ON THE EXISTENCE AND UNIQUENESS OF GENERALIZED SOLUTION OF NONSTATIONARY PROBLEM OF VISCOUS INCOMPRESSIBLE LIQUID MOTION IN CLOSED REGION AT THE PRESENCE OF TEMPERATURE DISTRIBUTION

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The definition of generalized solution of nonstationary problem of viscous incompressible liquid motion in closed region at the presence of temperature distribution is given in the paper. The theorem about existence and uniqueness of generalized solution is proved.

Let's consider the equation system, describing the viscous incompressible liquid motion in closed region at the presence of temperature distribution on region boundary.

$$\begin{aligned}&\frac{\partial\theta}{\partial t}+(\mathrm{v},\nabla)\theta-\vartheta\Delta\theta=H,\\&-\Delta\mathrm{v}+\nabla p=Gr\theta\vec{\gamma}-(\mathrm{v},\nabla)\mathrm{v},.\\&div\mathrm{v}=0.\end{aligned}\qquad(1)$$

Equations are written in dimensionless quantities: θ - temperature, v - velocity, H - function of temperature sources, p - pressure, Gr – Grasgoph reduced number and ϑ - reduced thermal conducting $\vec{\gamma}$ - unit vector, which is codirectional to acceleration vector of free fall.

The equation system (1) is considered in closed region $\Omega\subset R^2$ – with fixed, impermeable picewise smooth boundary $\partial\Omega$ (i.e. consisting of finite number of smooth arcs, crossing under zero angles). The following conditions are given on the boundary:

$$\begin{aligned}&v|_{\partial\Omega}=0,\\&\left\{\theta,\frac{\partial\theta}{\partial n}\right\}|_{\partial\Omega}=0.\end{aligned}\qquad(2)$$

Let's designate $\overset{\circ}{J}{}^1(\Omega)$ the space being locking in $\overset{\circ}{W}{}_2^1(\Omega)$ of infinite differentiable solenoidal vector-functions. Let's introduce the conception of generalized solution of the problem.

Definition

The generalized solutions of the problem (1), (2) are such functions as $\theta\in L_\infty(0,T;\,L_2(\Omega))$ and $u\in\overset{\circ}{J}{}^1(\Omega)$, that for $H\in L_\infty(0,T;\,L_2(\Omega))$ the equations are correct:

$$\begin{aligned}&\int_0^T\left\{(\theta,-\varphi_t-(u,\nabla)\varphi)_{L_2}+\vartheta[\theta,\varphi]_{W_2^1}+\sigma\int_{\partial\Omega}\theta\varphi dS-(H,\varphi)_{L_2}\right\}dt+\\&+(\theta(x,0),\varphi(x,0))_{L_2}=0,\end{aligned}\qquad(3)$$

$$\sum_{i,k}-(u_iu_k,\frac{\partial}{\partial x_i}\Phi_k)_{L_2}+[u,\Phi]_{\overset{\circ}{J}{}^1}-Gr(\theta\gamma,\Phi)_{L_2}=0,\ t\in[0,T]\qquad(4)$$

For any functions $\varphi(x,t)$ and $\Phi(x,t)$ such, as $\varphi(x,t)\in C^1(0,T;\,W_2^1(\Omega))$, $\varphi(T)=0,\Phi(x,t)\in\overset{\circ}{J}{}^1(\Omega)$ almost everywhere in [0,T].

$\sigma(x)|_{x\in\partial\Omega}$ has the values either 0, either 1, in the dependence on the type of boundary conditions. Further, let's understand the sum $[\theta,\varphi]_{W_2^1}+\frac{1}{\vartheta}\sigma\int_{\partial\Omega}\theta\varphi dS.$ under the designation $[\theta,\varphi]_1$.

The generalized solution, introduced by us, takes place

Theorem 1

Let's $H(x,t)\in L_\infty(0,T;L_2(\Omega)),\theta(x,0)=\theta_0(x)\in L_2(\Omega)$.

Then even one generalized solution of the problem (1)-(2) exists.

Demonstration

Let's carry out the demonstration by Galerkin method.

