# INVESTIGATION OF CORRELATION FUNCTIONS FOR LASER'S HAVING LONGITUDINAL MODE

**ABDULLAYEV R., ŞENTÜRK Ş., ÖZKIRIM İ.,[*] KÜÇÜKBURSA A.**

*Dumlupınar University, Department of Physics, Kütahya-Turkey*
*[*]Hacettepe University, Department of Physics Engineering, Ankara-Turkey*

## Abstract

In this paper, coherence properties of laser with longitudinal mode regime were studied through semi-classical approach, and effect of mode structure to these properties are investigated.

Analytical form of normalized correlation function obtained for longitudinal modes having some amplitude exhibits that coherence degree of laser decreases as the mode number increases. The correlation functions were also estimated for the case of modes propagate uniformly. It was seen that results obtained experimentally are in agreement with the prediction of correlation functions.

## Introduction

One of the most important properties of optical quantum generators is to have coherent radiation. This property is defined via correlation function: that is

$$\Gamma\,(x_1, x_2, \tau) = \langle E(x_1 + t + \tau)\, E^*(x_2, \tau)\rangle\,. \qquad (1)$$

This function is used to explain the coherence when $x_1$ equals to $x_2$ ($x_1 = x_2$) and space coherence when the time equals to zero (t=0). On the other hand, since coherence of radiation leads to interference, the function describes this duration is:

$$v = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}\,. \qquad (2)$$

It can be shown easily that there is a direct relation between function described above and normalized correlation function, that reads:

$$v = |\gamma\,(x, \tau\,)|\,. \qquad (3)$$

Investigation of coherence and radiation statistic properties posses significant importance for the laser's production. In general, theoretical and experimental investigations were performed mostly for the lasers with fixed amplitude. However, such studies for lasers having high power and operates well above the threshold level are limited, and these studies are highly in need for the technical and scientific applications. It is observed that this case results in decrease of degree of longitudinal and transverse modes of lasers appeared [1].

Purpose of this study is to explain effect of longitudinal mode operation on the coherence. Firstly, for the case of the modes produced at equal distance, assuming that amplitude of modes is constant, analytical equation of correlation function was formed, then the correlation function was calculated for the case where amplitude of modes propagate appropriately along the line of the powering and the results obtained were compared.

## Results and Discussion

The number of independent modes appear for lasers having open resonators are determined through number of static waves occupied along the resonator, that is; if the L is the length of a resonator, possible wavelength within the resonator can be found from the following relation:

$$n\frac{\lambda_n}{2} = L \quad . \qquad (4)$$

The frequencies associated to these reads:

$$\omega_n = n\frac{\pi c}{L} \quad . \qquad (5)$$

The distance between the modes is:

$$\omega = \omega_{n+} - \omega_n = \frac{\pi c}{L} \quad . \qquad (6)$$

This equation shows that the modes are at same distance to each other, in other words they posses equ-distant. In the laser, only N mode dropped to the powering counter will remain out of the n mode produced by resonator. In this case, total electric field occurred in the laser will be superposition of electric field of N number mode and that is [2];

$$E(t) = \Sigma E_n\, e^{i[(\omega_0 + n\omega)t + \varphi_n]} , \qquad (7)$$

where $\omega_0$ is the mid-frekans selected, as reference, $E_n$ and $\varphi_n$ are amplitude and phase of $n^{th}$ mode, N is the number of modes produced by laser. One of the properties of this relation is the periodicity according to the $T = \frac{2\pi}{\omega} = \frac{2L}{c}$ .

This can be seen directly by putting the $(t+T) = (t + \frac{2\pi}{\omega})$ into equation (7) for the t, then we obtain the $E(t+T) = E(t)$. In order to explain how the number of mod produced in the laser effects the correlation function, knowing modes are at equ-distant, assuming the amplitudes possess same value, we shall put together complex electric field equation (7) as:

$$E(t) = a\, e^{i(\omega_0 + \frac{N\omega}{2})t} \Sigma e^{(n\omega_t + \varphi_n)} \quad . \qquad (8)$$

The autocorrelation function from this can be obtained as:

$$\Gamma(\tau) = \langle E(t + \tau)\, E^*(t)\rangle = a^2 e^{i(\omega_0 - \frac{N\omega}{2})\tau} \Sigma e^{in\omega\tau} ,$$

$$= a^2 e^{i(\omega_0 - \frac{N\omega}{2})\tau} \frac{e^{i N\omega\tau} - 1}{e^{i\omega\tau} - 1} = a^2 e^{i(\omega_0 - \frac{N\omega\tau}{2})\tau} \frac{\sin\frac{N\omega\tau}{2}}{\sin\frac{\omega\tau}{2}} \quad . \qquad (9)$$

