AZƏRBAYCAN MİLLİ ELMLƏR AKADEMİYASININ XƏBƏRLƏRİ
Fizika-riyaziyyat və texnika elmləri seriyası, fizika və astronomiya, 2004 №2

ENTROPY OF STATES AND NUCLEAR TEMPERATURE FOR LARGE DEFORMED HEAVY NUCLEI

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In this paper we represent a new method of calculation of the entropy and nuclear temperature of large deformed heavy nuclei. The essence of the method used is similar to that for the "entropy of integer numbers" given by the Hardy-Ramanujan formula. The obtained entropy and nuclear state equations are turned into the corresponding equations of the nuclear Fermi-gas model in the limit of corresponding nuclear excitations.

Energy levels of the ground state bands as well as excited bands in the large deformed heavy nuclei in the rare earths and actinides demonstrate the "equidistant" property in a large sense. To define this symmetry property in the energy bands one can choose in general the unit energy as $E_1(i_1^\pi) - E_0(i_0^\pi)$, where $E_0(i_0^\pi)$ is the energy of the lowest level with spin quantum number i_0 and parity π and $E_1(i_1^\pi)$ is the energy of first excited state with quantum numbers i_1 and π over the level $E_0(i_0^\pi)$. For the ground state bands in the even-even nuclei we have $E_0(i_0^\pi) = 0(0^+)$ while for the excited bands of these nuclei $E_0 \neq 0$ and these bands are to be characterized by the conserved quantum number K that is the projection of the nuclear angular momentum on the nuclear symmetry axis. The energy bands of the even-odd, odd-even and odd-odd nuclei have similar peculiarities. The excited level of a single particle (proton, or neutron) in such nuclei forms a head of collective excitation band [1]. Introducing the ratio

$$R_n \equiv \frac{E_n(i_n^\pi) - E_0(i_0^\pi)}{E_1(i_1^\pi) - E_0(i_0^\pi)}, \tag{1}$$

for the levels of an appropriate band, one arrives at

$$\frac{E_2(i_2^\pi) - E_0(i_0^\pi)}{E_1(i_1^\pi) - E_0(i_0^\pi)}, \frac{E_3(i_3^\pi) - E_0(i_0^\pi)}{E_1(i_1^\pi) - E_0(i_0^\pi)}, \frac{E_4(i_4^\pi) - E_0(i_0^\pi)}{E_1(i_1^\pi) - E_0(i_0^\pi)}, \ldots \tag{2}$$

for the different energy levels, starting from second over the $E_0(i_0^\pi)$ level. Using experimental data [2] for the nuclear energy levels in the corresponding bands, one finds an important symmetry property [3,4]

$$R_2 : R_3 : \ldots : R_n : \ldots \cong r : 2r : \ldots : (n-1)r : \ldots \ , \tag{3}$$

where $r = \frac{E_2(i_2^\pi) - E_0(i_0^\pi)}{E_1(i_1^\pi) - E_0(i_0^\pi)}$ and this quantity changes from2 to 3,3. For the ground state bands of even-even rare earth and actinide elements $r \approx 3.3$, whereas for the same bands of the even-odd, odd-even and odd-odd nuclei $r \approx 2$. Eq.(3) can be put in the form

$$R_2 : R_3 : R_4 : R_5 ... \approx 1:2:3:4... \ . \tag{4}$$

This result means that the ratios of the sequential excited level energies accounted from first energy level, to the unit energy is expressible as ratios of the integer numbers, starting from unity. This symmetry property in the energy bands of the nuclei considered may be used in order to calculate the entropy of energy states in the large deformed heavy nuclei. Indeed, the well-known equation in the numbers theory, the Hardy-Ramanujan formula [3] allows for calculating the number of possible representations of any integer as combinations of its lower integers (subintegers). This is, in fact, the "entropy of an integer", which is expressed as

$$P(n) \cong \frac{1}{4\sqrt{3} \cdot n} \exp(\pi \sqrt{\frac{2}{3} \cdot n}) . \tag{5}$$

This asymptotic equation has well accuracy. For $n > 10$ the error in this equation becomes less than 10%, by increasing n. For $n = 4$ the error is about 20%. In the region of nuclear excitations close to the neutron binding energy the ratio of nuclear excitation energy U, to the unit energy $\varepsilon_0 = E_1(i_1^\pi) - E_0^\pi$, U/ε_0 satisfies condition $U/\varepsilon_0 > 10$. Then the entropy of a nuclear state of the energy U can be calculated by

$$P(U, \varepsilon_0) = \frac{1}{4\sqrt{3}\frac{U}{\varepsilon_0}} \exp(\pi \sqrt{\frac{2}{3} \cdot \frac{U}{\varepsilon_0}}) . \tag{6}$$