In $L_2(\Omega)$ space let's choose the orthonormalized basis $\{\theta^l\}$ of eigen functions of Laplace operator in Ω region:

$$\begin{cases}-\Delta\theta^l=\mu_l\theta^l,\\\{\theta^l,\dfrac{\partial\theta^l}{\partial n}\}\,|_{\partial\Omega}=0.\end{cases}$$

Let's seek the approximate solutions $\theta^{n)}(x,t)$ in the form of finite sums

$$\theta^{(n)}(x,t)=\sum_{i=1}^{n}C_l{}^{(n)}\theta^{(l)}(x,t),$$

where $C_l^n\in C^1(0,T),\ l=\overline{1,n}.$

The identities (3) and (4) should be correct for $Q(n)(x,t)$.

$$\int_0^T \{(\theta^{(n)}, -\varphi_t - (u,\nabla)\varphi) + \vartheta[\theta^{(n)}, \varphi]_1 - (H, \varphi)\}dt + (\theta_0, \varphi(x,0)) = 0,$$

$$(u_i^{(n)}, u_k^{(n)} \frac{\partial}{\partial x_i}\Phi_k) + [u,\Phi]_{\overset{\circ}{J}{}^1} Gr(\theta^{(n)}\vec{\gamma},\Phi) = 0.$$

The index *(n)* of the velocity means that $u^{(n)}$ corresponds only to $\theta^{(n)}$.

In order to obtain the system of ordinary differential equations, we require that the identity (3) should be correct for any functions $\varphi^j(x,t) = h(t)\theta^j(x), \; j = \overline{1,n},$, where $h(t) \in C^1(0,T), \;\; h(T) = 0$.

This means, that following identity takes place:

$$(\dot{\theta}^{(n)}, \theta^j) + ((u^{(n)},\nabla)\theta^{(n)}, \theta^j) + \vartheta[\theta^{(n)}, \theta^j]_1 - (H, \theta^j) = 0. \quad (5)$$

Substituting the representation *Q(n)(x,t)* in the sum form in the obtained identity, we obtain Cauchy problem for the system of ordinary differential equations:

$$\dot{C}_l^{(n)}(t) + \beta_{lk}^{(n)} + \vartheta\delta_{lk}C_k^{(n)} = h_l, \; k,l = \overline{1,n}. \quad (6)$$

where $\delta_{lk} = \begin{cases} 0, & k \neq l \\ 1, & k = l \end{cases}$.

$$\beta_{lk}^{(n)} = (u^{(n)}\theta^k, \nabla\theta^l) = -(u^{(n)}\theta^l, \nabla\theta^k) = -\beta_{lk}, \quad (7)$$

$$\begin{aligned} h_l &= (H, \theta^l), \\ C_l^{(n)}(0) &= (\theta_0, \theta^l). \end{aligned} \quad (8)$$

The following Lemma takes place:

Lemma 1

Even if

$(x,t) \in L_\infty(0,T;\; L_2(\Omega)), \; \theta \in C^1(0,T;\; W_2^1(\Omega)), \; u \in \overset{\circ}{J}{}^1(\Omega).$,

then

$$\max_{0\le t\le T}\{\|\theta\|_{L_2}^2\} + \vartheta\int_0^T \|\theta\|_1^2 \, dt \le \widetilde{C}.$$

Demonstration

Let's require that for $\theta \in C^1(0,T; W_2^1(\Omega))$ identity (3) should be correct for any trial functions $\varphi(x,t)$ of $\varphi(x,t) = h(t)\psi(x)$ form, where $h(t) \in C^1(0,T), \; h(T) = 0$. This means that following equality should be correct:

$$(\theta_t, \Psi) + ((u,\nabla), \Psi) + \vartheta[\theta, \Psi] - (H, \Psi) = 0. \quad (9)$$

Let's $\Psi = \theta$, then

$$\frac{1}{2}\frac{\partial}{\partial t}\|\theta\|^2 + \vartheta\|\theta\|_1^2 = (H, \theta).$$

Let's use Young's inequality

$$(H,\theta) \le \frac{\|H\|_{-1}}{2} + \frac{\|\theta\|_{-1}}{2},$$

where $\|H\|_{-1} = \sup\limits_{q\in W_2^1} \dfrac{(H,q)}{\|q\|_1} \le \sqrt{\mu_1}\,\|H\|_{L_2}$.

Let's integrate equality on *t*, take maximum and estimate the right part from above.

We obtain:

$$\max_{0\le t\le T}\|\theta\|^2 + \vartheta\int_0^T \|\theta\|_1^2 \, dt \le \frac{T}{2}\max_{0\le t\le T}\|H\|_{-1}^2 + \|\theta(x,0)\|^2 = \widetilde{C},$$

that is required to demonstrate.