As a final step, analytical relation of normalized correlation function module can be written as:

$$|\gamma(\tau)| = \frac{|\Gamma(\tau)|}{|\Gamma(0)|} = \frac{\sin\frac{N\omega\tau}{2}}{N\sin\frac{\omega\tau}{2}} \quad . \qquad (10)$$

It can be seen from equations (10) and (3) that as the number of modes produced by laser increases, degree of coherence decreases. The changes of s parameter ($s=\frac{\omega\tau}{2\pi}=\frac{\tau}{T}$) with periodic field function, $|\gamma(\tau)|$, is presented in figure 1.

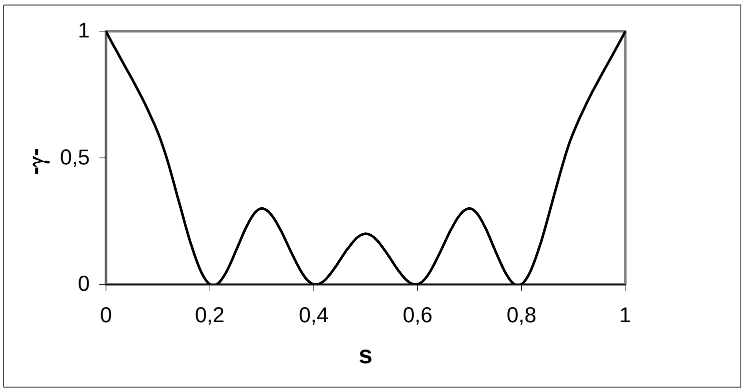


Figure 1. Normalized correlation function for the five equal-intensity modes (N=5)

As can be seen from the graph, the function consists of pulses changes with $T=\frac{2\pi}{\omega}=\frac{2L}{c}$ period and number of them changes depending on the mode number N, amplitude of them are small in between maximums, but the (N-2) number mid-maximums have same value. Characteristic time slope of $|\gamma(\tau)|$ function (coherence time) is smaller N times from the period $\tau_k=\frac{T}{N}=\frac{2\pi}{N\omega}=\frac{2\pi}{\Delta\omega}$, and it is inversely proportional to the line wide of the powering spectrum $\Delta\omega$. Hence, increase of mode numbers also causes decrease of coherence time of laser. This study shows that equal distribution of modes reflects the idealized case. It was observed experimentally that main-maximums enlarges and amplitude of the mid-maximums are different from each of other and becomes smaller for the determined correlation function of laser's having same N number longitudinal mode [3]. In order to explain such changes, in real case amplitude of modes equal distant produced by laser should not be same and assuming that longitudinal modes propagates uniformly to the counter of the laser powering, for the N=3 and for N=5 cases normalised correlation function can be calculated through following formula:

$$|\gamma(s)| = \frac{\sqrt{[\Sigma\langle a_k^2\rangle\cos 2\pi ks]^2+[\Sigma\langle a_k^2\rangle\sin 2\pi ks]^2}}{\Sigma\langle a_k^2\rangle} \quad . \qquad (11)$$

The plots of $|\gamma(s)|$ function for s parameter are given in Figures 2 and 3.

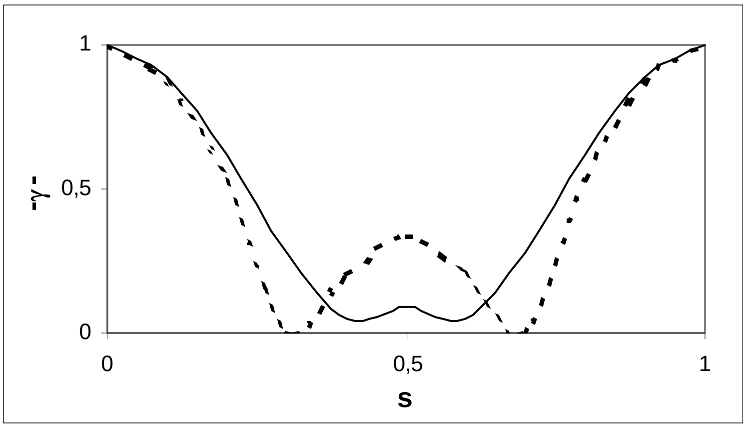


Figure 2. Normalized correlation function for three modes (N=3)
a) the modes are at equal-intensity, b) $I_1 = I_3 = 0{,}6I_2$

