# MESONS DISTRIBUTION FUNCTIONS IN THE "NAIVE-NON-ABELIANIZATION" APPROXIMATION AND POWER-SUPPRESSED CORRECTIONS TO $F_K(Q^2)$

**Y.V. MAMEDOVA**
*Baku State Univercity, Institute for Applied Mathematics*
*AZ 1148, Z. Khalilov st., 23*

Power suppressed corrections to the kaon electromagnetic form factor $F_M(Q^2)$ is estimated by means of the running coupling constant method. In calculating the mesons distribution amplitudes (DAs) found, the "naive non-abelianization" approximation is used. Comparisons are made with $F_M(Q^2)$ obtained using the "ordinary" DAs and running coupling constant method, as well as with frozen coupling approximation's results.

1. Investigation of mesons electromagnetic (e.m.) form factors (ffs) $F_M(Q^2)$ is one of the interesting and long-standing problems in perturbative QCD (pQCD) [1-3]. The form factors are a source of information on the structure of mesons, on their DAs $\Phi_M(x,\mu_F^2)$, which are universal, nonperturbative quantities characterizing meson *M*. These DAs can be used to explain and compute other exclusive processes involving *M*. Therefore, comparing the calculated form factors with experimental data, one can deduce the information concerning the shape of DA. But before inferring such information and making conclusions about a meson DA, one has to be sure that all corrections, are least those calculable in the context of pQCD, are taken into account. There are some sources of such corrections to the meson e.m. form factor $F_M(Q^2)$, considered in the literature [4-8]. First of all, these are the next-to-leading order correction to the hard-scattering amplitude $T_H(x,y;Q^2,\mu_F^2,\mu_R^2)$ of the subprocess $q\overline{q}'+\gamma*\to q\overline{q}'$ found in [4], the two-loop correction to the Brodsky-Lepage evolution kernel $V\left[x,y;\alpha_S(Q^2)\right]$ and to the distribution amplitude itself obtained in [5]. The form factor $F_M(Q^2)$ with effects of transverse momenta of the meson constituents on the one-gluon exchange hard scattering amplitude $T_H$ and on the distribution amplitude, including the Sudakov form factor and unconventional helicity components $(h_1+h_2=\pm1)$ of the meson wave-function, has been computed in [6-8].

Another source of contributions to the form factor $F_M(Q^2)$ is the power-suppressed corrections, which in the present experimentally-accessible regime of momentum transfer $(Q^2\sim a\ few\ GeV^2)$, may play an important role in explaining the experimental data. In order to estimate these corrections in [9] and [10], the running coupling constant method and infrared matching scheme have been used.

In this letter we calculate the hard-scattering e.m. form factor of the kaon using the running coupling constant method and the mesons DAs, recently obtained in [11] in the "naive non-abelianization" (NNA) approximation.

2. It is well known that at large momentum transfer the meson *M* electromagnetic form factor $F_M(Q^2)$ is given by the expression [1]

$$F_M(Q^2)=\int_0^1\int_0^1 dxdy\Phi_M^*\left(y,\mu_F^2\right)T_H\left(x,y;Q^2,\mu_F^2,\mu_R^2\right)\Phi_M\left(x,\mu_F^2\right), \tag{1}$$

where $Q^2=-q^2$ is the momentum transfer in the process ($q^2$ is the square of the four-momentum of the virtual photon $\gamma*$), $\mu_R^2$, $\mu_F^2$ are the renormalization and factorization scales, respectively.