Let's show, that system of equations (6)-(8) has at least one solution.

For this purpose let's consider set K, consisting on *n*-dimensional vector-functions *b(t)*, such as

$$b_i(t) \in C^1(0,T), \; i = \overline{1,n}, \; |b(t)| = \sqrt{\sum_{i=1}^n b_i^2(t)} \le \widetilde{C}, \;\; b_i(0) = C_i(0), \; i = \overline{1,n},$$

where $\widetilde{C}$ is constant from Lemma 1.

Let's fix $b^0 \in K$, only one element $h^{(n),0} = \sum\limits_{i=1} n b_i^0 \theta^i, \;\; h^{(n),0} \in C^1((0,T)$ corresponds to element b^0; in $W_2^1\Omega$ at least one vector $u^{(n),0}$ exists. We define $\beta_{lk}^{(n),0}$ on $u^{(n),0}$ on formula (7).

The system of ordinary differential equations, defined by such way, has unit solution $b_i^1(t) = C_i(t); \; b_i^1(t)$, that corresponds to $h^{(n),1} = \sum\limits_1^n b_i^1(t)\theta_i(x)$. according to ODE theory

According to Lemma 1 $\max\limits_{0\le t\le T}\left\|h^{(n),1}\right\| + \vartheta\int_0^T \left\|h^{(n),1}\right\|_1 dt \le \widetilde{C}$.

Therefore $b^1 \in K$. Thus, mapping Λ of set *K* itself to one's is constructed.

The following Lemma takes place:

Lemma 2

If conditions of Lemma 1 and $\|\theta\|_1 \le C_1$ fulfil, then the following inequality is correct

$$\int_0^T \|\dot{\theta}\| \, dt \le C .$$

Demonstration

Let's take $\Psi = \dot{\theta}$ in the capacity of trial function Ψ in identity (9)

$$(\dot{\theta},\ \dot{\theta} + (u,\nabla)\theta) + \vartheta[\theta,\ \dot{\theta}] = (H,\dot{\theta}) .$$

Let's transfer all items, besides the first one, to the right part and estimate by the module sum from above.

$$\|\dot{\theta}\|^2 \le \frac{\vartheta}{2}\frac{d}{dt}\|\theta\|_1^2 + |(H,\dot{\theta})| + |(u,\nabla)\theta,\dot{\theta})| . \quad (10)$$

Choosing in the capacity of the trial function $\Phi = u$ from formula (4), we obtain the estimation for $\|u\|_1$:

$$\|u\|_1 \le \frac{Gr}{\mathcal{M}_1}\|\theta\|_1 \le C_2 .$$

Then

$$|(u,\ \nabla)\theta,\ \dot{\theta})| \le \|(u,\nabla)\theta\|_{L_2}\|\dot{\theta}\|_{L_2} .$$

As $\theta \in C^1(0,T;\ W_2^1(\Omega))$, so u belongs to $W_2^2(\Omega)$ at least, therefore according to enclosure theorem $u \in C(\Omega)$.

From continuity condition of *u* we have:

$$\max_{x\in\Omega}|u| \le M(C_1) .$$

Thus, $\left|(u,\nabla)\theta,\ \dot{\theta}\right| \le M\|\theta\|_1\|\dot{\theta}\|_{L_2} \le C_3\|\theta\|_1^2 + \frac{\|\dot{\theta}\|_{L_2}^2}{4}$.

Analogically, using Young's inequality:

$$\left|(u,\nabla)\theta,\ \dot{\theta}\right| \le M\|\theta\|_1\|\dot{\theta}\|_{L_2} \le C_3\|\theta\|_1^2 + \frac{\|\dot{\theta}\|_{L_2}^2}{4} .$$

Substituting the obtained results in the initial equality (10) and integrating on *t*, we obtain

$$\int_0^T \|\dot{\theta}\|_{L_2}^2 dt \le \vartheta(\|\theta\|_{W_2^1}^2(t) - \|\theta\|_{W_2^1}^2(0)) + C_4\int_0^T \|\theta\|_{W_2^1}^2 dt + C_5\int_0^T \|H\|_{L_2}^2 dt \le C ,$$

That is required to demonstrate.

It is followed from Lemma 2 and Rellikh's theorem, that Λ is entirely continuous on Λ, therefore Λ on K has at least one stationary point on Shauder's theorem.

