\nu_2}$ - are vertex functions.

We shall calculate eq. (2) in both $\chi_\mu A_\mu=0$ and in Lorenz's gauge. The gluons Green function in $\chi_\mu A_\mu=0$ gauge is determined by us in work [3] and has the following form:

$$G^{(0)}_{\mu\nu} = -\frac{1}{k^2}\left(\delta_{\mu\nu} - \frac{1+\alpha}{k^2}k_\mu k_\nu\right)$$

It is seen from this expression, that it differs from Green functions in Lorenz's gauge by a longitudinal part. Note that Lorenz's gauge corresponds to transverse gauge in momentum space, and gauge $\chi_\mu A_\mu=0$ corresponds to transverse gauge in "x-space". The expression (2) can be written in the following form:

$$\begin{aligned}\Pi^{ab}_{\mu\nu} = &-\frac{ig^2}{(2\pi)^4}\int\frac{d^4k}{k^2(p-k)^2}\left(g_{\nu_1\mu_1} - \frac{1\pm\alpha}{k^2}k_{\mu_1}k_{\nu_1}\right)\cdot\left(g_{\mu_2\nu_2} - \frac{1\pm\alpha}{(p-k)^2} *\right.\\ &* (p-k)_{\mu_2}(p-k)_{\nu_2}\Big)\Big[(p+k)_{\mu_2}g_{\mu\mu_1} + (p-2k)_\mu g_{\mu_1\mu_2} + (k-2p)_{\mu_1}g_{\mu\mu_2}\Big] *\\ &* \Big[(p+k)_{\nu_2}g_{\nu\nu_1} + (k-2p)_{\nu_1}g_{\nu\nu_2} + (p-2k)_\nu g_{\nu_1\nu_2}\Big]\delta^{a_1b}\delta^{a_2b_2}\end{aligned} \quad (3)$$

Here the top sign concerns to gauge $\chi_\mu A_\mu=0$, and the bottom sign concerns to Lorenz's gauge. Let to present eq. (3) in more convenient form:

$$\begin{aligned}\Pi^{ab}_{\mu\nu} = \frac{ig^2}{(2\pi)^4}\Bigg\{&\int\frac{d^4k}{k^2(p-k)^2}\Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2}g_{\mu_1\nu_1}g_{\mu_2\nu_2} - \\ &-(1\pm\alpha)\int\frac{d^4k}{k^4(p-k)^2}k_{\mu_1}k_{\nu_1}\Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2}g_{\mu_2\nu_2} - \\ &-(1\pm\alpha)\int\frac{d^4k}{k^2(p-k)^4}(p-k)_{\mu_2}(p-k)_{\nu_2}\Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2}g_{\mu_1\nu_1} + \\ &+(1\pm\alpha)\int\frac{d^4k}{k^4(p-k)^4}k_{\mu_1}k_{\nu_1}(p-k)_{\mu_2}(p-k)_{\nu_2}\Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2}\Bigg\}\end{aligned}$$

where

$$\Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2} = \left[(p+k)_{\mu_2} g_{\mu_1\mu_2} + (p-2k)_{\mu} + (k-2p)_{\mu_1} g_{\mu\mu_2}\right] *$$

$$* \left[(p+k)_{\mu_2} g_{\nu\nu_1} + (p-2k)_{\nu} g_{\nu_1\nu_2} + (k-2p)_{\nu_1} g_{\nu\nu_2}\right]$$

The first integral in this expression has been calculated in a number of works (see [1,2]) and looks like:

$$J_1 = \int \frac{d^4k}{k^2(p-k)^2} \Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2} g_{\mu_1\nu_1} g_{\mu_2\nu_2} = \int \frac{d^4k}{k^2(p-k)^2} \left[\left(2k^2 - 2kp + 5p^2\right) g_{\mu\nu} - 2p_\mu p_\nu + 10k_\mu k_\nu - 5\left(p_\mu k_\nu + p_\nu k_\mu\right)\right]$$

