ELECTRON VISCOSITY OF LIQUID ALKALI METALS

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The specific character of molten metals has been analyzed and it is shown that electronic gas gives the essential contribution in the general viscosity of molten metal "because of" Thomas-Fermi screening. The bond between viscosity and such metal characteristics as conductivity, electron concentrations, Fermi energy and electron effective mass have been established.

The liquid viscosities are investigated in many works. However, none of them has universal character even qualitatively; it is traditionally considered that molten metals are the simplest liquids (they are so called) and their viscosity can be described by traditional method which is classical one. The presence of two interactions which are ion and electron of subsystems in molten metals isn't practically considered. The present paper is dedicated to investigation of molten metal viscosities taking into consideration the possible contribution of electron gas. The supposed approach is more thoroughly analyzed in the example of molten metals. The X-ray investigations show that coordination number in first coordination sphere doesn't practically change only on (2-3%) and is equal to the number of closest neighbors [1]. Besides, it is established that the essential change of structure and bond don't take place at metal melting point (entropy at melting point increases at less than two units) (see table 1). The above mentioned fact gives us the foundation to present the liquid metal as complex system consisting from two interacting subsystems: ion and electron ones. It is obvious that the subsystem of free electrons practically doesn't change at melting. By this reason the theory of free electrons is useful for description of crystalline and liquid states [2]. All physical properties in molten metals will take place because of change in ion and electron subsystems or the interaction of these subsystems. The above mentioned is the specific feature of molten metals, i.e. the subsystem is absent in all other liquids (table 1).

Table 1.

l is the average electron free length in liquid phase; d is average interatomic distance at T_{melt}

Metals	Structure [2]	$\left(\frac{\Delta V}{V}\right)\cdot 10^3$ [1]	Entropy changes: At melting: *cal/mol·K* [1]	Coordination Number, [6]	Bond Energy eV/atom, [4]	$\frac{m_e^*}{m_{oe}}$, [5]	$\frac{l}{d}$ [2]
Li	VCC	16,5	1,59	9,5-9,8	1,58	1,19	164
Na	VCC	25,0	1,68	7-8	1,13	1,0	158
K	VCC	25,5	1,65	9-10	0,95	0,99	177
Rb	VCC	25,0	1,79	9,5	0,89	0,97	118
Cs	VCC	26,0	1,69	8-9	0,81	0,98	85

The ionic interactions in molten metals are totally collective ones and are carried out through combined interactions of all ions with free electron subsystem. The free electrons further totally shield the Coulomb repulsion between ions and simultaneously provide effective ion attraction on more close distances. The repulsion exchange effects of ion electron membranes play the important role on short distances. The effective coupled attraction strongly depending on density and ionic distance appears on big distances.

The damped oscillation potential behavior of coupled interaction connected with screening properties by electrons of direct ion interactions appears for metals on very far distances.

In theory of free electrons it is supposed that potential energy and electron density are constant ones. However, as it is followed from above mentioned, this can't take place in crystalline and also molten metals. Indeed, according to Thomas-Fermi theory the electron density and impulse in metal depends on coordinates [3]:

$$n(r) = \frac{8\pi}{3h^3}\left[P_F(r)\right]^3 \tag{1}$$

where P_F is Fermi impulse, r is electron radius-vector.

The changes of electron densities appear because of the action of space-heterogeneous potential on electrons. The presence of positive ion frames causes the charge redistribution in electron gas, i.e. electrons attract to them and the local electron cloud forms around positive ion. According [3] the field potential surrounding the positive ion frame has the form of modified Coulomb potential exponentially decreasing with distance. Physically this means that free electrons shield the interactrions between positive ions. The electron distribution around ion frames can be in following expression:

$$n(r) = n_o\left(1 + \frac{3qe}{2E_F \cdot r}e^{-\frac{r}{\lambda_{F-T}}}\right) \tag{2}$$

where E_F is Fermi energy at OK, q is charge of ion frame, n_o is electron number in volume unit in metal part undisturbed by positive ion. As it is seen from (2) the influence of ion frame is insignificant on distances essentially exceeding λ_{T-F}, λ is measure of screening effectiveness of ion frame by free electrons and is called Thomas-Fermi screening radius. This value is defined as [4]:

$$\lambda^2_{T-F} = \frac{E_F}{G\pi e^2 n_o} \tag{3}$$

Fermi energy $E_F = \frac{\hbar}{m^*}\left(3\pi^2 n_o\right)^{2/3}$ in the case of Fermi spherical surface as it takes place in the alkali metal, then after simple transformations we have from (3):

$$\lambda_{T-F} = \left(\frac{\pi}{4m^*}\right)^{1/2} \cdot \left(\frac{\hbar}{e}\right)\left(\frac{1}{3\pi n_o}\right)^{1/3} \sim \frac{1}{\left(m^*\right)^{1/2}} \cdot \frac{1}{n_o^{2/3}} \tag{4}$$

where m^* is electron effective mass in metal. If we neglect the weak temperature dependence m^* and take under consideration that $n_0=const$ in metals, then from (4) we can conclude on temperature independence of Thomas-Fermi screening radius. λ_{T-F} will also different because of difference of Fermi energy and electron concentration in different metals. For alkali metals the values of screening radiuses calculated on (4) are given in the table. As it is followed from the table the screening radiuses for alkali metals is almost the same by the order ~$1\mathring{A}$. The insignificant increase λ_{T-F} at transition from Li to Cs is connected with decrease of effective mass and electron concentration (see table 2).

Table 2.

Metals	E_F, eV [2]	$n_e \cdot 10^{-22}$ [4]	$\rho_{liq.} \cdot 10^6$ *Om·cm* [1]	$\eta_0 \cdot 10^3$ *Pa·s* [6]	$\eta_0 \cdot 10^4$ *Pa·s*	$\lambda_{T-F} \cdot 10^9$, *cm*
Li	4,72	4,70	24,17	1,98	2,95	6,23
Na	3,23	2,65	9,6	4,79	3,96	6,68
K	2,12	1,40	13	6,33	3,71	7,43
Rb	1,85	1,15	22,5	6,60	1,87	7,68
Cs	1,58	0,91	37	8,54	0,97	7,99

The above mentioned spherical structure of alkali metals assumes the participation of both ion and electron subsystems in appearance of physical phenomenon including dynamic viscosity. As the given subsystems are interconnected then it is obvious that it is impossible to evaluate the contribution of each system separately. However, viscosity electron share η_e can be evaluated on the base of gas – kinetic theory in first approximation neglecting by the interaction between conductivity electrons and ion frames in molten metals. Then dynamic electron viscosity will be written as follows:

$$\eta_e \approx \frac{1}{3}\rho_e \overline{\vartheta_e} \cdot \overline{l_e} = \frac{1}{3} m_e n_e \overline{\vartheta_e} \cdot \overline{l_e} \tag{5}$$

where m_e is electron mass; n_e are concentrations of almost free electrons; $\overline{\vartheta_e}$ is average electron thermal rate; $\overline{l_e}$ is average length of free electron in molten metal.

As it is known the average squared rate on Fermi surface (ϑ_F) [5] in model of free electrons with Fermi spherical surface. Then we have from [5]:

$$\eta_e \approx \frac{1}{5} m_e^* n_e \vartheta_F^2 \left(\frac{\overline{l_e}}{\vartheta_F}\right) = \frac{1}{5} m_e^* n_e \vartheta_F \tau \tag{6}$$

In (6) $\tau = \overline{l_e}/\vartheta_F$ is electron relaxation time. As $\tau = m_e\sigma/n_e e^2$ (τ is specific conductivity), then from (6) it is followed:

$$\eta_e \approx \frac{2}{5e^2}\left(\frac{m_e\vartheta_F^2}{2}\right)\sigma m_e^* = \frac{2}{5e^2} m_e^* E_F \sigma \tag{7}$$

$E_F = \frac{\hbar^2}{2m_e^*}(3\pi^2 n_e)^{2/3}$. from elastic side. Then for electron viscosity from (7) we obtain:

$$\eta_e \approx \frac{1}{5}\left(\frac{\hbar}{e}\right)^2 (3\pi^2 n_e)^{2/3} \cdot \sigma \tag{8}$$

According (8) the electron viscosity in metals (including molten metals) is defined by electron concentration and electric conductivity. As the concentration of free electrons is constant value and doesn't change with temperature, then insignificant decrease of electric conductivity at temperature increase can lead to corresponding decrease of electron viscosity. However, metal electric conductivity practically doesn't depend on temperature, excluding low-temperature region $T \leq 30K$. That's why η_e is also constant value for the given metal in the given temperature region.