The following Lemma takes place:

Lemma 3

The inequality is correct for the functions satisfying conditions of Lemma 2:

$$\forall\delta : 0 < \delta < T \qquad \int_0^{T-\delta} \|\theta(t+\delta) - \theta(t)\|_{L_2}^2 dt \le C\sqrt{\delta} ,$$

where *C* doesn't depend on *δ*.

Demonstration

Let's rewrite the identity (9) in the following form

$$\frac{d}{d\tau}(\theta(\tau),\varphi(t)) = ((u(\tau),\nabla)\Psi,\ \theta(\tau)) - \vartheta[\theta(\tau),\Psi(t)] + (H(\tau)),\Psi(t)) .$$

Integrating the obtained identity on τ on the interval $[t, t+\delta]$, $t \in [0, T-\delta]$ and taking $\theta(t+\delta) - \theta(t)$. in the capacity of $\Psi(t)$ we obtain as result the following equality:

$$\|\theta(t+\delta) - \theta(t)\|^2 = \int_t^{t+\delta} d\tau\{((u(\tau),\nabla)(\theta(t+\delta) - \theta(t)),\theta(\tau)) - \cdot$$
$$- \vartheta[\theta(\tau),\theta(t+\delta) - \theta(t)]_1 + (H(\tau),\theta(t+\delta) - \theta(t))\}$$

Estimating the right part of the equality as module sum, we obtain the following inequality:

$$\|\theta(t+\delta) - \theta(t)\|^2 \le \sum_{k=1}^{6} I_k(t),$$

where

$$I_1(t) = \left| \int_t^{t+\delta} (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))d\tau \right|,$$

$$I_2(t) = \left| \int_t^{t+\delta} (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t),\theta(\tau))d\tau \right|,$$

$$I_3(t) = \vartheta\left| \int_t^{t+\delta} [\theta(\tau),\theta(t)]_{W_2^1} d\tau \right|,$$

$$I_4(t) = \vartheta\left| \int_t^{t+\delta} [\theta(\tau),\theta(t+\delta)]_{W_2^1} d\tau \right|,$$

$$I_5(t) = \left| \int_t^{t+\delta} (H(\tau),\theta(t+\delta))d\tau \right|,$$

$$I_6(t)=\Big|\int_t^{t+\delta}(H(\tau),\theta(t))d\tau\ \Big|..$$

Let's integrate on t on the interval $[0,T-\delta]$ and show, that

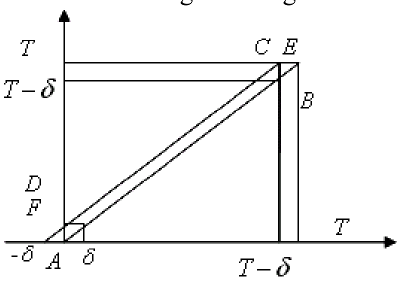
$$\int_0^{T-\delta} I_k(t)dt\le\alpha_k\sqrt{\delta},\ \ k=\overline{1,6}$$

with constant α_k, doesn't depending on δ.

Let's consider $I_1(t)$

$$\int_0^{T-\delta} I_1(t)dt\le\int_0^{T-\delta}d\tau\int_t^{t+\delta}\Big|\ (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))\ \Big|=J_1.$$

We change the order of integration and estimate J_1 integral and consider the integration region



$$J_1=\int_\delta^T d\tau\int_{\tau-\delta}^{\tau}dt\Big|\ (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))\ \Big|-$$
$$-\int_{\Delta CBE}d\tau dt\Big|\ (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))\ \Big|+$$
$$+\int_{\Delta ADG}d\tau dt\Big|\ (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))\ \Big|.$$

Suppose $\theta(t')\equiv 0$, if $t'\overline{\in}[0,T]$. Then

$$J_1=\int_0^T d\tau\int_{\tau-\delta}^{\tau}dt\Big|\ (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))\ \Big|-$$
$$-\int_{\Delta CBE}d\tau dt\Big|\ (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))\ \Big|-$$
$$-\int_{\Delta FDA}d\tau dt\Big|\ (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))\ \Big|.$$

Because of the nonnegativeness of integrand function

$$J_1\le\int_0^T d\tau\int_{\tau-\delta}^{\tau}dt\Big|\ (u_k(\tau)\frac{\partial}{\partial x_k}\theta(t+\delta),\theta(\tau))\ \Big|\le$$
$$\le C\int_0^T d\tau\|u(\tau)\|_2\|\theta(\tau)\|_1\int_{\tau-\delta}^{\tau}dt\|\theta(t+\delta)\|_1.$$