Eq.(6) implies the number of ways for representation the excitation energy U as being combinations of the energy parcels $\varepsilon_0, r\varepsilon_0, 2r\varepsilon_0, 3r\varepsilon_0 ...$, that is the entropy of nuclear state of the energy U. Clearly, instead of using Eq.(6), its logarithm ($\ln P$) is used in statistical thermodynamics. Nuclear entropy is then expressed as

$$S = \ln P . \tag{7}$$

Thus, starting with the possibility of representation of the nuclear excitation energy as combination of the energies of microstates we can calculate the nuclear entropy at an excitation energy U, by the use of Eqs.(6) and (7). Introducing the level density parameter $a_0 = \pi^2 / 6\varepsilon_0$, of the nuclear collective states, which is discussed in detail in Refs.[3,4] and $s = 2\sqrt{a_0 E}$ one can rewrite Eq.(6) as

$$P = \frac{6\sqrt{3}}{\pi^2} \cdot \frac{\exp(s)}{s^2} . \tag{8}$$

Then entropy of a nucleus can be expressed as

$$S = \ln P = \ln 6\sqrt{3} / \pi^2 + s - 2\ln s . \tag{9}$$

For not very small excitations we may neglect the constant term in Eq.(9) to find

$$S = \ln P = s - 2\ln s . \tag{10}$$

Here we note that the expression $s = 2\sqrt{a_0 E}$ including into the above equations (8)-(10), is simply the excitation energy dependence of the entropy in the Fermi-gas

model of a nucleus. However, in this model, instead of the parameter a_0 should be written the single-particle level density parameter, a.

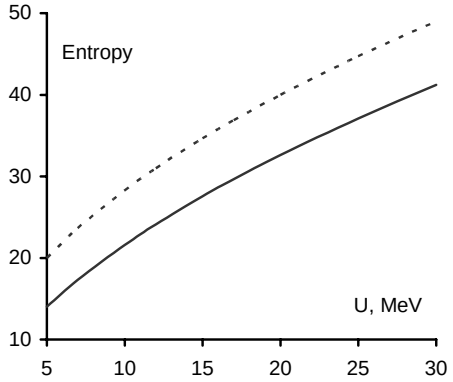

Fig.1.

Calculation results of nuclear entropy versus nuclear excitation energy ($a_0 = a = 20MeV^{-1}$): — Eq.(10) , - - Fermi-gas model.

In figure 1 it is illustrated entropy of a nucleus versus excitation energy. Dashed line indicates the Fermi-gas calculation result, solid line is the calculation result given by the equation (10). In calculations we have chosen $a_0 = a = 20MeV^{-1}$. As it follows from the figure absolute value of the entropy in our model is regularly less and changes slower with the energy, than that of in Fermi-gas model. At the higher excitation energies the contribution coming from logarithmic term in Eq.(10) becomes negligible. From Eq.(10) we obtain nuclear thermodynamic temperature as

$$\frac{1}{T} = \frac{\partial S}{\partial U} = \sqrt{\frac{a_0}{U}}\left(1 - \frac{1}{\sqrt{a_0 U}}\right). \qquad (11)$$

Resolving this algebraic equation, we easily find nuclear state equation in the form

$$U = \frac{1}{2}a_0 T^2 - T + \frac{1}{2}a_0 T^2\sqrt{1 - \frac{4}{a_0 T}}\,. \qquad (12)$$

The parameter a_0, in the deformed nuclei of interest takes large values ($a_0 \geq 20Mev^{-1}$) [3,4]. Then, instead of Eq.(12) we obtain

$$U = a_0 T^2 - 2T\,. \qquad (13)$$

Note that the empirical equations similar with that of Eq.(11) were formerly used in the literature[5-7]. Eq.(13) differs from nuclear Fermi-gas state equation by $-2T$. This indicates that the nuclear excitation energy changes slower than that of the Fermi-gas model due to the temperature term. At the higher temperatures ($T>1MeV$) Eq.(13) becomes

$$U = a_0 T^2, \qquad (14)$$

like the Fermi-gas model [8,9]. Note that the parameter a in Eq.(14), that is the single particle level density parameter, has the same or close values with the parameter a_0 [3,4]. In figure 2 we represent the calculation results of nuclear excitation energy versus temperature. Solid dots, solid line and dashed line are respectively the results of the calculations by Eqs.(12), (13) and (14),being with the parameters a_0 and where as $a_0 = a = 20MeV^{-1}$. As it is evident from the figure, the calculation results through Eqs.(12) and (13) coincide, and Eq.(13) may be used instead of Eq.(12) for sufficiently higher

accuracy. Slower behavior of the excitation energy against temperature, comparing with Fermi-gas model given by Eq.(14)is also clear in figure 2.

