

621. 311

## OBTAINING OPTIMAL SLIDING SURFACE SLOPE BY USING GENETIC ALGORITHMS AT SLIDING MODE CONTROLLER APPLIED TO INDUCTION MOTOR

SENOL I., DEMIRTAS M., RUSTEMOV S.

Dept. of Electrical Eng.  
Yıldız Technical University  
Istanbul, Turkey  
e-posta: [senol@yildiz.edu.tr](mailto:senol@yildiz.edu.tr)

Dept. of Electrical Eng.  
Dicle University  
Diyarbakır, Turkey  
e-posta: [mdtas@dicle.edu.tr](mailto:mdtas@dicle.edu.tr)

Dept. of Electrical Eng.  
Dicle University  
Diyarbakır, Turkey  
e-posta: [rustemov@dicle.edu.tr](mailto:rustemov@dicle.edu.tr)

**Abstract** -The application of sliding mode control for improving the dynamic response of the induction motor (IM) based position servo system is presented. It provides attractive features such as fast response, good transient performance, insensitivity to variations in plant parameters and external disturbance. This paper presents a GA-based sliding mode controller for a vector controlled IM servo system. Optimal sliding surface slope is obtained by using genetic algorithms. The performance of the controller is presented graphically. MATLAB program is used to achieve simulations.

### 1. INTRODUCTION

By considering an induction motor servo drivers, the drive inertia and load characteristics change dramatically. Following features are essential that are; a drive system that can provide fast dynamic response, a parameter insensitive control feature, and rapid recovery from speed drop caused by impact loads [1].

Recently, interest in the study of Sliding Mode Control (SMC) applied to AC drive systems has grown. The reason for much attention is that SMC is capable of providing many good properties such as robustness against system uncertainties, disturbances and fast dynamic response [2-4].

The sliding mode control law is inherently discontinuous naturally. When the control is directly applied to check the developed torque with the inner loop of a vector-controlled drive, the system results in torque pulsation, current ripple and the corresponding speed ripple and acoustic noise [1].

Switching between two discrete control structures is the most definite property of a variable structure system. The design of variable structure controller generally consists of two steps, which are hitting and sliding phases. First, the system is directed towards a switching surface by a feedback control law, and then sliding mode occurs. When the system states enter the sliding mode, the dynamic of the system is determined by the choice of sliding surface. The mentioned situations are independent of parametric uncertainties and external disturbances. Hence, SMC has been employed to the position and speed control of AC drive systems.

Variable structure systems (VSS) such as motor control, robotic manipulators and indefinite systems have an important place in the control of modern non-linear system. However, there are some difficulties such as occurring so many switches between the control bounds, which cannot be realized by real controllers. To overcome this problem, a thin boundary layer neighbouring the switching surface is introduced for smoothing out the control discontinuity. The boundary layer can be achieved by a saturation function. However, introducing boundary layer, steady-state errors may occur.

## 2. MATHEMATICAL MODEL OF VECTOR CONTROLLED INDUCTION MOTOR

For a vector controlled induction motor, the following mechanical equation is applied [3].

$$T_e - T_l = \frac{2}{p} J \frac{dw_r}{dt} + Bw_r \quad (1)$$

For ideal decoupling control, the electric-magnetic torque equation becomes

$$T_e = \frac{3}{2} \frac{p}{2} \frac{L_m}{L_r} i_{qs} \Psi_r \quad (2)$$

If we define  $K_t$  as

$$K_t = \frac{3}{2} \frac{p}{2} \frac{L_m}{L_r} \Psi_r \quad (3)$$

Then, the torque equations can be written as follows:

$$T_e = K_t i_{qs} \quad (4)$$

Where,

$i_{qs}$  : Stator instantaneous current in q-axis ;  $L_m$  : Mutual inductance,  
 $L_r$  : Rotor inductance referred to the stator ;  $w_r$  : Rotor angular velocity,  
 $p$  : Number of pole pairs ;  $J$  : Mass torque of inertia,  
 $B$  : Viscous friction constant ;  $T_e$  : Electromagnetic torque,  
 $T_l$  : Load torque.