Applying Cauchy inequality to inner interval, we obtain

$$\int_{\tau-\delta}^{\tau}dt\|\theta(t+\delta)\|_1\le\sqrt{\delta}\Big(\int_{\tau-\delta}^{\tau}dt\|\theta(t+\delta)\|^2_{W_2^1}\Big)^{\frac{1}{2}}\le\sqrt{\delta}\Big(\int_0^T dt\|\theta(t+\delta)\|^2_{W_2^1}\Big)^{\frac{1}{2}}=$$
$$=\sqrt{\delta}\Big(\int_0^T dt\|\theta(t)\|^2_{W_2^1}\Big)^{\frac{1}{2}}.$$

Let's estimate $\|u\|_{\overset{\circ}{J}{}^1}$ from identity (4)

$$\|u\|_{\overset{\circ}{J}{}^1}\le C_9\|\theta\|_{W_3^1}.$$

Substituting the obtained inequalities, we have

$$J_1\le C_9C\Big(\int_0^T dt\|\theta(t)\|^2_{W_2^1}\Big)^{\frac{3}{2}}\sqrt{\delta}.$$

According to Lemma 1 we have:

Repeating all discussions, we obtain the following estimation:

$$\int_0^{T-\delta} I_2(t)dt\le\alpha_2\sqrt{\delta}.$$

Let's estimation $\int_0^{T-\delta} I_3(t)dt=J_3$.

$$J_1\le\alpha_1\sqrt{\delta}.$$

$$J_3=\vartheta\int_0^{T-\delta}dt\int_t^{t+\delta}d\tau\big|[\theta(\tau),\theta(t+\tau)]_1\big|\le\vartheta\int_0^{T-\delta}dt\int_t^{t+\delta}\|\theta(\tau)\|_1\|\theta(t+\tau)\|_1 d\tau.$$

$$J_3\le\widetilde{C}\sqrt{\delta}\int_0^T\|\theta(t)\|^2_{W_2^1}dt\le\alpha_3\sqrt{\delta}.$$

It is estimated analogically the following expression:

$$\int_0^{T-\delta} I_4(t)dt\le\alpha_4\sqrt{\delta}.$$

Let's consider $J_5 = \int_0^{T-\delta} I_5(t)dt$:

$$J_5 = \int_0^{T-\delta} dt \int_t^{t+\delta} d\tau \left|(H(\tau), \theta(t+\delta))\right| \le \int_0^{T-\delta} \|\theta(t+\delta)\|_1 J_5 = \int_t^{t+\delta} d\tau \|H(\tau)\|_{-1},$$

$$J_5 \le \sqrt{\delta} T^{\frac{3}{2}} \int_0^T \|\theta(t+\delta)\|_1 dt \max_{0\le t\le T} \|H(t)\|_{-1}^2 C \le \alpha_5 \sqrt{\delta}.$$

It is estimated analogically the following expression:

$$J_6 = \int_0^{T-\delta} I_6(t)dt \le \alpha_6 \sqrt{\delta}.$$

Thus, we obtain

$$\int_0^{T-\delta} \|\theta(t+\delta) - \theta(t)\|_{L_2}^2 dt \le C\sqrt{\delta}, \quad C = \sum_{k=1}^{6} \alpha_k.$$

That is required to demonstrate.

According to Lemma 1 $\{\theta^{(n)}\}$ is uniformly limited in $L_\infty(0,T;\, L_2(\Omega)) \cap L_2(0,T;\, W_2^1(\Omega))$. According to Lemma 3 $\{\theta^{(n)}\}$ are equipower continuities, therefore set of all $\{\theta^{(n)}\}, \theta^{(n)} \in C^1(0,T; W_2^1(\Omega))$, satisfying formulas (5) and (4), is compact.