At the leading order of pQCD the hard-scattering amplitude $T_H\left(x,y;Q^2,\mu_R^2,\mu_F^2\right)$ does not depend on the factorization scale $\mu_F^2$ and depends on $\mu_R^2$ only through the running coupling constant $\alpha_S\left(\mu_R^2\right)$. At the next-to-leading order, $T_H$ depends on $\mu_R^2$, $\mu_F^2$ explicitly due to terms proportional to $\ln\left(Q^2/\mu_F^2\right)$ and $\ln\left[(1-x)(1-y)Q^2/\mu_R^2\right]$ (see [4]). The proper choice of these scales, i.e. the choice which minimizes the higher-order corrections to $F_M(Q^2)$ and at the same time allows one to estimate the power-suppressed corrections to $F_M(Q^2)$, is an important problem in pQCD [4,9,10,12]. For the factorization scale $\mu_F^2$ a natural choice is $\mu_F^2=Q^2$ which eliminates the logarithms of $Q^2/\mu_F^2$. In Ref.9 the renormalization scale $\mu_R^2$ has been chosen as

$$\mu_R^2=(1-x)(1-y)Q^2,\quad \overline{\mu}_R^2=xyQ^2, \tag{2}$$

and in [10] as

$$\mu_R^2=(1-x)Q^2/2,\quad \overline{\mu}_R^2=xQ^2/2. \tag{3}$$

Equation (2) describes the case with two running variables $(x,y)$, whereas in (3) we freeze one of the variables by taking its mean value. In (3), we take $\langle y\rangle=1/2$, $x$ is the running variable. Alternatively, one can take $\langle x\rangle=1/2$, and $y$ as the running variable or the mean value of the sum of the form factors, calculated using both of these possibilities; due to symmetry of $T_H$ and (1) with respect to $x, y$, one will obtain the same result. In all cases the choice of $\mu_R^2$, $\overline{\mu}_R^2$ depends on the Feynman diagram for $T_H$ under consideration. Of course, the second choice, (3), leaves in the NLO correction some logarithmic terms, but it allows us to compare our predictions with results obtained by means of the infrared matching scheme [13], and also leads to better agreement with experimental data [10]. Therefore, we shall use (3) in our computations.

At the leading order of pQCD, $T_H$ has the following form:

$$T_H(x,y;Q^2,\alpha_S(\mu_R^2)) = \frac{16\pi C_F}{Q^2}\left[\frac{2}{3}\frac{\alpha_S(\mu_R^2)}{(1-x)(1-y)} + \frac{1}{3}\frac{\alpha_S(\overline{\mu}_R^2)}{xy}\right], \tag{4}$$

where $C_F = 4/3$ is the color factor.

3. An important moment in our study is the choice of the mesons DAs $\phi_M(x,Q^2)$ in Eq.(1). The meson DAs are phenomenological model functions, the information about shapes of which should be taken either from experimental data, or from nonperturbative calculations. The evolution of $\phi_M(x,Q^2)$ as a function of the factorization scale $Q^2$ can be found by means of pQCD methods [1]. In the literature for the pion, kaon and $\rho_L$-meson various model DAs have been proposed [2,14,15]. They have been obtained using QCD sum rules method. But from the very beginning these DAs, enhanced in the endpoint region (for example, the Chernyak-Zhitnitsky DA of the pion) have been met with criticism [16], intensified recently in the light of new experimental data on the transition form factor $F_{\pi\gamma}(Q^2)$ reported by CLEO collaboration [17]. In [18] and [19] the authors have concluded that these data can be explained by the pion asymptotic or asymptotic-like DA and that model DA from [2] (Chernyak-Zhitnitsky DA) disagrees with the data. At the same time an asymptotic-like pion DA employed for computation of the electromagnetic form factor $F_\pi(Q^2)$ in the hard-scattering approach (1) gives result lying below the experimental data on $F_\pi(Q^2)$. As it was proven in [9] and [10], the power-suppressed corrections, estimated using the running coupling constant method, enhance the “ordinary” pQCD result approximately by a factor 2 and can help in solution of problems with $F_\pi(Q^2)$.

