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NUMERICAL METHOD OF CALCULATION OF DINAMIC PROCESSES IN THE DRILL PIPE STRING BY ROTARY DRILLINGS

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To establish automatic control over oil well drilling, it is necessary to study complicated nature of movement of all parts of drilling unit which is observed in the process of rotary drilling. In the process of rotary drilling torsion vibrations of the drill string result in complex movement pattern and violate operation of automatic operation of electric drive.

Calculation of dynamic processes going on at the drill string is required for development of mechanism of protection and control against excess or lowered rotation rate and torsion moment in conditions of drill string parting, jumping or sticking [1-3]. Study of transient processes in the drill sting is very important for supervisory control over rotary drilling of oil wells as it allows finding out emergency situations and eliminating them [10-11].

Methods of calculations of transient processes in drill string in the case torsion vibrations are not developed well enough, thus certain difficulties arise in the course of the design drilling systems and their operation, and also when developing automatic control systems.

Thus, as rotary drilling becomes most widely used drilling method, efficient engineering methods of calculation of transient processes in the drill string are especially important because of torsion vibrations. As the same time study of dynamics of drilling systems in conditions of rotary drilling allows establishing new relationships and regularities, to carry out complete analysis of the problem, to make conclusions and to give recommendations.

Specific feature of dynamics of drill string is that it is an object with distributed parameters [1-5, 11].

Analysis shows that use of analytical methods of calculation of dynamic processes in the drill string as an object with distributed parameters, methods of operational calculus [1, 5-8] first, results in complex expressions containing infinite series and second creates serious mathematical difficulties when changing from images to original functions, and third, arbitrariness of boundary conditions complicates calculations.

At presents, numerical methods seem to be most efficient means for solution of this class of problems.

Numerical methods [1-6] based on the theory of pulsed systems and mathematical description of Laplace transform [1,2] is one of efficient specialized numerical method of calculation of transient processes in systems with distributed by partial differential equation of hyperbolic type.

Specific feature of calculation of transient processes in systems with distributed parameters based on said method [1-6] is the fact that transition from images to original functions is implemented based on convolution theorem.

Such approach allows identifying transient processes in the objects with distributed parameters, without findings roots of characteristic equation, expansion of operator coefficient of wave propagation and operator wave resistance into series. This simplifies mathematical calculations and improves their accuracy; infinite series of Bessel’s functions are excluded from telegrapher’s equation, thus reducing calculation volume. Obtained recurrent relationships can be used for computation of transient poetesses in the objects with distributed parameters.

In accordance with such approach, solution of dynamic problems in the objects with distributed parameters consists of three stages [1-6]:

1) at the first stage, expressions for desired functions M (x, t) and ω (x, t) in the Laplace images domain are determined;
2) at the second stage, based on theory of pulsed systems transition from obtained Laplace images to domain of equivalent digital images is done;
3) at the third stage, based on the convolution theorem transition from obtained discrete images to domain of original functions is implemented.

This paper describes numerical method of calculation of dynamic processes in the drill string in case of torsion vibrations, taking into account distribution of parameters of the objects. This method is modification of existing methods [1-6].

Proposed method is advantageous as it allows establishing dynamic processes in the object with distributed parameters, without transition to digital images domain.

Such approach allows to improve existing methods [1,2], to simplify and formalize calculation algorithms.

Dynamic model of drill with torsion vibrations

In general case dynamic processes taking place in the drill string (Fig. 1) when torsion vibrations appear are described by partial differential equations of hyperbolic type [1, 5, 7-9]:

$$-\frac{\partial M}{\partial X} = \theta \frac{\partial \omega}{\partial t} + R\omega$$
$$-\frac{\partial \omega}{\partial X} = C\frac{\partial M}{\partial X}, \qquad 0 \le x \le L \tag{1}$$

where M = M (x, t), ω = ω (x, t) – change of torque and angular velocity in any points x of the string at arbitrary time; θ – moment of inertia of the string per unit of length; R-loss factor; L-length of drill string; c-elasticity factor.

In the system of equations (1) under M (x, t) and ω (x, t) we shall mean their excess values against steady-state ones.

Actual values of angular velocity and torque are defined using the following formula:

$$\omega_1(x,t) = \omega(x,t) + \omega_0, \tag{2}$$
$$M_1(x,t) = M(x,t) + M_0 \tag{3}$$

where ω_0, M_0 – initial velocity and torque.