## 3. SLIDING MODE CONTROL

VSS control is developed that all trajectories in the state space are directed toward some switching surface, it slides along switching surface, i.e. repeatedly crosses and immediately recrosses the surface. The system response depends there after only on the gradients of the switching surfaces and remains insensitive to bounded disturbance and parameter variations [1]. In case of 2nd order system, the control has the form

$$u = \begin{cases} u^+(x_1, x_2, t) & \text{for } S(x, t) > 0 \\ u^-(x_1, x_2, t) & \text{for } S(x, t) < 0 \end{cases} \quad (5)$$

where S is switching hyper plane which satisfy

$$S = Cx_1 + x_2 = 0 \quad (6)$$

The conditions of the occurrence of the sliding motion on the hyper plane may be written in numerous ways, i.e.

$$\lim_{s \rightarrow 0^+} \dot{S} < 0 \quad \text{and} \quad \lim_{s \rightarrow 0^-} \dot{S} < 0 \quad (7)$$

or equivalently

$$\lim_{s \rightarrow 0} S \dot{S} < 0 \quad (8)$$

## 4. IM POSITION CONTROL BY BOUNDED SLIDING MODE CONTROLLER

Fig.1 shows the block diagram of the ideal vector controlled servo system with Bounded Sliding Mode (BSM) controller. The actual position  $\theta_r$  is compared to the reference position  $\theta_r^*$  to produce an error signal ( $x_1$ ) that is used with the motor angular velocity ( $x_2$ ) to determine the motor control action. The 2nd order state equation of Fig.1 can be given as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -B/J \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -K_t/J \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{T_l}{J} \quad (9)$$

Where  $K_t$ =torque constant,  $J$ =shaft moment of inertia and  $B$ =motor damping constant.

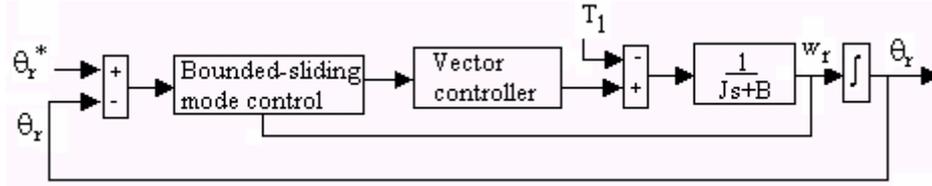


Fig.1. Induction motor position control system.

The stability is analysed using Lyapunov theorem and the results become the design tool for the proposed control law [1,3].

Let  $V$  be semi positive definite function as

$$V = (Cx_1 + x_2)^2 = S^2 \quad (10)$$

The derivative of  $V$  becomes:

$$\dot{V} = 2S \dot{S} < 0$$

The Eq. (10) is a Lyapunov function along a line perpendicularly intersecting to the stable switching plane by Eq.(2). Consequently, the BSM controller makes the closed loop system stable. If  $\dot{V}$  is negative, the system is stable.

That is, the following inequality equations are satisfied,

$$\alpha x_2 - \frac{T_l}{K_t} < u(t) \text{ if } x_2 > -Cx_1 \text{ (Region I) (11); } \alpha x_2 - \frac{T_l}{K_t} > u(t) \text{ if } x_2 < -Cx_1 \text{ (Region II) (12)}$$

Where  $\alpha = (B - JC) / K_t$ . (13).

Recalling the mathematical model of the induction motor represented by Eq.(1), we need two variables to apply SMC to the induction motor. The state variables  $x_1$  and  $x_2$  are the position error  $\theta_r^* - \theta_r$  and the angular velocity  $\omega_r$ , respectively. Where,  $\theta_r^*$  is the desired angular position.  $\theta_r$  can be obtained as

$$\theta_r = \int w_r dt \quad (14)$$

The mathematical forms of state variable are following:

$$x_1 = \theta_r^* - \theta_r \quad (15)$$

$$x_2 = -w_r \quad (16)$$

If we rewrite Eq.(15),(16), we obtain

$$\dot{x}_1 = -\dot{\theta}_r = -w_r = x_2 \quad (17)$$

$$\dot{x}_2 = -\frac{dw_r}{dt} = (-bT_e + bT_l + \alpha x_2)$$

$$a = \frac{pB}{2J}, b = \frac{p}{2J} \quad (18)$$

Then, the state equations can be represented in matrix form as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -K_t b \end{bmatrix} i_{qs} + \begin{bmatrix} 0 \\ b \end{bmatrix} T_l \quad (19)$$

The chosen sliding surface in the phase plane is shown in Fig. 2.