Recently, in [11] the authors have calculated the contribution of “bubble chain” diagrams to the Brodsky-Lepage evolution kernel $V[x,y;\alpha_S(Q^2)]$ in the “naive non-abelianization” (NNA) approximation and, as a result, have got new, infrared (ir) renormalon improved DA for the meson

$$\phi_M(x,Q^2) = f_M[x(1-x)]^{1+\alpha}\sum_{n=0}^{\infty} b_n(Q^2)A_n(\alpha_S)C_n^{3/2+\alpha}(2x-1), \tag{5}$$

where $\{C_n^{3/2+\alpha}(2x-1)\}$ are the Gegenbauer polynomials, $A_n(\alpha_S)$ are normalization constants, $b_n(Q^2)$ define the evolution of $\phi_M(x,Q^2)$ with $Q^2$ and $\alpha \equiv \alpha_S(Q^2)\beta_0/4\pi$. Here, the $n$ takes odd values in the case of the kaon, because the kaon DA contains an antisymmetric under replacement $2x-1 \leftrightarrow 1-2x$ [2]. In (5) $f_M$ is the meson $M$ decay constant, for the kaon it equals to $f_K = 0.112 GeV$. In accordance with this normalization of DA and decay constant $f_M$, which differs from that of [11], $A_n(\alpha_S)$ are given by the expression

$$A_n(\alpha_S) = \frac{\Gamma(3+2\alpha)}{\sqrt{3}\Gamma(1+\alpha)\Gamma(2+\alpha)}\frac{n!}{(2+2\alpha)_n}\frac{3+2\alpha+2n}{2+2\alpha+2n}, \tag{6}$$

where $\Gamma(z)$ is the Euler gamma function, $(\alpha)_n$ is the Pochhammer symbol, $(\alpha)_n = \Gamma(\alpha+n)/\Gamma(\alpha)$.

In this work we neglect the dependence of $\phi_M(x,Q^2)$ on the factorization scale $Q^2$, therefore we do not write down the expression for $b_n(Q^2)$. It is worth noting that (5) and (6) are valid for both even and odd values of $n$.

4. As it has been emphasized above, here we choose the renormalization scale $\mu_R^2$ as in (3). But the electromagnetic form factor (see (1) and (4)) with $\alpha_S[(1-x)Q^2]$ (and $\alpha_S(xQ^2/2)$) suffers from ir singularities associated with the behavior of $\alpha_S$ in the soft regions $x \to 1;\ 0$ . Thus, the form factor $F_M(Q^2)$ can be found after regularization of $\alpha_S(\mu_R^2)$ in this endpoint regions. To solve this problem it is convenient to express the running coupling constant $\alpha_S(\lambda Q^2/2)$ in terms of $\alpha_S(Q^2/2)$, which can be done by means of the renormalization group equation [20]

$$\alpha_S(\lambda Q^2/2) = \frac{\alpha_S}{1+\ln\lambda/t} - \frac{\alpha_S^2\beta_1}{4\pi\beta_0}\frac{\ln[1+\ln\lambda/t]}{[1+\ln\lambda/t]^2}. \tag{7}$$

Here, $t = 4\pi/\beta_0\alpha_S(Q^2/2)$, $\alpha_S \equiv \alpha_S(Q^2/2)$ is the one-loop QCD coupling constant and $\beta_0$, $\beta_1$ are the QCD beta-function one- and two-loop coefficients, respectively,

$$\alpha_S(\mu^2) = \frac{4\pi}{\beta_0\ln(\mu^2/\Lambda^2)}, \quad \beta_0 = 11 - \frac{2}{3}n_f, \quad \beta_1 = 102 - \frac{38}{3}n_f, \tag{8}$$

where $n_f$ is the number of quark flavors, $\Lambda$ is the QCD parameter $\Lambda = 0.2\,GeV$.

For our purposes it is convenient to rewrite the meson DA in the following form:

$$\phi_M\left(x,Q^2\right) = f_M\left[x(1-x)\right]^{1+\alpha}\sum_{n=0}^{\infty}K_n\left(\alpha_S\right)x^n. \quad (9)$$

The explicit expressions of new coefficients $K_n\left(\alpha_S\right)$ can be found in the Appendix.