Parameters of drill string can be calculated as follows:

$$\theta = \frac{\gamma}{g} j10^{-5}, \tag{4}$$
$$c = \frac{10^4}{jQ}, \tag{5}$$

where γ is specific weight of the string material;

g is free fail acceleration, m/san;

j is sectional moment of inertia, cm;

Q is shear modules.

Wave velocity of torsion vibrations will be:

$$v = \frac{1}{\sqrt{c\theta}} = \sqrt{10\frac{g}{\gamma}Q}$$

To solve the system of equation (I) it is necessary to choose initial and boundary conditions.

According to (2),(3) initial conditions are zero.

Boundary conditions in depending from conditions of measurement parameters of control (of speed, of moment) in the processes of rotary boring may be different.

Take into account that, at present time rotary boring in the process of measurement parameters of control, meaning of speed and of torque moment in basis measuring in initial of the drill pipe string and of the speed on the bit, that convenient to choose as boundary conditions drilling engine speed ω(t) and drilling bit rotation speed $\omega_k(t)$:

$$\omega(x,t)\big|_{x=0} = \omega(0,t) = \omega_H(t), \tag{6}$$

$$\omega(x,t)\big|_{x=e} = \omega(\ell,t) = \omega_k(t), \tag{7}$$

where $\omega_H(t)$, $\omega_k(t)$- arbitrarily chosen laws of change of drilling shaft engine and drilling bit rotation speed.

Solution of differential equations.

The task is to find solution of system of equations (1) at accepted initial and boundary conditions.

In accordance with proposed approach, when solving this problem, at the first stage it is necessary to obtain Laplace image for functions ω(x,t), M(x,t).

Using this method we will obtain formula describing functions in operator form:

$$\omega(x,p) = \frac{sh\gamma(\ell - x)}{sh\,v\ell}\omega_H(p) + \frac{sh\gamma x}{sh\gamma\ell}\omega_k(p), \tag{8}$$

$$M(x,p) = \sqrt{\frac{\theta}{c}}\frac{\gamma_1}{p}\frac{ch\gamma(\ell - x)}{sh\gamma\ell}\omega_H(p) - \sqrt{\frac{\theta}{c}}\frac{\gamma_1}{p}\frac{ch\gamma x}{sh\gamma\ell}\omega_k(p), \tag{9}$$

where $\gamma = \frac{1}{v}\sqrt{p(p+2a)}$, $\gamma_1 = \gamma v$, γ- constant of operator, v- wave propagation, $a = \frac{R}{2\theta}$ -wave damping factor; p –operator of Laplace transform; M(x,t), ω(x,p) $\omega_I(p)$, $\omega_k(p)$- image of functions M(x,t), ω(x,t), $\omega_H(t)$, $\omega_k(t)$.

Next stage for solution of the task is connected with transition from Laplace images (8), (9) to originals domain.

Unlike works [1-6], we used different approach. Equation (8) might be presented in the following form:

$$\omega(\delta,p)\left[\frac{1}{p} - k_1(p)\right] = \left[k_2(p) - k_3(p)\right]\omega_H(p) + \left[k_4(p) - k_5(p)\right]\omega_k(p) +$$

$$+\left[k_4(p)-k_5(p)\right]\omega_k(p), \qquad (10)$$

where

$$k_1(p)=\frac{e^{-2\gamma e}}{p}, \qquad k_2(p)=\frac{e^{-2\gamma\delta e}}{p},$$

$$k_3(p)=\frac{e^{-2\gamma e(1-\delta)}}{p}, \qquad k_4(p)=\frac{e^{-\gamma e(1-2\delta)}}{p},$$

$$k_5(p)=\frac{e^{-\gamma e(1+2\delta)}}{p}, \qquad \delta=\frac{x}{2\ell}$$

Going from equation (10) for images to equation for originals, we shall obtain:

$$\int_0^t \omega(t-\eta,\delta)1(\eta)d\eta=\int_{\frac{2\ell\delta}{v}}^{t}\omega_H(t-\eta)k_2(\eta)d\eta-$$

$$-\int_{\frac{2\ell(1-\delta)}{v}}^{t}\omega_H(t-\eta)k_3(\eta)d\eta+\int_{\frac{2\ell(1-2\delta)}{v}}^{t}\omega_k(t-\eta)k_4(\eta)d\eta-$$

$$-\int_{\frac{2\ell(1+2\delta)}{v}}^{t}\omega_k(t-\eta)k_5(\eta)d\eta-\int_{\frac{2\ell}{v}}^{t}k_1(\eta)\omega(t-\eta,\delta)d\eta, \qquad (11)$$

Integral equation (11) could be solved numerically if we replace integrals with sums.