The second integral after convolution of indices looks like:

$$J_2 = \int \frac{d^4k}{k^4(p-k)^2} \Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2} k_{\mu_1} k_{\nu_1} g_{\mu_2\nu_2} =$$

$$= \int \frac{d^4k}{k^4(p-k)^2} \left[p_\mu p_\nu k^2 + k_\mu k_\nu \left(p^2 + 2pk - k^2\right) + \left(p_\mu k_\nu + p_\nu k_\mu\right)\left(k^2 - 3pk\right) + \left(k^2 - 2pk\right) g_{\mu\nu}\right]$$

The third and fourth integrals are equal to:

$$J_3 = \int \frac{d^4k}{k^2(p-k)^4} (p-k)_{\mu_2} (p-k)_{\nu_2} \Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2} g_{\mu_1\nu_1} =$$

$$= \int \frac{d^4k}{k^2(p-k)^4} \left[\left(p^2 - k^2\right) g_{\mu\nu} + p_\mu p_\nu \left(-p^2 + 2k^2\right) + k_\mu k_\nu \left(-k^2 + 2p^2\right) - \left(p_\mu k_\nu + k_\mu p_\nu\right)(pk)\right]$$

and

$$J_4 = \int \frac{d^4k}{k^4(p-k)^4} (p-k)_{\mu_2} (p-k)_{\nu_{21}} \Gamma^{\nu\nu_1\nu_2}_{\mu\mu_1\mu_2} k_{\mu_1} k_{\nu_1} = \int \frac{d^4k}{k^4(p-k)^4} \left[k_\mu k_\nu p^4 + p_\mu p_\nu (pk)^2 - \left(p_\mu k_\nu + k_\mu p_\nu\right) p^2 (pk)\right]$$

Now it is necessary to calculate the integrals J_1, J_2, J_3 and J_4 over the momentum and angles variables.

Integration over the k we carry out in n-dimensional space with the further transition $n = 4$ thus we using the following formula [1]:

$$\int \frac{\left(k^2\right)^m d^nk}{\left[k^2 + p^2 x(1-x)\right]^l} = \frac{i\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)} (-1)^{m+l} \cdot \left[-p^2 x(1-x)\right]^{m+\frac{n}{2}-l} \cdot \frac{\Gamma\left(m+\frac{n}{2}\right)\cdot\Gamma\left(l-\frac{n}{2}-m\right)}{\Gamma(l)}$$

For application of this formula to J_1, J_2, J_3 and J_4 we use Feyman's parameterization:

In this formula it was done in to Euclidean space.

$$\int \frac{d^4k}{k^2(p-k)^2} \to \int_0^1 dx \int \frac{d^nk}{\left[k^2 - p^2 x(1-x)\right]^2}$$

And we carry out integration over x by use of formula

$$\int_0^1 x^\alpha (1-x)^\beta dx = \frac{\Gamma(1+\alpha)\Gamma(1+\beta)}{\Gamma(2+\alpha+\beta)}$$

Having integrated over the k and x we receive the following expression:

$$\Pi^{ab}_{\mu\nu}=-\frac{ig^2}{(2\pi)^4}i\pi^2\left\{\left[\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\frac{19}{6}-\frac{1}{2}p_\mu p_\nu\right]\Gamma\left(2-\frac{n}{2}\right)\left(-\frac{p^2}{\mu^2}\right)^{\frac{n}{2}-2}-\right.$$
$$-(1\pm\alpha)\left[\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\frac{11}{12}-\frac{1}{4}\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\Gamma\left(2-\frac{n}{2}\right)+\frac{1}{4}p_\mu p_\nu\Gamma\left(2-\frac{n}{2}\right)\right]\cdot\left(-\frac{p^2}{\mu^2}\right)^{\frac{n}{2}-2}-$$
$$-(1\pm\alpha)\left[\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\frac{11}{12}-\frac{1}{4}\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\Gamma\left(2-\frac{n}{2}\right)+\frac{3}{4}p_\mu p_\nu\Gamma\left(2-\frac{n}{2}\right)\right]\cdot\left(-\frac{p^2}{\mu^2}\right)^{\frac{n}{2}-2}-$$
$$\left.-\frac{(1\pm\alpha)}{2}\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\cdot\left(-\frac{p^2}{\mu^2}\right)^{\frac{n}{2}-2}\right\}\delta_{ab}$$