Let's multiply the formula (5) on $h_l(t)$, where $h_l(t) \in C^1(0,T)$ and $h_l(T) = 0$ and integrate on t on $[0,T]$.. We obtain

$$\int_0^T \{(\theta^{(n)}, \varphi_t) - ((u^{(n)}, \nabla)\varphi) + \vartheta[\theta^{(n)}, \varphi]_1 - (H, \varphi)\}dt + (\theta_0, \varphi(0)) = 0,$$

where $\varphi = \sum h_l \theta^l$,

$$-(u_i^{(n)} u_k^{(n)}, \frac{\partial}{\partial x_i}\Phi_k) + [u^{(n)}\Phi]_{\overset{\circ}{J}{}^1} - Gr(\theta^{(n)} \vec{\gamma}, \vec{\Phi}) = 0.$$

As $\theta^{(n)} \to \theta$ in $L_\infty(0,T; L_2(\Omega)) \cap L_2(0,T;\, W_2^1(\Omega))$ and $u^{(n)} \to u$ (weak) in $\overset{\circ}{J}{}^1(\Omega)$, we go to the limit in two written identities.

For this purpose we define:

$$1)\ \lim_{n\to\infty} \int_0^T \{(\theta^{(n)},\, (u^{(n)},\, \nabla)\varphi) - (\theta^{(n)},\, (u^{(n)}\nabla)\varphi)\}dt =$$

$$= \lim_{n\to\infty} \int_0^T \{(\theta^{(n)},\, (u^{(n)} - u,\, \nabla)\varphi) - (\theta^{(n)} - \theta,\, (u^{(n)}\nabla)\varphi)\}dt = 0,$$

as $\lim_{n\to\infty} \int_0^T \{(\theta^{(n)},\, (u^{(n)} - u, \nabla)\varphi\}dt = 0$ because of the weak convergence of $u^{(n)} \to u$ and limitation of $\int_0^T \|\theta\|_1^2 dt$.

$\lim_{n\to\infty} \int_0^T (\theta^{(n)} - \theta,\, (u,\, \nabla)\varphi)dt = 0$ because of convergence $\theta^{(n)} \to \theta$ and limitation of $\|u\|_{\overset{\circ}{J}{}^1}$ on $[0,T]$. almost everywhere.

2) The convergence of rest scalar products are caused by the linearity of scalar products

We argue analogically at the transition to the limit in the equation for the velocity. The demonstration of this theorem is finished

Theorem 2 takes place for the generalized solution, introduced by us.

Let's $H(x,t) \in L_\infty(0,T; L_2(\Omega)), \theta(x,0) = \theta_0(x) \in L_2(\Omega)$ and $\frac{2^{\frac{3}{2}}}{\mu_1} \frac{Gr}{\vartheta} \widetilde{C}^{\frac{3}{2}} < 1$ where $\widetilde{C}$ is constant from Lemma 1, depending only on initial task data. Then the generalized task solution (1), (2) will be defined by the single way.

Demonstration

Let's use the identity (5) as at the demonstration of Theorem 1.

Let's permit, that two solutions $\theta^{(n),1}$ and $\theta^{(n),2}$, to which correspond velocity vectors u^1 and u^2, are possible.

Let's introduce the following designations:

$$\chi^{(n)} = \theta^{(n),1} - \theta^{(n),2}, \ \text{v} = u^1 - u^2.$$

Let's $\chi^{(n)} = \sum_1 n h_i \theta^i$.

The formulas (5) and (4) fulfil for $(\theta^{(n),1}, u^1)$ and $(\theta^{(n),2}, u^2)$, and $(\chi^{(n)}, \text{v})$ satisfy the following identities:

$$(\dot{\chi}^{(n)} + (u^1, \nabla)\chi^{(n)} + (\text{v}, \nabla)\theta^{(n),2}, \varphi) + \vartheta[\chi^{(n)}, \varphi]_1 = 0, \quad (11)$$

$$((u^1, \nabla)\Phi) + [\text{v}, \Phi]_{\overset{\circ}{J}^1} - Gr(\chi^{(n)} \vec{\gamma}, \Phi) = 0. \quad (12)$$

Suppose $\varphi = \chi^{(n)}, \Phi = \text{v}.$. It is followed from the formula (8):

$$\frac{1}{2}\frac{d}{dt}\left\|\chi^{(n)}\right\|_{L_2}^2 + \vartheta\left\|\chi^{(n)}\right\|_1 \le (\text{v}\theta^{(n),2}, \nabla\chi^{(n)}) \le \le \sqrt{\frac{2}{\mu_1}}\left\|\text{v}\right\|_{\overset{\circ}{J}^1}\left\|\theta^{(n),2}\right\|_{W_2^1}\left\|\chi^{(n)}\right\|_{W_2^1}. \quad (13)$$