Thus, following papers [1,2] and using relationship between continuous time t and discrete (digital) n in the form of $t={nT}/{\lambda}$ (where λ- arbitrary integer T=2τ, $\tau={1}/{v}$ - time of wave travel to one end of drill string; n=0,1,2, …) we shall digitize equation (11), replacing operation of continuous integration with rectangular integration technique.

Instead of (11) we will obtain

$$\sum_{m=0}^{n}\omega[n-m,\delta]1[m]=\left(\sum_{m=\lambda s}^{n}k_2[n-m]-\sum_{m=\lambda(1-\delta)}^{n}k_3[n-m]\right)\omega_H[m]-$$

$$-\left(\sum_{m=0,5\lambda(1-2\delta)}^{n}k_4[n-m]-\sum_{m=0,5\lambda(1+2\delta)}^{n}k_5[n-m]\right)\omega_k[m]+\sum_{m=\lambda}^{n}k_1[n-m]\omega[m,\delta], \qquad (12)$$

where if n<λ

$$k_1[n]=\left\{e^{-2a\tau}+2a\tau\sum_{m=\lambda+1}^{n}e^{-\frac{amT}{\lambda}}\frac{I_1\left(\frac{aT}{\lambda}\sqrt{m^2-\lambda^2}\right)}{\sqrt{m^2-\lambda^2}}\right\}$$

if n>λ

if n<λδ

$$k_2[n]=\left\{e^{-2a\tau\delta}+2a\tau\delta\sum_{m=\lambda\delta+1}^{n}e^{-\frac{amT}{\lambda}}\frac{I_1\left(\frac{aT}{\lambda}\sqrt{m^2-(\lambda\delta)^2}\right)}{\sqrt{m^2-(\lambda\delta)^2}}\right\}$$

if n>λδ

if n<λ(1-δ)

$$k_3[n]=\left\{e^{-2a\tau(1-\delta)}+2a\tau(1-\delta)\sum_{m=\lambda(1-\delta)+1}^{n}e^{-\frac{amT}{\lambda}}\frac{I_1\left(\frac{aT}{\lambda}\sqrt{m^2-[\lambda(1-\delta)]^2}\right)}{\sqrt{m^2-[\lambda(1-\delta)]^2}}\right\}$$

if n>0,5λ(1-2δ),

$$k_4[n]=\left\{e^{-a\tau(1-2\delta)}+a\tau(1-2\delta)\sum_{m=0,5\lambda(1-2\delta)+1}^{n}e^{-\frac{amT}{\lambda}}\frac{I_1\left(\frac{aT}{\lambda}\sqrt{m^2-[0,5\lambda(1-2\delta)]^2}\right)}{\sqrt{m^2-[0,5\lambda(1-2\delta)]^2}}\right\}$$

if n<0,5λ(1+2δ),

$$k_5[n]=\left\{e^{-a\tau(1+2\delta)}+a\tau(1+2\delta)\sum e^{-\frac{amT}{\lambda}}\frac{I_1\left(\frac{aT}{\lambda}\sqrt{m^2-[0,5\lambda(1+2\delta)]^2}\right)}{\sqrt{m^2-[0,5\lambda(1+2\delta)]^2}}\right\}$$

if n<0,5λ(1+2δ),

ω[n,δ], $\omega_n[n]$, $\omega_k[n]$- value of initial functions ω[t,δ], $\omega_n[t]$, $\omega_k[t]$ in a grid form; $K_i[n]$ (where I =1,5)- known grid function , obtained from relevant continuous functions $K_i[t]$ by replacement of t for nT/λ and operation of continuous integration with rectangular summing; $I_i(n)$ – Bessel function of the 1-st order.

Here

$$\sum_{m=0}^{n} 1[m]\omega[n-m,\delta] = \omega[n,\delta] + \sum_{m=1}^{n}\omega[n-m,\delta], \tag{13}$$

Formula (12), accounting for (13) will look as follows:

$$\omega[n,\delta] + \sum_{m=1}^{n}\omega[n-m,\delta] = \left(\sum_{m=\lambda\delta}^{n} k_2[n-m] - \sum_{m=\lambda(1-\delta)}^{n} k_3[n-m]\omega_H[m]\right) -$$
$$-\left(\sum_{m=0,5\lambda(1-2\delta)}^{n} k_4[n-m] - \sum_{m=0,5\lambda(1+2\delta)}^{n} k_5[n-m]\right)\omega_k[n-m] + \sum_{m=\lambda}^{n} k_1[m]\omega[n-m,\delta], \tag{14}$$