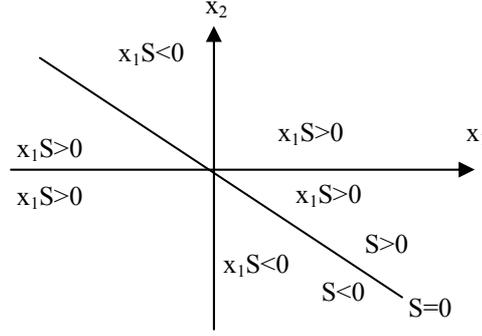


Fig. 2. Chosen sliding surface in the phase plane.

The mathematical form of the sliding surface is defined as

$$S = Cx_1 + x_2 \quad (20)$$

After we define the sliding surface, the switching control law should satisfy the Lyapunov stability criterion, which is resulted in the following two conditions:

$$S \dot{S} < 0 \quad \text{for } \forall S \quad (21)$$

$$x_1 \rightarrow 0, \quad x_2 \rightarrow 0 \quad S(t) = 0 \quad (22)$$

If Eq.(21) satisfies, the system trajectory will tend to the sliding surface and if Eq.(22) satisfies, the system applied SMC is asymptotically stable. Therefore the basic switching control can be selected in the form of:

$$i_{qs} = i_{eq} + d \operatorname{sgn}(S) \quad (23)$$

Where  $i_{eq}$  is the equivalent control to compensate the estimated undesirable dynamics,  $\operatorname{sgn}$  is a sign function and  $d$  is a constant.  $i_{eq}$  can be obtained by Eq.(20) and the derivative of the Eq.(20) equating to zero.

$$\dot{S} = C \dot{x}_1 + \dot{x}_2 = 0 \quad (24)$$

Then, substituting Eq.(18) into Eq.(24) and rearranging the obtained equation, we have

$$i_{eq} = \frac{a+C}{bK_t} x_2 + \frac{T_l}{K_t} \quad (25)$$

In real systems, it is difficult to determine load torque. Hence, Eq.(23) can be rearranged as follows if  $K$  is the maximum load torque.

$$i_{qs} = \alpha_2 x_2 + d \operatorname{sgn}(S) \quad (26)$$

Where,

$$i_{eq} = \alpha_2 x_2 \quad (27)$$

$$\alpha_2 = \frac{a+C}{bK_t} \quad (28)$$

If we substitute Eq.(18) into Eq.(24), we achieve

$$\dot{S} = C \dot{x}_1 + \frac{p}{2J} (Bx_2 + T_l - T_e) = 0 \quad (29)$$

Using the above equation and the sliding condition represented by Eq.(21), We obtain the following condition:

$$\frac{T_l}{K_t} < d \quad (30)$$

If the above condition satisfies, the sliding condition will also meet. Then Eq.(26) can be rearranged with the position errors as follows

$$i_{qs} = \alpha_1 x_1 + \alpha_2 x_2 + d \operatorname{sgn}(S) \quad (31)$$

If the procedure followed for Eq.(26) is applied to Eq.(31), one can illustrate

$$-\frac{p}{2J}SK_t\alpha_1x_1 - \frac{pK_t d|S|}{2J} + \frac{pSK}{2J} < 0 \quad (32)$$

Eq.(32) can be analysed into two parts:

$$-\frac{p}{2J}SK_t\alpha_1x_1 < 0 \quad (33)$$

$$-\frac{pK_t d|S|}{2J} + \frac{pSK}{2J} < 0 \quad (34)$$

It is seen in Fig.2 that the inequality represented by Eq.(33) satisfies. For satisfying the condition represented by Eq.(34), the following inequality should be justified.