Substituting (4), (7) and (9) into (1), performing integration over $x$ using the inverse Laplace transformations [21]

$$\frac{1}{(t+z)^{\nu}} = \frac{1}{\Gamma(\nu)}\int_0^{\infty}\exp[-u(t+z)]u^{\nu-1}du, \ \mathrm{Re}\,\nu>0 \quad (10)$$

and

$$\frac{\ln(t+z)}{(t+z)^2} = \int_0^{\infty}\exp[-u(t+z)](1-C-\ln u)udu, \quad (11)$$

where $z=\ln(1-x)$, and after integration over $y$ we find the following expressions for the e.m. form factor for the kaon:

$$\left[Q^2F_M\left(Q^2\right)\right]^{res} = \frac{(16\pi f_K)^2}{9\beta_0}\{2\sum_{n=0}^{4}K_nB(2+n+\alpha,\,1+\alpha)$$
$$\times\sum_{l=0}^{3}K_l\int_0^{\infty}\exp(-tu)R(u,t)B(2+l+\alpha,\,1+\alpha-u)du$$
$$+\sum_{n=0}^{3}K_nB(2+\alpha,1+n+\alpha)$$
$$\times\sum_{l=0}^{4}K_l\int_0^{\infty}\exp(-tu)R(u,t)B(2+\alpha,\,1+l+\alpha-u)du\}, \quad (12)$$

with $R(u,t)$ defined as

$$R(u,t) = 1-\frac{\beta_1}{\beta_0^2}u(1-C-\ln t-\ln u).$$

In (12), $B(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the Beta function, $C\simeq 0.577216$ is the Euler-Mascheroni constant.

As it was demonstrated in [9] the integration in (1) in the framework of the running coupling constant method using the inverse Laplace transforms allows us to obtain the Borel transforms $B\left[Q^2F_M\right](u)$ and the resumed expressions for $Q^2F_M\left(Q^2\right)$.

The inverse Borel transformation (12) have the infinite number of infrared renormalon poles at the points $u=N+\alpha$ in the Borel plane. Indeed, this is evident from the following formula for $B(\alpha,\beta)$

$$B(\alpha,\beta) = \frac{\alpha+\beta}{\alpha\beta}\prod_{k=1}^{\infty}\frac{k(k+\alpha+\beta)}{(k+\alpha)(k+\beta)}, \quad (13)$$

and $N=1;\ k+1, k+l+1$.

After regularization of these ir renormalon poles in accordance with the principal value prescription (see Refs. 20 and 22) Eq. (12) became the resumed form factor $[Q^2F_M\left(Q^2\right)]^{res}$.

It is instructive to compare our recent result with its obtained in the context of the same method, but using the "ordinary" DA ($\alpha\equiv 0$, in (5)) [9,10],

1) we have the infinite number of ir renormalon poles, instead of finite one,

2) each infrared renormalon pole in (12) is shifted to a value $\alpha$.

It is known [9, 10] that an ir renormalon pole at $u=u_0$ corresponds to a power-suppressed contribution ~ $\left(\Lambda^2/Q^2\right)^{u_0}$ to the form factor. Even if the pole is located at $u=u_0+\alpha$, its contribution is of order $\left(\Lambda^2/Q^2\right)^{u_0}/e$. Therefore, our formula (12) take into account the power-suppressed corrections $C_p\left(Q^2\right)\left(\Lambda^2/Q^2\right)^p$, $p=1,2,3,...$ to the meson e.m. form factor $Q^2F_M\left(Q^2\right)$, the coefficients $C_p\left(Q^2\right)$ of which depend on the meson DA under consideration. It is worth noting that the principal value prescription itself produces the higher twist ir renormalon ambiguities $\delta C_p\left(Q^2\right)\left(\Lambda^2/Q^2\right)^p$ which has to be canceled exactly by uv-renormalon ambiguities of higher twist corrections to $Q^2F_M\left(Q^2\right)$. In our work we neglect these effects and do not estimate $\delta C_p\left(Q^2\right)$.