This expression can be written in a more foreseeable form, separating divergent parts:

$$\Pi^{ab}_{\mu\nu}=-\frac{g_1^2\delta_{ab}}{16\pi^2}\left\{\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\left[\frac{19}{6}+\frac{1}{2}(1\pm\alpha)\right]\Gamma\left(2-\frac{n}{2}\right)-\frac{1}{2}p_\mu p_\nu\left[1-\frac{1}{2}(1\pm\alpha)\right]\Gamma\left(2-\frac{n}{2}\right)-\right.$$
$$\left.\left.-(1\pm\alpha)\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\cdot\left(\frac{11}{6}+\frac{1}{2}(1\pm\alpha)\right)\right\}\cdot\left(\frac{\mu^2}{-p^2}\right)^{2-\frac{n}{2}}\right\} \tag{4}$$

In order to keep correct dimension of polarization operator, we have used a dimensionless constant of interaction $g_1^2=g^2\left(\frac{\mu^2}{-p^2}\right)^{4-n}$

According eq.(4) we can write out expression for the polarization operator in both gauge:
a) Lorentz's gauge:

$$\Pi^{ab}_{\mu\nu}=-\frac{g_1^2\delta_{ab}}{16\pi^2}\left\{\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\cdot\left[\frac{11}{3}-\frac{1}{2}\alpha\right]\Gamma\left(2-\frac{n}{2}\right)-\frac{1}{4}p_\mu p_\nu(1\pm\alpha)\Gamma\left(2-\frac{n}{2}\right)-\right.$$
$$\left.\left.-(1-\alpha)\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\cdot\left(\frac{7}{3}+\frac{\alpha}{2}\right)\right\}\cdot\left(-\frac{\mu^2}{p^2}\right)^{\frac{n}{2}-2}\right\} \tag{5}$$

b) $\chi_\mu A_\mu=0$ gauge

$$\Pi^{ab}_{\mu\nu}=-\frac{g_1^2\delta_{ab}}{16\pi^2}\left\{\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\left[\frac{11}{3}+\frac{1}{2}\alpha\right]\Gamma\left(2-\frac{n}{2}\right)-\frac{1}{4}p_\mu p_\nu(1-\alpha)\Gamma\left(2-\frac{n}{2}\right)-\right.$$
$$\left.\left.-(1+\alpha)\left(p^2g_{\mu\nu}-p_\mu p_\nu\right)\cdot\left(\frac{7}{3}+\frac{\alpha}{2}\right)\right\}\cdot\Gamma\left(2-\frac{n}{2}\right)\right\} \tag{6}$$

The analysis of the formula (5) shows, that in Lorentz's gauge ($\alpha=1$) the polarization operator is not transverse. And in gauge $\chi_\mu A_\mu=0$ at $\alpha=1$ it has (the formula (6)) the form of transverse polarization operator. Thus, it is possible to obtain the transverse polarization operator not introducing Faddeev-Popov's ghost. It is necessary to choose successful gauge. Let us use the renormalization procedure for expressions (5) and (6).

The preliminary factor $\left(\frac{\mu^2}{-p^2}\right)^{2-\frac{n}{2}}$ we present in the following form:

$$\left(\frac{\mu^2}{-p^2}\right)^{2-\frac{n}{2}}=1+(2-\frac{n}{2})\ln\frac{\mu^2}{-p^2}+...$$

a $\Gamma(2-\frac{n}{2})$ we represent as,

$$\Gamma(2-\frac{n}{2}) = \frac{1}{2-\frac{n}{2}} - \gamma$$

where $\gamma = 0{,}5772$ - Euler's constant.