$$\frac{K}{K_t} < d \quad (35)$$

However, ideal sliding mode rarely occurs because of switching delay in real systems. That's why, the trajectories chatter along the switching surface due to rapidly changing values of control. To eliminate this defect, we introduce a thin boundary layer around the switching surface to smooth out the control discontinuity. Then, Eq.(31) can be modified as,

$$i_{qs} = \alpha_1x_1 + \alpha_2x_2 + d \text{ sat}(S / \varepsilon) \quad (36)$$

Where,

$$\text{sat}\left(\frac{S}{\varepsilon}\right) = \begin{cases} \text{sgn}(S) & \text{if } |S| > \varepsilon \\ \frac{S}{\varepsilon} & \text{if } |S| \leq \varepsilon \end{cases}$$

And  $\varepsilon$  is the thickness of the boundary layer.

## 5. GENETIC ALGORITHMS

The genetic algorithm is a derivative-free stochastic optimisation method based on the concepts of natural selection and evolutionary procedure. It is a rule of survival of the fittest will win [5]. GA is based on an analogy to the genetic code in our own DNA (deoxyribonucleic acid) structure, where its coded chromosome is composed of many genes [6,8]. The GA approach involves a population of individuals represented by strings of characters or digits. Each string is, however, coded with a search point in the hyper search-space. From the evolutionary theory, only the most suited individuals in the population are likely to survive and generate off-spring that pass their genetic material to the next generation. In general GAs run repeatedly by using three basic operators such as reproduction, crossover and mutation, to find the best parameters in the whole parameter searching space [7].

In the genetic search, a two-way generation gap has been created in which a portion of the population in the new generation is being replaced by the old one during each generation. This helps to keep good genetic materials to be passed on to the next generation, leading to better solutions and thus improving the convergence rate of the genetic algorithms. The scheme is proceeded in two ways:

- (a) Replacing one string, i.e., replacing the best string from the old generation with the worst string in the new generation when the maximum fitness in the old generation is lower than the new generation,
- (b) Otherwise, replacing a group of strings such that a group of good strings in the old generation is replacing the equally sized group of worst strings in the new generation according to their fitness.

GAs generally encode each parameter set (or solution set) into a bit string called a chromosome, each bit of the chromosome is called the gene. We usually denote a set of chromosomes as a population. Each point, in a parameter set is evaluated using fitness function and each point associated with a fitness value used to reflect the degree of goodness

of the chromosome for solving a problem. A fitter chromosome has the tendency to produce good quality offspring, which means a better solution to the problem.

## 6. OBTAINING OF OPTIMAL C BY USING THE GA

The genetic algorithm (GA) is a derivative-free stochastic optimisation method based on the concepts of natural selection and evolutionary procedure. It is a rule of survival of the fittest will win. The paradigm of genetic algorithms appears to offer an effectively searching for a set of controller parameters for achieving better performance. The major components of GAs include encoding parameter, fitness evaluation, parent's selection, crossover operator and mutation operator [6,8].

There are two considerations to be made on the application of genetic algorithms to the optimising problem. Firstly, the choice of fitness function to be used for the measure of performance in the optimisation/searching process and second, the choice of coding to be used to code the design parameters into the chromosome.

A simple fitness function that reflects small steady-state errors, a short rise-time, low oscillation and overshoots with a good stability is given by:

$$f = \exp\left(-\frac{A}{T} \sum_{i=0}^T \{x_i^2 + \Delta x_i^2\} * i\right) \quad (37)$$

Where  $T$  is the duration of the simulation in evaluating  $C$ ;  $A$  is a positive number used to scale the maximum fitness up or down;  $i$  is the time index in simulation;  $x_i$  is the error between the measured signal and the desired signal at simulation step  $i$ ; and  $\Delta x_i$  is the change of error at simulation step  $i$ .

In this paper, the probability of crossover operator is  $P_c=0.09$  for both binary-type and real-type coding, the probability of mutation operator is  $P_m=0.08$ , the population size is  $P_s=40$ .

## 7. SIMULATION STUDY

In order to verify the control strategy as discussed above, the simulation study of the total system was completed using the MATLAB. The simulink model of closed-loop system with BSM controller is shown in Fig.3.