From the above the following recurrent relationship is found, which allows calculating functions ω[n,δ]:

$$\omega[n,\delta] = \left(\sum_{m=\lambda\delta}^{n} k_2[n-m] - \sum_{m=\lambda(1-\delta)}^{n} k_3[n-m]\right)\omega_H[m] - \left(\sum_{m=0,5\lambda(1-2\delta)}^{n} k_4[n-m] - \sum_{m=0,5\lambda(1+2\delta)}^{n} [n-m]\right)\omega_k[m] +$$
$$+\sum_{m=\lambda}^{n} k_1[m]\omega[n-m,\delta] - \sum_{m=1}^{n}\omega[n-m,\delta] \tag{15}$$

To determine torsion moment M[n,δ], formula (9) may be presented in the following form:

$$M(\delta,p)[k_1''(p) - k_1'(p)] = \sqrt{\frac{\theta}{c}}[k_2(p) + k_3(p)]\omega_H(p) -$$
$$-\sqrt{\frac{\theta}{c}}[k_4(p) + k_5(p)]\omega_k(p), \tag{16}$$

where $k_1'(p) = \dfrac{e^{-2ve}}{\sqrt{p(p+2a)}}$, $k_1''(p) = \dfrac{1}{\sqrt{p(p+2a)}}$

Going from equation (16) to the original domain we will obtain:

$$\int_0^e k_1''(\eta)M(t-\eta,\delta)d\eta - \int_{\frac{2\ell}{v}}^{t} k_1'(\eta)M(t-\eta,\delta)d\eta =$$

$$=\sqrt{\frac{\theta}{c}}\left\{\int_{\frac{2\ell\delta}{v}}^{t}\omega_H(t-\eta)k_2(\eta)d\eta + \int_{\frac{2\ell(1-\delta)}{v}}^{t}\omega_H(t-\eta)k_3(\eta)d\eta - \int_{\frac{2\ell(1-2\delta)}{v}}^{t}\omega_k(t-\eta)k_4(\eta)d\eta - \right.$$

$$\left. - \int_{\frac{2l}{V}(1+2\delta)}^{t}\omega_k(t-\eta)k_5(\eta)d\eta\right\} \tag{17}$$

Equation (17) in a grid form will look as follows:

$$\sum_{m=0}^{n} k_1''[m]M[n-m,\delta] - \sum_{m=\lambda}^{n} k_1'[m]M[n-m,\delta] = \tag{18}$$

$$=\sqrt{\frac{\theta}{c}}\left\{\left(\sum_{m=\lambda\delta}^{n}k_2[m] + \sum_{m=\lambda(1-\delta)}^{n}k_3[m]\right)\omega_H[n-m] - \left(\sum_{m=0,5\lambda(1-2\delta)}^{n}k_4[m] + \sum_{m=0,5\lambda(1+2\delta)}^{n}k_5[m]\right)\omega_k[n-m]\right\},$$

where

$$k_1''[n] = e^{-\frac{anT}{\lambda}} I_0\left(\frac{anT}{\lambda}\right), \qquad k_1'[n] = e^{-\frac{anT}{\lambda}} I_0\left(\frac{aT}{\lambda}\sqrt{n^2-\lambda^2}\right),$$

$I_0(n)$- Bessel function of zero-order.

Solving equation (18) with respect to function M[n,δ], we will obtain the following recurrent relationship, which allows to calculate functions M[n,δ]:

$$M[n,\delta] = \sqrt{\frac{\theta}{c}}\left(\sum_{m=\lambda\delta}^{n}k_2[m] + \sum_{m=\lambda(1-\delta)}^{n}k_3[m]\right)\omega_H[n-m] - \sqrt{\frac{\theta}{c}}\left(\sum_{m=0,5\lambda(1-2\delta)}^{n}k_4[m] + \sum_{m=0,5\lambda(1+2\delta)}^{n}k_5[m]\right)\omega_k[n-m] +$$

$$+\sum_{m=\lambda}^{n}k_1'[m]M[n-m,\delta] - \sum_{m=1}^{n}k_1''[m]M[n-m,\delta], \tag{19}$$

Error in calculation is related with value of λ. If λ is more large, the characteristics of continuous function less differ from respective characteristics of grid functions [1-5].

Thus, obtained recurrent relationships (15), (19) allow determining dynamic processes in any point of drill string at arbitrary time.

Example: Let us investigate the character of torsion vibration at instantaneous –stepwise increase of torque by ΔM. We will determine change of rotation rate of drilling bit $\omega_n[n]$ and torque at the output of drilling engine M[n] (Fig. 1).