We obtain from formula (11)

$$\left\|\text{v}\right\|_{\overset{\circ}{J}^1} \le \frac{Gr}{\sqrt{\mu_1}}\left\|\chi^{(n)}\right\|_{L_2}. \quad (14)$$

Let's substitute the result of formula (14) into formula (13) and integrate obtained inequality on time, using evident identity:

$$\left\|\chi^{(n)}\right\|_{L_2}\Big|_{t=0} = 0,$$

$$\left\|\chi^{(n)}\right\|_{L_2}\Big|_{t=t_1} + \vartheta\int_0^{t_1}\left\|\chi^{(n)}\right\|_1^2 dt \le \frac{\sqrt{2}Gr}{\mu_1}\int_0^{t_1}\left\|\theta^{(n),2}\right\|_{W_2^1}\left\|\chi^{(n)}\right\|_{W_2^1}\left\|\chi^{(n)}\right\|_{L_2} dt. \quad (15)$$

Taking the $\max\limits_{0\le t_1\le T}$ operation from both parts of inequality (15), making the amplification of the obtained inequality and deducting the non-negative value $\max\limits_{0\le t_1\le T}\left\|\chi^{(n)}\right\|_{L_2}^2$ from the left part we obtain as result:

$$\int_0^T\left\|\chi^{(n)}\right\|_1^2 dt \le \frac{\sqrt{2}}{\mu_1}\frac{Gr}{\vartheta}\int_0^T\left\|\theta^{(n),2}\right\|_{W_2^1}\left\|\chi^{(n)}\right\|_{W_2^1}\left\|\chi^{(n)}\right\|_{L_2} dt.$$

From the definition $\chi^{(n)} = \theta^{(n),1} - \theta^{(n),2}$ and from Lemma 1 we have:

$$\max_{0\le t_1\le T}\left\|\chi^{(n)}\right\|_{L_2}^2 \le 2\tilde{C}.,$$

then

$$\int_0^T\left\|\chi^{(n)}\right\|_1^2 dt \le \frac{2^{\frac{3}{2}}}{\mu_1}\frac{Gr}{\vartheta}\tilde{C}\int_0^T\left\|\chi^{(n)}\right\|_1^2 dt.$$

If $\frac{2^{\frac{3}{2}}}{\mu_1}\tilde{C} \le 1$, (where $\Re = \frac{Gr}{\vartheta}$ - Releу number), then $\int_0^T\left\|\chi^{(n)}\right\|_1^2 dt = 0$, i.e. equation system (5)has the unit solution.

The theorem statement follows from above mentioned.

Let's introduce the statement of generalized Releу number, which is equal on the definition to:

$$\Re^* = \Re\ \frac{2^{\frac{3}{2}}}{\mu_1}\tilde{C}$$

Taking into the consideration the initial task data from $\mu_1, \tilde{C}$: geometry, initial and edge conditions. Then theorem 2 is formulated by the following way: if the generalized Releу number is less than 1, then generalized task solution (1)-(1) is unit one.

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PAYLANMIŞ TEMPERATURLU QAPALI OBLASTDA SIXILMAYAN ÖZLÜ MAYENİN HƏRƏKƏTİNİN QEYRİ-STASİONAR MƏSƏLƏSİNİN ÜMUMİLƏŞMİŞ HƏLLİNİN VARLIĞI VƏ YEGANƏLİYİ HAQQINDA

Məqalədə özlü, sıxılmayan mayenin paylanmış temperatura malik qapalı oblastda qeyri stasionar hərəkətinin sərhəd məsələsinin ümumiləşmiş həlli anlayışı daxil edilir. Ümumiləşmiş həllin varlığı və yeganəliyi teoremləri isbat olunur.

Ф.Б. Имранов

О СУЩЕСТВОВАНИИ И ЕДИНСТВЕННОСТИ ОБОБЩЕННОГО РЕШЕНИЯ НЕСТАЦИОНАРНОЙ ЗАДАЧИ ДВИЖЕНИЯ ВЯЗКОЙ НЕСЖИМАЕМОЙ ЖИДКОСТИ В ЗАМКНУТОЙ ОБЛАСТИ ПРИ НАЛИЧИИ РАСПРЕДЕЛЕНИЯ ТЕМПЕРАТУР

В работе вводится определение обобщенного решения нестационарной задачи движения вязкой несжимаемой жидкости в замкнутой области при наличии распределения температур. Доказывается теорема о существовании и единственности обобщенного решения.

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