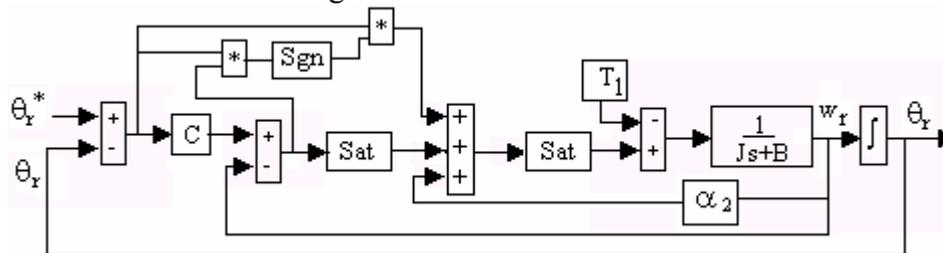


Fig.3. The simulink model of the closed-loop system with BSM controller.

The performance of the system is then compared with that of conventional sliding mode control and bounded sliding mode control. Fig. 4 shows the step response of the system with conventional sliding mode control. Fig.4 (a) represents the phase plane ( $x_1$ - $x_2$ ) trajectory. Fig.4 (b) and (c) give the corresponding time domain response of speed and position loop error, respectively. Fig.4 (d) shows also the corresponding torque component of current. With negative position error command, the system initially accelerates freely in the negative direction. Then, as the phase plane trajectory tends to cross the sliding line, BSM controller increase torque current quickly. The positive torque current decelerates the drive until it comes to a rest. Evidently, the chattering appears to have vanished altogether.

If the trajectory is on the switching surface, there is no need to apply the control torque to the system. However, if the trajectory leaves from the switching surface with a small angle, the trajectory will need a small control torque to return the switching surface. Table shows induction motor parameters used to simulation.

Table. Parameters of induction motor.

Mot. Pow	3 (hp)	$L_m$	0.08316 (H)
Mot. Vol.	220 (V)	$R_r$	0.842 (ohm)
Pole Num.	4	$L_r$	0.08525 (H)
$R_s$	0.687(ohm)	$J$	0.03 (kg-m <sup>2</sup> )
$L_s$	0.08397 (H)	$B$	0.001(Nm/sec)

## 8. DISCUSSION

Sliding surface slope effecting dynamic response of system is founded to be test at previous studies. This process takes for a long time.

Fig. 4 shows step response of induction motor servo system with conventional sliding mode controller. It has chattering problem. This problem are eliminated a using fuzzy logic control [1]. However, settle time of system is the same as two second as shown in Fig. 4. It does not become shorter.

Fig. 5 shows step response of induction motor servo system with boundary layer sliding mode controller. This method use saturation function for soft switching. Thus, settle time reduces from 2 second to 1 second as shown in Fig. 5 [5].

In this study, we simulate the system applied SMC with the boundary layer, represented by Eq.(36). Optimal C is obtained by using the GA. This process is made MATLAB. In the model, saturation function is used as a boundary layer. As a result of program, optimal  $C=11.0695$  is obtained. Obtained simulation results are shown in Fig. 6.