5. In this section we compare our result for the kaon electromagnetic ff $F_K\left(Q^2\right)$ obtained in the context of the running coupling constant method using the ir renormalon improved (9) and the ordinary DAs with each other, as well as with $F_K\left(Q^2\right)$ found by means of the frozen coupling approximation. It is worth noting that in this approximation $F_M\left(Q^2\right)$ with new DAs (9) can be easily calculated.

Equation (12) together with (13) is our final expression, which can be used for computation of power-suppressed corrections to $F_M\left(Q^2\right)$. In numerical calculations we have used $N=120$ ir renormalon poles in (13); this is enough for correct estimation of integrals in (12). Our results for the kaon electromagnetic form factor $F_K\left(Q^2\right)$ are depicted in Figs. 1 and 2. It is interesting to compare ffs found by means of the ordinary and ir renormalon improved DA. The same DAs in the framework of the frozen coupling approximation lead to predictions shown also in Fig.1. In this approximation the ir renormalon effects reduce the perturbative QCD contribution to $F_K\left(Q^2\right)$. The kaon electromagnetic ff $F_K\left(Q^2\right)$ found by means of different model DAs are plotted in Fig.2.

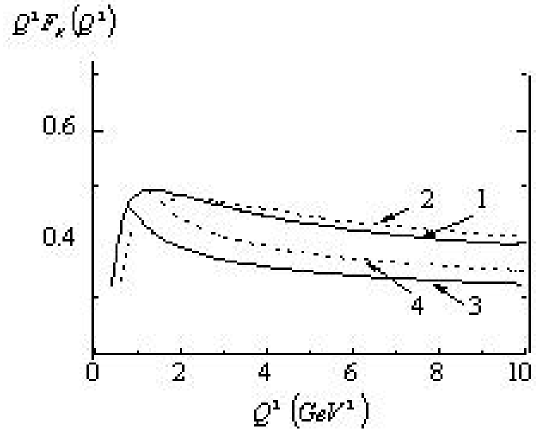


*Fig. 1.* The kaon electromagnetic form factor $F_K\left(Q^2\right)$ as a function of $Q^2$. All curves are found using the asymptotic DA. Curves 1, 2 correspond to ff obtained using the running coupling constant method; curve (1) – by means of infrared renormalon improved DA, curve (2) – using the ordinary DA. Curves 3 and 4 describe ff calculated in the framework of the frozen coupling approximation, with (curve 3) and without (curve 4) ir renormalon corrections.

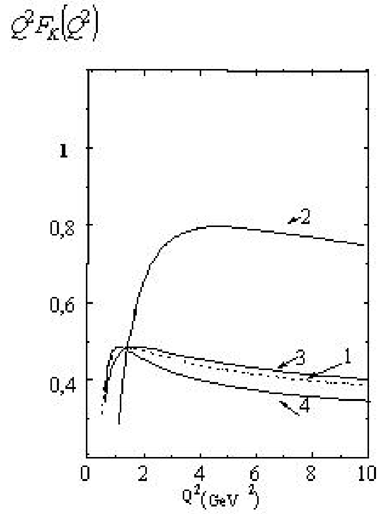


*Fig.2.* The kaon em formfactor $F_K\left(Q^2\right)$, computed in the context of the running coupling constant method and ir renormalon improved DAs vs $Q^2$. Curve 1 is found using the asymptotic DA, curve 2- by means of the Chernyak-Zhitnitsky DA, curve 3 corresponds to DA with parameters $b_0 = 1$, $b_2 = 0.1$, curve 4 – to DA with parameters $b_0 = 1$, $b_2 = -0.2$.