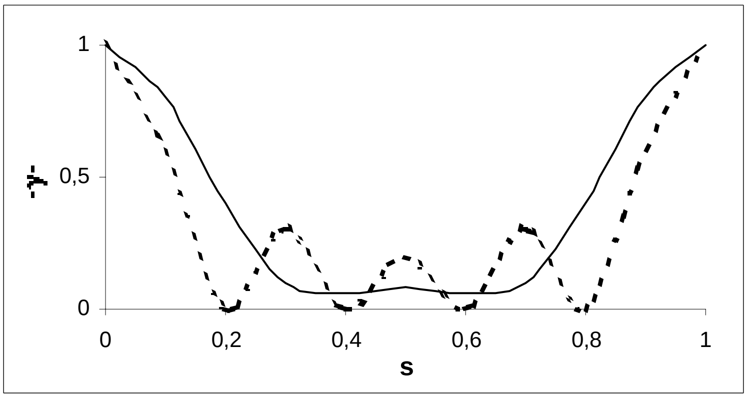


Figure 3. Normalized correlation function for five modes (N=5) laser beam
a) The modes are at equal-intensity, b) $I_1 = I_5 = 0{,}2I_3$, $I_2 = I_4 = 0{,}6I_3$

In the figures dashed lines are for the case of the mode with equal in intensity. As can be seen from the figures that although general structure of the $|\gamma(s)|$ function does not change main maximums enlarges, and amplitude of the mid-maximum are different from each other and becomes smaller for the later one. As a result of this, normalized correlation function and coherence of laser decreases. The increasing mode numbers results in decrease of coherence degree much more. This will generally cause changes of statistical properties of laser radiation [4].

**Conclusion**

One of methods for decreasing the number of longitudinal modes to prevent coherence of laser is to decrease the resonator length that is short use laser. In this case, according to the equation (6), while the L decreases the modes separate from each other and the mode number in the counter of laser decreases. Second method is to operate laser just above the threshold level. In this case, only the mode fulfilled the threshold condition will be produced within the mid-section of the counter. For both methods laser power will decrease. Accordingly this case requires optimisation depending on laser output power and also modes number produced.

Finally, the study on correlation functions helps us to acquire knowledge about laser working regime. This technique is especially most efficient for the super short laser pulses measurements.

---

[1] İ. A. Deryugin, R. A. Abdullayev, V. N. Kurashov, A. T. Mirzayev, V. N. Nastich, Radioteknik α Elektronik XVII, 8, 1623, Moskova, 1972.

[2] S. Sanders, L. Eng, J. Passalski, A. Yariv, Appl. Phys. Lett., 56, 310, 1990.

[3] İ. A. Deryugin, V. N. Kurashov, R. A. Abdullayev, V. N. Nastich, İzvestiya AK. Nauk SSSR, fiz.ser. XXXVII, 10, 2115, Moskova, 1973.

[4] R.A. Abdullayev, M.İ. Cenik, Bakü Üniversitesinin Haberleri, fiz.mat. serisi, 2, 80, 2001.

## UZUNUNA MODLAR REJİMİNDƏ İŞLƏYƏN LAZERİN KORRELYASİYA FUNKSİYALARININ TƏDQİQİ

**ABDULLAYEV R., ŞƏNTÜRK Ş., ÖZKIRIM İ., KİÇİKBURSA A.**

Məqalədə yarımklassik metodla uzununa modlar rejimində işləyən lazerin koherentlik xassələri tədqiq edilir. Modların amplitudalarının eyni olduğu hal üçün normallaşdırılmış korrelyasiya funksiyasının modların sayına bağlı olan analitik ifadəsi hesablanmışdır. Modların amplitudalarının bərabər olmadığı hal üçün normallaşdırılmış korrelyasiya funksiyaları hesabı yöntəmlə əldə edilmişdir. Bu nəticələr təcrübədən alınanlara uyğun gəlir.

## ИССЛЕДОВАНИЕ КОРРЕЛЯЦИОННЫХ ФУНКЦИЙ ЛАЗЕРА, РАБОТАЮЩЕГО В РЕЖИМЕ ПРОДОЛЬНЫХ МОД

**АБДУЛЛАЕВ Р., ШЕНТЮРК Ш., ОЗКЫРЫМ И., КИЧИКБУРСА А.**

В сообщении полуклассическим методом исследованы когерентные свойства лазера, работающего в режиме продольных мод. Получено аналитическое выражение нормированной корреляционной функции в зависимости от количества мод для равномерного распределения амплитуд мод. В случае неравномерного распределения амплитуды мод нормированные корреляционные функции вычислены численными методами. Результаты вычисления находятся в согласии с экспериментальными данными.