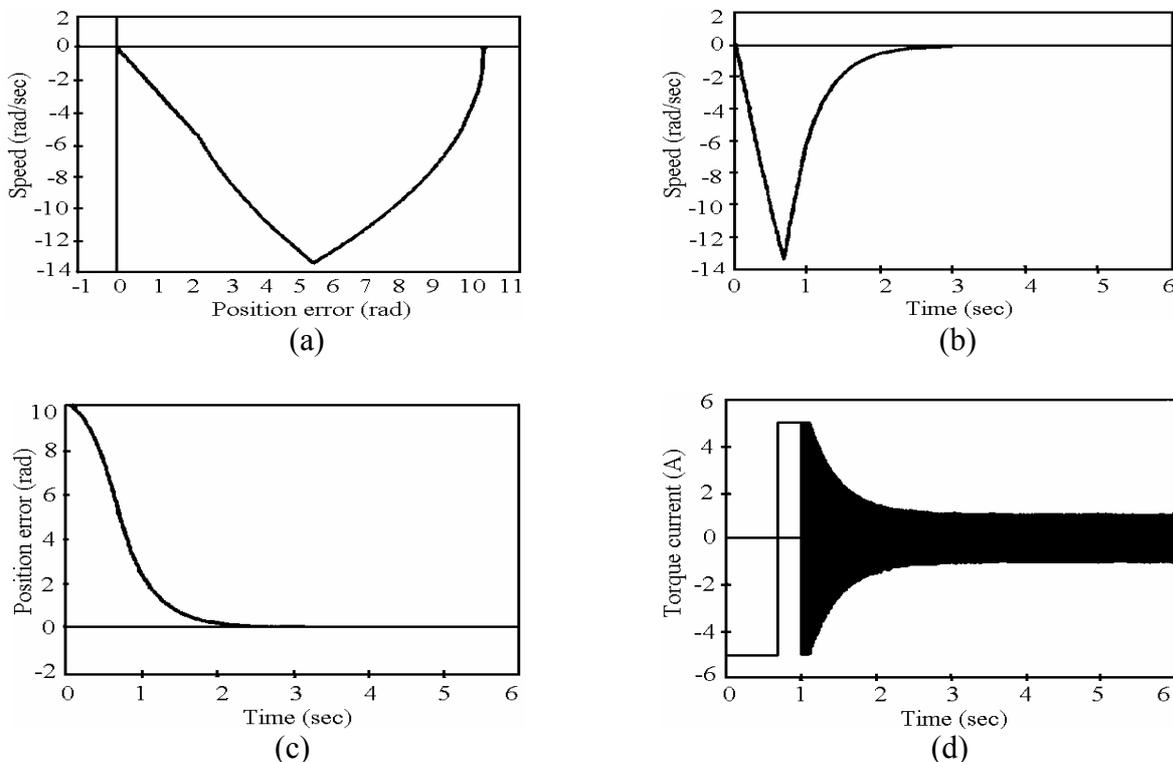


Fig.4. Step response of induction motor servo system with conventional sliding mode controller. (a) trajectory on the phase-plane (b) speed response (c) position error (d) torque current wave.