Taking under consideration that in SGS system $\sigma = 9 \cdot 10^{11} \rho^{-1}$ from (8) we obtain:

$$\eta_e \approx 34{,}43 \cdot 10^{11} \left(\frac{\hbar}{e}\right)^2 \cdot \frac{n^{2/3}}{\rho} \tag{9}$$

We can write through Fermi energy:

$$\eta_e \approx \frac{18}{5}\left(\frac{m_e^*}{e^2}\right) \cdot 10^{11} \rho \frac{E_F}{\rho} \approx 1{,}422 \cdot 10^3 \frac{E_F}{\rho} \tag{10}$$

where in (9) and (10) ρ - is specific resistance in molten metals, E_F is Fermi energy in erg. The values η_e calculated on (10) are given in table (2).

As it is seen from the table (2) the electron viscosity is the essential part of general viscosity of molten metal η_e and decreases at transition from Li to Cs.

The electron viscosity can be expressed through screening length. It is necessary to take under consideration in (9) as $\eta_{F-T} \approx \frac{a_o}{4n_e^{1/3}}$ ($a_o = 0{,}529 \cdot 10^{-10} m$ – is radius of first Bohr orbit). Then after simple transformations from (9) we obtain:

$$\eta_e \approx 6{,}17 \cdot 10^{-15} \left(\frac{n_e}{\rho}\right) \cdot \lambda_{T-F}^2 \tag{11}$$

where n_e is electron number in 1 cm^2; ρ - is specific resistance in molten metals. The electron viscosity calculated on (11) well agrees with its solution obtained from (10).

Thus, the specifics of molten metals is analyzed in given paper and it is shown that electron gas in Thomas-Fermi screening gives the essential contribution in general viscosity of molten metal. The carried out analysis is considered in detail in application to molten alkali metals. In the paper the connection between electron viscosity and such metal characteristics as specific conductivity, electron concentration, Fermi energy and electron effective mass is also established.

[1] *A. Ubbelode*. Plavlenie i kristallicheskaya struktura. M . 1969. (in Russian)

[2] *J. Zaymam*. Fizika metallov, M. 1972. (in Russian)

[3] *V. Geyne, M. Koen, D. Ueyn*. Teoriya psevdopotentsiala. M. 1975. (in Russian)

[4] *Ch. Kitel*. Statisticheskaya termodinamika, M. 1978. (in Russian)

[5] *S.V. Vonsovskiy, M.I. Katsnelson*. Kvantovaya fizika tverdikh tel, M., 1983. (in Russian)

[6] *M.E. Frits*. Svoystva elektronov. M., 1985. (in Russian)

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QƏLƏVİ METAL MAYELƏRİNİN ELEKTRON ÖZLÜLÜYÜ

İşdə bir-biri ilə qarşılıqlı əlaqədə olan ion və elektron altsistemlərinin varlığını nəzərə almaqla qələvi metal mayelərinin özlülüyü araşdırılır. Xüsusi halda göstərilmişdir ki, elektron altsisteminin özlülüyə əlavəsi temperaturdan asılı deyil və bütün hallarda o, nəzərə alınmalıdır.

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ЭЛЕКТРОННАЯ ВЯЗКОСТЬ ЖИДКИХ ЦЕЛОЧНЫХ МЕТАЛЛОВ

Анализирована специфика жидких металлов и показано, что "из-за" Томас - Фермий экранировки электронный газ дает значительный вклад в общую вязкость жидкости металла. Установлена связь между электронной вязкостью и такими характеристиками металла, как удельная электропроводность, концентрация электронов, энергия Ферми и эффективная масса электронов.

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