AZƏRBAYCAN MİLLİ ELMLƏR AKADEMİYASININ XƏBƏRLƏRİ
Fizika-riyaziyyat və texnika elmləri seriyası, 2001 №2,5

621.315.592

# ELECTRIC FIELD DEPENDENCE OF INTRASUBBAND INTERFACE ROUGHNESS SCATTERING IN A QUANTUM WELL

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The scattering rates in the lowest subband in a quantum well are calculated for interface roughness scattering when an electric field is applied perpendicular to the layer plane. It is found that the interface roughness scattering rate increases with increase in the electric field. The electric field changes the interface roughness scattering rates drastically in thick $QW_s$ compared with those for the zero-field case.

## INTRODUCTION

There has been considerable interest in the study of intersubband transitions in a quantum well (QW) both in the presence and in the absence of an electric field applied perpendicular to the QW layer plane [1-12]. The reason behind the interest is practical device application [3-4]. In [1] proposed a new type of infrared laser amplifier using the intersubband transition and resonant tunneling. An infrared detector in the 10μm has been proposed and fabricated [5] that relies on the intersubband transition and resonant tunneling between adjacent QWs.

Theoretical studies [6,7] on intersubband optical absorption under an electric field have pointed out an increase in the oscillator strength. In all the theories [6,7] the intra - and intersubband relaxation rates come into the calculation of the absorption coefficient. Various scattering events determine the intra and intersubband relaxation rates, of which polar optic phonon scattering has been major scattering mechanism over a considerable temperature range. The rates for such scattering have already been calculated [7]. In [8] have calculated the alloy-disorder scattering rate for two-dimensional electrons in the lowest subband of a quantum well an applied electric field.

It is established that interface - roughness scattering dominates the low temperature mobility of two - dimensional (2Д) electrons in thin QWs [13,14]. The mobility and relaxation rates due to interface roughness scattering in QWs have been formulated previously in the absence of an applied electric field [15,16]. In the present work we report similar calculations when an electric field is applied perpendicular to the QW layer, and investigate how the scattering rates a modified. As a first step, we consider intrasubband relaxation for the lowest subband.

## CALCULATIONS

The electrons are assumed to be quantizied along the Z direction and the wave function is taken to be of the following form [2].

$$\Phi(R)=\Psi(Z)\exp(ikr)=N(\beta)\left(1+\frac{\beta z}{L}\right)\Psi_0(z)\exp(ikr), \qquad -L/2<z<L/2 \qquad (1)$$

where $\psi_0(Z)$ is the unperturbed ground-state wave function. The normalization factor of the electron wave function are

$$N^2(\beta)=\frac{\pi^3}{\pi^3+\beta^2(\pi^2-8)} \qquad (2)$$

$L$ is the thickness of the well, and k and r are, respectively the 2Д wave vector and position vector of the electron in the plane of free motion (x-y plane). In Eg (1) $\beta$ is variational parameter related to the electric field F as [2]

$$\beta = \frac{2|e|Fm^* L}{\hbar^2} < z^2 >_0 \ , \qquad \beta^2 < z^2 >_0 << L^2 \tag{3}$$

where $< z^2 >_0 = < \psi | z^2 | \psi_0 >$.

The interface roughness is characterized by the height $\Delta$ and the lateral correlation length $\wedge$ of the Gaussian fluctuation. For the perturbing potential due to interface roughness we use the standard model [10,17], for which the random potential is assumed to have Gaussian distribution and expressed as the autocorrelation function

$$< V(R)V(R') >= V_0^2 \Delta^2 \delta(Z - L/2)\delta(Z' - L/2)\exp\left(-\frac{(r - r')^2}{\Lambda^2}\right) \tag{4}$$

where $\langle ...... \rangle$ means an ensemble average, $V_0$ is the barrier height

Using Eq. (4) we get for the square of the matrix element for roughness scattering from the k state to the k` state

$$|M(K,K')|^2 = \pi\Lambda^2\Delta^2 V_0^2 \psi^4\left(\frac{L}{2}\right)\exp\left(-\frac{q^2\Lambda^2}{4}\right), \tag{5}$$

where q=k`-k is the 2Д scattering wave vector and q=$|q|$. Also, as the scattering is elastic

$k` = k, \quad q^2 = 2k(1-\cos\theta), \quad \cos\theta = kk`/k^2$

The transition probability from a state k to all other state k` is then given by

$$W_F = \frac{2\pi}{\hbar}\sum_{K'} |M(K,K')|^2 \delta(E_{K'} - E_K) = \frac{\pi\, m^* \Lambda^2\Delta^2 V_0^2 \psi^4 (L/2)}{\hbar^3} G(\Lambda, K), \tag{6}$$

where

$$G(\Lambda, K) = \frac{1}{2\pi}\int_0^{2\pi} \exp[-\Lambda^2 K^2 (1-\cos\theta)/2] d\theta .$$

The interface roughness scattering rate is proportional to $V_0^2\psi^4(L/2)$, and using [9]

$$\psi(L/2) = \left(\frac{\hbar^2}{2m^*(V_0 - E)}\right)^{1/2} \frac{d\psi}{dZ}\Big|_{Z=L/2} \tag{7}$$

We obtain

$$W_F = \frac{\pi^5\Lambda^2\Delta^2\hbar}{m^* L^6}\cdot\frac{V_0^2}{(V_0 - E)^2} N^4(\beta)\left(1+\frac{\beta}{2}\right)^4 \cdot G(\Lambda, K). \tag{8}$$

As shown in Eg.(8), the scattering rate is proportional to $L^{-6}$, this means that interface roughness scattering rate is much more important for narrow wells and the interface roughness scattering limited mobility in quantum wells is proportional to $L^6$ [15].