To simplify analysis, we shall assume that during the time of transient process rotation rate of the shaft of drilling engine remains constant, i.e. $\omega_n[t]=0$. According to (9) change of rotation rate of drilling bit is determined from the following equation:

$$\omega_k(p) = -\sqrt{\frac{c}{\theta}} \cdot \frac{\Delta M}{\sqrt{p(p+2a)}} \cdot \frac{sh\gamma e}{ch\gamma e} \qquad (\Pi.1)$$

Equation (Π.1) in the original domain, in a grid form looks as follows:

$$\omega_k[n] = -\Delta M \sqrt{\frac{c}{\theta}} \left(\sum_{m=0}^{n} k_2[m] - \sum_{m=\lambda}^{n} k_3[m] \right) - \\ - \sum_{m=\lambda}^{n} k_1[m]\omega_k[n-m] - \sum_{m=0}^{n-1} \omega_k[m], \qquad (\Pi.2)$$

where

$$k_2[n] = e^{-\frac{\alpha T n}{\lambda}} I_0\left(\frac{\alpha T n}{\lambda}\right)$$

$$k_3[n] = ^{\frac{\alpha T n}{\lambda}} I_0\left(\frac{\alpha T}{\lambda}\sqrt{n^2-\lambda^2}\right)$$

Obtained recurrent relationship (Π.2) for each tome moment n=0,1,2… allows to determine rotation rate of drilling bit at any moment.

At x=0, from equation (9) we shall define change of torque at the beginning of drill sting:

$$M_H(p) = \frac{\Delta M}{p} \cdot \frac{1}{ch\gamma e} \qquad (\Pi.3)$$

Representing formula (Π.3) in a grid form we will obtain:

$$M_H[n] = 2\sum_{m=0,5\lambda}^{n} k_2'[m] - \sum_{m=\lambda}^{n} k_1[m] M_H[n-m] - \sum_{m=0}^{n-1} M_H[m], \qquad (\Pi.4)$$

where if $n<0,5\lambda$

$$k_2'[n] = \left\{ e^{-\alpha r} + \alpha r \sum_{m=0,5\lambda+1}^{n} e^{-\frac{\alpha T m}{\lambda}} \frac{I_1\left(\frac{\alpha T}{\lambda}\sqrt{m^2 - (0,5\lambda)^2}\right)}{\sqrt{m^2 - (0,5\lambda)^2}} \right\}$$

if n>0,5λ

To understand how drilling mud influences dynamic processes, we must determine the effect of value of friction coefficient R (friction of drill string with mud).

In this example, according [1] we assume that damping factor α is equal to

$$\alpha = \frac{R}{2\theta} = 0{,}031/5$$

Length of drill string L=5000 m, and rate of propagation of torsional vibrations within the string is v=3·10^3 m/sec then $r = \frac{\ell}{v} = 1{,}67\,\text{sec}$.

Calculations were done on computer at λ=10, in compliance with the programs in basic language.

Results of calculations of drilling engine torque are in the following form of stipples lines given ni Fig. 1, rotation rate of drilling bit –in Fig. 2.

On Figs. 1 and 2 gets results well consents with of results gets by method of Dalamber.

It could be seen from Fig. 1 that when after time τ wave front reaches the engine, then wave of torque will be reflected having been combined with direct wave. As a result we will obtain double torque on the engine comparing to increase of torque ΔM on drilling bit.

It can be seen from Fig. 1 that on increase of torque on the drilling bit by value ΔM rotation rate of drilling bit decreases.

Besides, it follows from Fig. 1 and 2 that dynamic processes in the drill string are attenuated because of friction between the drill string and mud.

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ROTOR ÜSULU İLƏ QAZIMADA, QAZIMA KOLONNASINDA BAŞ VERƏN DİNAMİK RECİMLƏRİN ƏDƏDİ TƏYİN EDİLMƏSİ.

ƏLİYEV Y.A.

Məqalədə rotor üsulu ilə qazımada, qazıma kolonnasında baş verən dinamik recimlərin təyin edilməsi üçün ədədi üsul təklif edilmişdir.

ЧИСЛЕННОЕ ОПРЕДЕЛЕНИЕ ДИНАМИЧЕСКИХ РЕЖИМОВ РАБОТЫ КОЛОННЫ БУРИЛЬНЫХ ТРУБ ПРИ РОТОРНОМ БУРЕНИИ

АЛИЕВ Я.А.

В статье предложен численный метод расчета динамических режимов работы колонны бурильных труб при роторном бурении.