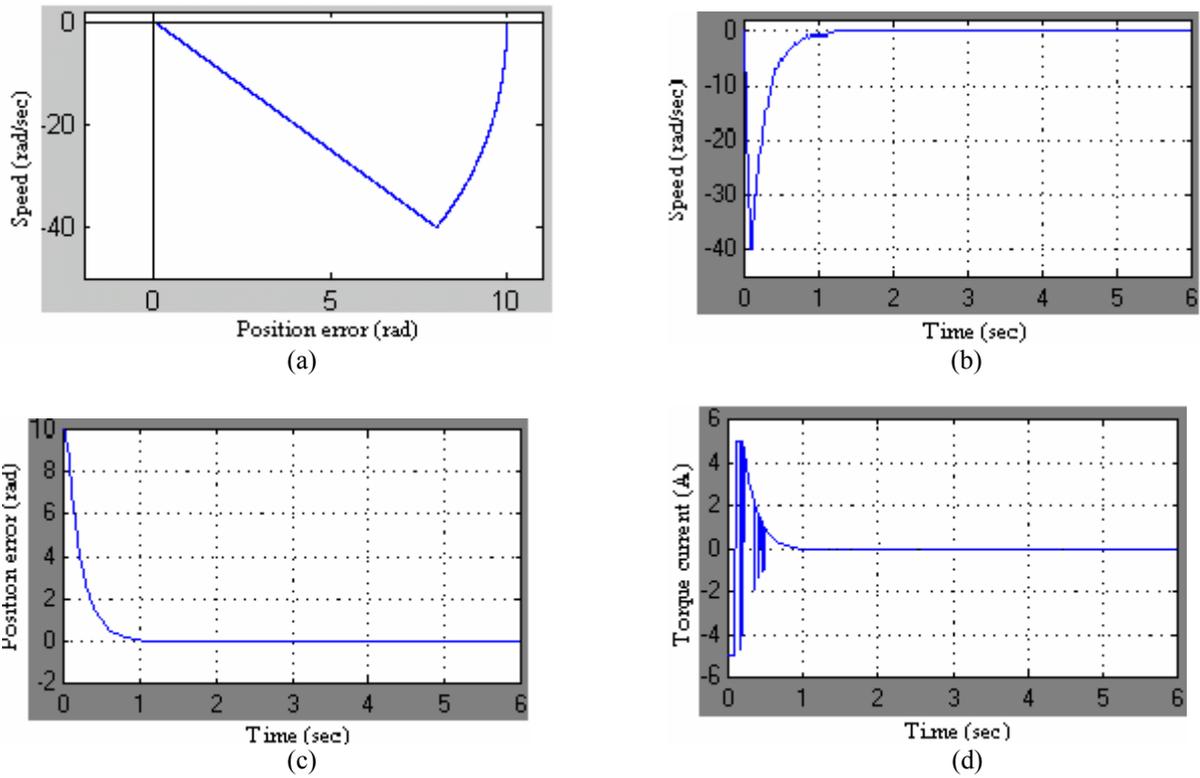


Fig. 5. Step response of induction motor servo system with bounded sliding mode controller. (a) trajectory on the phase-plane (b) speed response (c) position error (d) torque current wave.

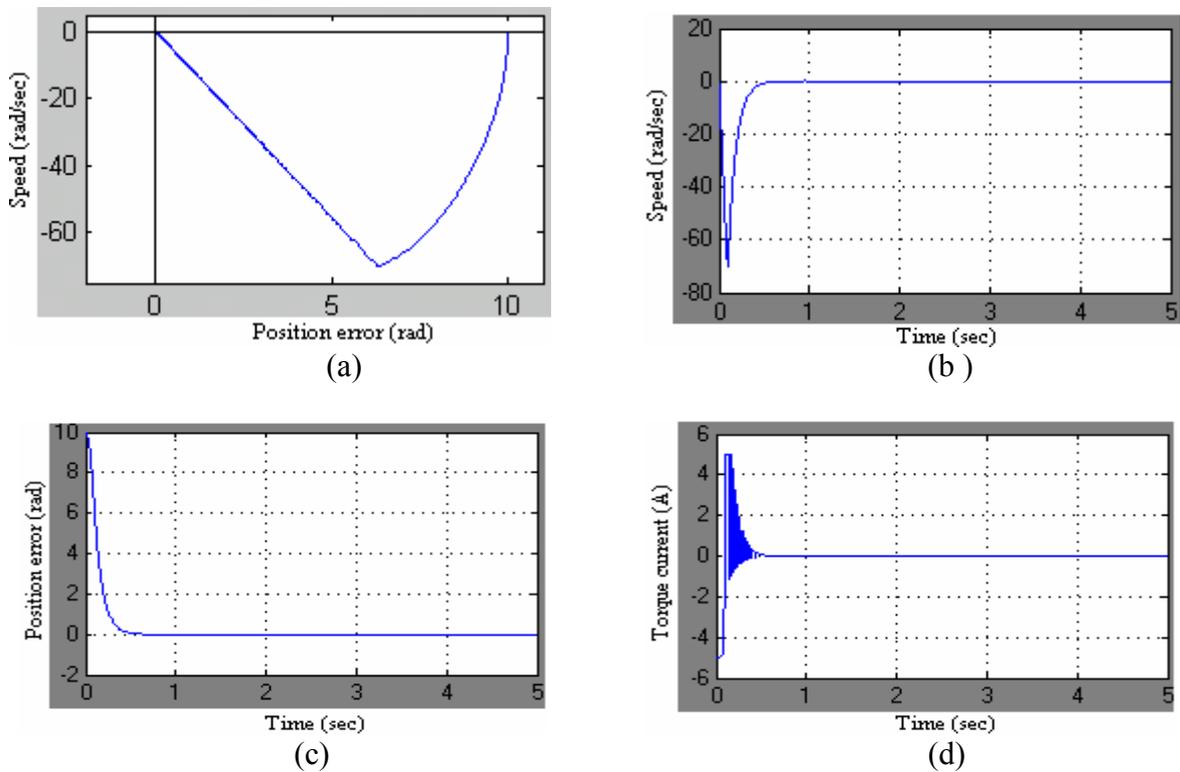


Fig. 6. Step response of induction motor servo system with GA-based sliding mode controller. (a) trajectory on the phase-plane (b) speed response (c) position error (d) torque current wave.