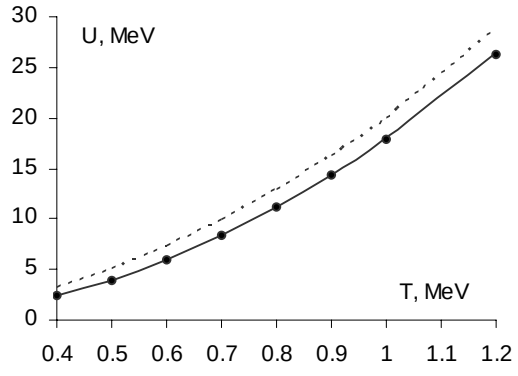

Fig.2.

Calculation results of nuclear excitation energy versus nuclear thermodynamic temperature ($a_0 = a = 20 MeV^{-1}$): • Eq.(12) , — Eq.(13) , - - Eq.(14).

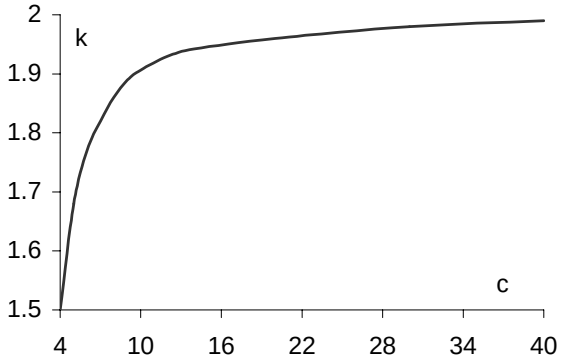

Fig.3.

The change of degree *k* in Eq.(15) with the factor $c = a_0 T$.

In some cases it is useful to express the nuclear excitation energy depending on temperature . For this reason we write Eq.(13) as

$$E = a_k T^k = a_0 T^2 - 2T , \qquad (15)$$

and attempt to estimate physically acceptable k values. For nuclear excitations close or higher than neutron binding energy one may take $2a_0 T \equiv 2c >> 1$, and then the coefficient a_k can be defined as $a_k = a_0^{k-1}$. The relationship between c and k can be obtained from Eq.(15) as

$$(k-1)\ln c = \ln(c-2) , \qquad (16)$$

If $c - 2 \approx c$, is a good approximation then $k \approx 2$, on the contrary $2 > k > 1/2$. In figure 3, we illustrate the behaviour of k versus c. As it is obvious from the figure, the value of k increases with the increase in production of a_0 and T. Identifying the quantities a and a_0 and taking into account that their values in the nuclear region of interest are change from 20 to 35 MeV^{-1}, one can conclude that same values of k may correspond to different values of nuclear excitations in the different nuclei. This result implies that the value of k in the Fermi-gas model, $k = 2$, is determined not with alone the large value of the nuclear excitation energy (or temperature) but with the value of the parameter a_0.

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DEFORMASİYA OLMUŞ AĞIR NÜVƏLƏRDƏ ENERJİ HALLARININ ENTROPİYASI VƏ NÜVƏ TEMPERATURU

H. ƏHMƏDOV

Bu məqalədə güclü deformasiya olmuş ağır nğvələrin entropiyasının və nüvə temperarturunun hesablanması ğzğn yeni metod verilmişdir. Bu məqsədlə, tam ədədlər nəzəriyyəsindən məlum olan və "tam ədədin entropiyası menasını verən Hardi-Ramanujan düsturu istifadə edilmişdir. Nüvə entropiyasının həyacanlanma enerjisindən asılılığı və nüvənin hal tenliyi üçün alınan neticələr limit halında nüvənin Fermi-gaz modelinin vermiş olduğu uygun tənliklərlə üst-üstə düşür.

ЭНТРОПИЯ СОСТОЯНИЙ И ЯДЕРНАЯ ТЕМПЕРАТУРА В ДЕФОРМИРОВАННЫХ ТЯЖЕЛЫХ ЯДРАХ

Г. АХМЕДОВ

В этой работе предлагается новый метод расчета энергетической зависимости энтропии и уравнения состояния в ядрах с большими деформациями. С этой целью в работе используется формула, известная из теории целых чисел как формула Харди–Раманужана, которая может быть интерпретирована как "энтропия целого". Найденные выражения для энтропии и уравнения состояния ядра являются оригинальными и в предельном случае больших энергий переходят в соответствующие уравнения для ядерной модели Ферми–газа.

Редактор:Ш.Нагиев