## Appendix

The coefficients $K_n(\alpha_S)$ of DA in (9) are given as

$$K_0(\alpha_S) = A_0(\alpha_S) - b_1 A_1(\alpha_S)(3+2\alpha) + b_2 A_2(\alpha_S)(2+\alpha)(3+2\alpha) - \\ -\frac{1}{3} b_3 A_3(\alpha_S)(2+\alpha)(3+2\alpha)(5+2\alpha) + \\ +\frac{1}{8} b_4 A_4(\alpha_S)(3+2\alpha)(5+2\alpha)\left[1+\frac{1}{3}(3+2\alpha)(7+2\alpha)\right],$$

$$K_1(\alpha_S) = 2b_1 A_1(\alpha_S)(3+2\alpha) - 2b_2 A_2(\alpha_S)(3+2\alpha)(5+2\alpha) + \\ +2b_3 A_3(\alpha_S)(3+\alpha)(3+2\alpha)(5+2\alpha) - \\ -\frac{2}{3} b_4 A_4(\alpha_S)(3+\alpha)(3+2\alpha)(5+2\alpha)(7+2\alpha),$$

$$K_2(\alpha_S) = 2b_2 A_2(\alpha_S)(3+2\alpha)(5+2\alpha) - 2b_3 A_3(\alpha_S)(3+2\alpha)(5+2\alpha)(7+2\alpha) + \\ +2b_4 A_4(\alpha_S)(4+\alpha)(3+2\alpha)(5+2\alpha)(7+2\alpha),$$

$$K_3(\alpha_S) = \frac{4}{3} b_3 A_3(\alpha_S)(3+2\alpha)(5+2\alpha)(7+2\alpha) - \\ -\frac{4}{3} b_4 A_4(\alpha_S)(3+2\alpha)(5+2\alpha)(7+2\alpha)(9+2\alpha),$$

$$K_4(\alpha_S) = \frac{2}{3} b_4 A_4(\alpha_S)(3+2\alpha)(5+2\alpha)(7+2\alpha)(9+2\alpha).$$

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**Y.V. Məmmədova**

## “ZƏİF QEYRİ-ABELİZASİYA” YAXINLAŞMASINDA MEZONLARIN PAYLANMA FUNKSİYALARI VƏ $F_K(Q^2)$ -Ə ÜSTLÜ DÜZƏLİŞLƏR

İşdə kaonun $F_K(Q^2)$ elektromagnit formfaktoruna dəyişən qarşılıqlı təsir sabiti üsulu çərçivəsində üstlü düzəlişlər qiymətləndirilmiş, hesablamalar zamanı “zəif qeyri-abelizasiya” yaxınlaşmasında tapılmış paylanma funksiyasından istifadə olunmuşdur. Alınmış nəticələr “adi” paylanma funksiyası və dəyişən qarşılıqlı təsir sabiti, eləcə də fiksə olunmuş qarşılıqlı təsir sabiti üsulunun və digər model paylanma funksiyalarının köməyi ilə tapılmış nəticələrlə müqayisə olunmuşdur.

**Е.В. Мамедова**

## ФУНКЦИИ РАСПРЕДЕЛЕНИЯ МЕЗОНОВ В ПРИБЛИЖЕНИИ «НАИВНОЙ НЕАБЕЛИЗАЦИИ» И СТЕПЕННО-ПОДАВЛЕННЫЕ ПОПРАВКИ К $F_K(Q^2)$

В работе оценены степенно-подавленные поправки к электромагнитному форм фактору $F_K(Q^2)$ каона, полученного с помощью метода бегущей постоянной взаимодействия. В вычислениях использована функция распределения мезонов в приближении «наивной неабелизации». Полученные результаты сравниваются как с результатами, полученными с помощью обычных ФР и метода бегущей постоянной взаимодействия, так и с использованием приближения фиксированной постоянной и различных модельных функций распределения.

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