For the interface roughness calculation we have not bothered to take screening into account. It seems evident that screening can only reduce an effect that is already very small.

In order to facilitate comparison, a similar expression for the transition probability or scattering rate without an applied field is needed. Thus if we take the limit $V_0 \to \infty$ and $\beta = 0$ in Eg.(8), we may write for the scattering rate without a field, $W_0(K)$ as [15]

$$W_0 = \frac{\pi^5\Lambda^2\Delta^2\hbar}{m^* L^6} G(\Lambda, K). \tag{9}$$

In the limit $V_0 \to \infty$ from Eg .(8) - (9) the ratio is expressed as

$$\gamma = \frac{W_F(K)}{W_0(K)} = N^4(\beta)\left(1+\frac{\beta}{2}\right)^4 . \tag{10}$$

## RESULTS

We have calculated the scattering rates in $QW_s$ by using the parameters characteristic of GaAs. The values of the ratio $\gamma$ calculated from Eg.(10) plotted against the electric field F in Fig.1. It is found from the figure that the scattering rate increases with an increase in the electric field. However, as seen from the figure, the electric field changes the interface roughness scattering rates drastically in thick $QW_s$ compared with those for zero-field case. These results predict qualitatively the trends observed [2]. Smaller electric field dependences can be explained by the fact that the wave function change due to the applied electric field of thin $QW_s$ is smaller than that thick $QW_s$. We find that $\gamma$ is also a strong function of field and this ratio becomes as small as 1.1~1.5 below 100kV/cm. Another observation is that although the value for $\gamma$ is 1.5÷4.5 field above 400kv/cm. In the case of polar optic phonon scattering $\gamma \approx 1.2$ (F=200kV/cm) [7] and for alloy-disorder scattering $\gamma \approx 4.5$ (F=400kV/cm) [8].

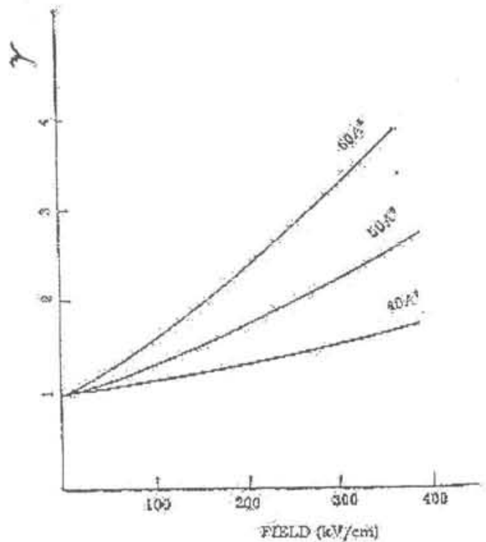


**Fig.1.**

Variation of scattering rate, $W_F(k)$, as a function of applied electric field for GaAs QW of 40, 50 and 60Å. The rate $W_F$ is normalized by the rate $W_0(k)$ in the absence of field and the ratio is denoted by $\gamma$.

## CONCLUSION

We have demonstrated that there was an increase of the scattering rate of electrons in the first subband of a QW, with increase in the perpendicular electric field, when the scattering is due to interface roughness. We have found that $W_f/W_0$ had a stronger dependence on the well width.


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## KVANT CUXURDA SƏTHİN KƏLƏKÖTURLÜYÜNDƏN ALTZONADAXİLİ SƏPİLMƏNİN ELEKTRİK SAHƏSİNDƏN ASILILIĞI

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Kvant cuxurun müstəvisinə perpendikulyar elektrik sahisində ölcülü kvantlanmış birinçi altzonadan elektronların səthin kələkötürlüyündən daxili altzona səpilmə vaxtı hesablanmışdır.

Göstərilmişdir ki, elektrik sahəsi, galın kvant cuxurlarda səthin kələkötürlüyündən alt daxili zona səpilmə vaxtına əsaslı tə'sir göstərir.

## ВНУТРИПОДЗОННОЕ РАССЕЯНИЕ НА ШЕРОХОВАТОСТЯХ ПОВЕРХНОСТИ В КВАНТОВОЙ ЯМЕ В ЭЛЕКТРИЧЕСКОМ ПОЛЕ

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Рассчитано время внутриподзонного рассеяния на шероховатостях поверхности для электронов первой подзоны размерного квантования в квантовой яме в электрическом поле, перпендикулярном плоскости ямы. Показано, что приложение электрического поля приводит к значительному увеличению рассеяния. Электрическое поле оказывает существенное влияние на время внутризонного рассеяния на шероховатостях поверхности в толстой квантовой яме.

Редактор: Ф.Гашимзаде