## 9. CONCLUSION

In this study, GA-based boundary layer sliding mode control is proposed and applied to an indirect vector controlled induction motor servo system. Optimal sliding surface slope is obtained by using GA. Thus, the settle time becomes shorter. That is to say settle time reduces from 1 second to 0.5 second as shown in Fig.6. In summary it has been shown that proposed GA-based sliding mode controller give superior performance.

- 
- [1] *C.Y. Won, D.H. Kim and S.C. Kim*, "Position control of induction motor with a new fuzzy-sliding mode controller", IEEE Proc. of APEC'93, 1993.
  - [2] *E.Y.Y. Ho and P.C. Sen*, "Control dynamics of speed drive system using sliding mode controllers with integral compensation.", IEEE Trans. Ind. Appl., vol.27, no.5, pp883-892, 1981.
  - [3] *I. Senol, M. Demirtas, S.A. Rustemov*, "Position Control of Induction Motor with Bounded Sliding Mode Controller", Power Engineering Problems, No:4, pp. 97-104, 2002.
  - [4] *M. Demirtaş*, "Position Control of Vector Controlled Induction Motor with Fuzzy Sliding Mode and Genetic Sliding Mode", Ph.D. Thesis, F. B. E., Yildiz Technical. Ü., 2002.
  - [5] *Chen T.T. and T.-H. S. Li*, "Integrated Fuzzy GA-Based Simplex Sliding Mode Control", International Journal of Fuzzy Systems, Vol. 2, No.4, December, 2000.
  - [6] *D.E. Goldberg*, Genetic Algorithms in Search, Optimization, and Machine Learning. Addison-Wesley, Reading, Ma, 1989.
  - [7] *K.C. NgY. Li, D.J. Murray-Smith and K.C. Sharman*, "Genetic Algorithms Applied to Fuzzy Sliding Mode Controller Design", Proc. First IEE/Ieee Int. Conf. On Genetic Algorithms in Eng. Syst, Innovations and Appl., Sheffied, pp.220-225 Sept., 1995.

### **GENETİK ALORİTMİDƏN İSTİFADƏ EDƏRƏK İNDUKSION ELEKTRİK MÜHƏRRİKİNİN SÜRÜŞMƏ REJİMİNDƏ NƏZARƏT EDƏN QURĞUNUN SÜRÜŞMƏ SƏTHİNİN OPTİMAL MEYLİNİN TƏYİNİ**

**ŞENOL İ., DƏMİRTAŞ M., RÜSTƏMOV S.**

Məqalədə genetik alqoritm əsasında servosistemin induksion elektrik mühərrikinin sürüşmə rejimini idarə edən nəzarət qurğusu şərh edilmişdir. Sürüşmə səthinin optimal meyli təyin edilmişdir. Modelləşdirmə MATLAB proqramından istifadə edərək yerinə yetirilmişdir

### **ОПРЕДЕЛЕНИЕ ОПТИМАЛЬНОГО НАКЛОНА ПОВЕРХНОСТИ СКОЛЬЖЕНИЯ КОНТРОЛЛЕРА РЕЖИМА СКОЛЬЖЕНИЯ ИНДУКЦИОННОГО ЭЛЕКТРОДВИГАТЕЛЯ С ИСПОЛЬЗОВАНИЕМ ГЕНЕТИЧЕСКИХ АЛГОРИТМОВ**

**ШЕНОЛ И., ДЕМИРТАШ М., РУСТАМОВ С.**

В данной статье описан контроллер для управления режимом скольжения индукционного двигателя сервосистемы на основе генетического алгоритма. Определен оптимальный наклон поверхности скольжения. Моделирование проводилось с использованием программы MATLAB.