Thus:

$$\Gamma\left(2-\frac{n}{2}\right)\left(\frac{\mu^2}{-p^2}\right)^{2-\frac{n}{2}} = \left(1+(2-\frac{n}{2})\ln\frac{\mu^2}{-p^2}\right)\cdot\left(\frac{1}{2-\frac{n}{2}}-\gamma\right)$$

Carrying out renormalization procedure we shall have:

a) For Lorenz's gauge:

$$\Pi^{ab}_{\mu\nu} = -\frac{g_1^2\delta_{ab}}{16\pi^2}\left\{\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)\cdot\left[\frac{11}{3}+\frac{1}{2}\alpha\right]\cdot\left(-\gamma + ln\frac{\mu^2}{-p^2}\right) - \frac{1}{4}p_\mu p_\nu (1+\alpha)\left(-\gamma + ln\frac{\mu^2}{-p^2}\right) - \right.$$
$$\left. -(1-\alpha)\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)\cdot\left(\frac{7}{3}+\frac{\alpha}{2}\right)\right\}$$

at $\alpha=1$, we have

$$\Pi^{ab}_{\mu\nu} = -\frac{g_1^2\delta_{ab}}{16\pi^2}\left\{\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)\frac{19}{6}\left(-\gamma + \ln\frac{\mu^2}{-p^2}\right) - \frac{1}{2}p_\mu p_\nu\left(-\gamma + \ln\frac{\mu^2}{-p^2}\right)\right\} \quad (7)$$

This formula coincides with earlier known formula (see [1,2]).

b) For gauge $\chi_\mu A_\mu=0$

$$\Pi^{ab}_{\mu\nu} = -\frac{g_1^2\delta_{ab}}{16\pi^2}\left\{\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)\left[\frac{11}{3}+\frac{1}{2}\alpha\right]\cdot\left(ln\frac{\mu^2}{-p^2} - \gamma\right) - \frac{1}{4}p_\mu p_\nu (1-\alpha)\left(ln\frac{\mu^2}{-p^2} + \gamma\right) - \right.$$
$$\left. -(1+\alpha)\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)\cdot\left(\frac{7}{3}+\frac{\alpha}{2}\right)\right\}$$

and at $\alpha=1$

$$\Pi^{ab}_{\mu\nu} = -\frac{g_1^2\delta_{ab}}{16\pi^2}\left\{\left(p^2 g_{\mu\nu} - p_\mu p_\nu\right)\left[\frac{25}{6}\left(-\gamma + \ln\frac{\mu^2}{-p^2}\right) - \frac{17}{3}\right]\right\} \quad (8)$$

The formula (8) in Feyman gauge is new result and it is transverse. Now we shall consider the second diagram. The second diagram contains the integral:

$$J = \int\frac{d^4k}{k^2}$$

In the n-dimensionally space the following formula takes place:

$$J = \int\frac{\left(k^2\right)^{\alpha-1} d^n k}{(2\pi)^n}$$

(Here $\alpha=0, 1, \ldots, m$)

And equal to zero.

Thus, the contribution of the second diagram to Green functions in the second order equal to zero.

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ƏSAS YAXINLAŞMADAN SONRAKI TƏRTİBDƏ QLÜONUN QRİN FUNKSİYASI

Məqalədə $\chi_\mu A_\mu=0$ və $\partial_\mu A_\mu=0$ kalibrovkalarında polyarizasiya operatoru hesablanıb. Göstərilmişdir ki, yalnız Landau kalibrovkasında hər iki hal üçün polyarizasiya operatoru eninəlik şərtini ödəyir. Feyman kalibrovkasında $\chi_\mu A_\mu=0$ və $\partial_\mu A_\mu=0$ kalibrovkaların verdiyi nətijələr bir-birində jiddi fərqlənir, və eninəlik şərti ödənmir.

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ФУНКЦИЯ ГРИНА ГЛЮОНА В СЛЕДУЮЩЕМ ПОРЯДКЕ ЗА ГЛАВНЫМ ПРИБЛИЖЕНИЕМ

В данной статье вычислен поляризационный оператор в калибровке $\chi_\mu A_\mu=0$ и $\partial_\mu A_\mu=0$. Было показано, что в калибровке Ландау поляризационный оператор для обоих состояний является поперечным. В калибровке Феймана полученные значения в калибровках $\chi_\mu A_\mu=0$ и $\partial_\mu A_\mu=0$ сильно отличаются друг от друга и не отвечают условию поперечности.

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