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THERMOPOWER OF NON-DEGENERATE SEMICONDUCTORS UNDER CONDITIONS OF STRONG ELECTRON-PHONON MUTUAL DRAG IN HIGH ELECTRIC FIELD

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The thermopower of non-degenerate semiconductors in high electric field is investigated taking into account both the heating of electrons and phonons, and their mutual drag. It is considered the conditions of strong mutual drag, when the electrons and phonons are scattered mainly by each other. It is shown that when the additional relaxation channel of the electrons (relaxation by impurities) becomes wider than that of the phonon momentum scattering by the crystal boundaries and by the phonons, the total thermopower (V) rapidly increases on raising heated electric field: $V \sim E^{10/3}$.

The interest to the studies of thermopower in different systems under the conditions of carrier heating at the high external electric field has recently been intensified [1-6]. Lei [1], E.M.Conwell and J.Zucker [2], Xing, Liu, Dong and Wang [3] were discussed the thermopower under the conditions of carrier heating at the external high electric field neglecting the contribution of the phonon drag, which is very important at low temperatures of lattice [7]. The role of the phonon drag in thermopower of hot carriers was studied by Wu, Horing and Cui [4], with taking into account only the drag of electrons by phonons (thermal drag), but the mutual drag of electrons and phonons was neglected. The influence of mutual drag of the carrier-phonon system on the thermopower and thermomagnetic effects of hot carriers were studied in [5-6].

In this work the thermopower of non-degenerate semiconductors under the conditions of strong electron-phonon mutual drag is investigated by taking into account the electron and phonon heating in high electric ($\vec{E}$) field. The spectrum of electrons is assumed to be parabolic:

$$\varepsilon = \frac{p^2}{m}. \tag{1}$$

The basic equations of the problem are the coupled Boltzmann transport equations for electrons and phonons. It is assumed that the electrons and phonons are scattered mainly by each other (the conditions of strong mutual drag) via the deformation interaction. The case of quasi-elastic electron scattering by acoustic phonons is considered. For the considered case the distribution functions of electrons $f(\vec{p},\vec{r})$ and phonons $N(\vec{q},\vec{r})$ may be presented in the form:

$$f(\vec{p},\vec{r}) = f_0(\varepsilon,\vec{r}) + \vec{f}_1(\varepsilon,\vec{r})\frac{\vec{p}}{p}, \qquad \left|\vec{f}_1\right| << f_0, \tag{2}$$

$$N(\vec{q},\vec{r}) = N_0(q,\vec{r}) + \vec{N}_1(q,\vec{r})\frac{\vec{q}}{q}, \qquad \left|\vec{N}_1\right| << N_0. \tag{3}$$

Here f_0 and $\vec{f}_1$, N_0 and $\vec{N}_1$ are the isotropic and anisotropic parts of the electron and phonon distribution functions, respectively.

If the inter-electronic collision frequency is much more than the collision frequency of the electrons for the energy transfer to lattice, then $f_0(\varepsilon,\vec{r})$ is the Fermi distribution function with an electron temperature T_e. We consider the case, when for long-wavelength (LW) phonons there is a “thermal reservoir” of short-wavelength (SW) phonons: $q_{max} \approx 2\overline{p} << \frac{T}{s_0}$, where s_0 is the sound velocity in the crystal, q_{max} is the maximum quasi-momentum of LW phonons. In this case $N_0(q,\vec{r})$ has the form

$$N_0(q,\vec{r}) \approx \frac{T_p(\vec{r})}{s_0 q}, \qquad (4)$$

where T_p is the effective temperature of the LW phonons. The electron and the phonon temperature gradients ∇T_e and ∇T_p can be produced by the gradient of strong electric field E or by lattice temperature gradient ∇T.

Starting from the Boltzmann transport equations we obtain the following equations for $\vec{f}_1$ and $\vec{N}_1$ in the steady state:

$$\frac{p}{m}\nabla f_0 - e\overrightarrow{E_c}\frac{p}{m}\frac{\partial f_0}{\partial \varepsilon} + \nu(\varepsilon)\vec{f}_1 + \frac{2\pi m}{(2\pi\hbar)^3 p^2}\frac{\partial f_0}{\partial \varepsilon}\int_0^{2p}\overrightarrow{N}_1(q)W(q)\hbar\omega_q q^2 dq = 0, \qquad (5)$$

$$s_0\nabla N_0(q) + \beta(q)\overrightarrow{N}_1(q) - \frac{4\pi m}{(2\pi\hbar)^3}W(q)N_0(q)\int_{q/2}^{\infty}\vec{f}_1 dp = 0. \qquad (6)$$

Here e is the absolute value of the electron's charge, $\vec{E}_c = \vec{E} + \vec{E}_T$, $\vec{E}_T$ is the thermoelectric field, m is the electron's effective mass, $\hbar\omega_q = s_0 q$ is the phonon energy, $W(q) = W_0 q$ is the square matrix element of the electron-phonon interaction, $\beta(q)$ and $\nu(\varepsilon)$ are the total phonon and electron momentum scattering rates, respectively.

Solving the coupled equations (5) - (6) by the same way as in [8] and using the conditions $j_z = 0$ ($\vec{E}\| oy, \nabla T_{e,p}\| oz$) we obtain the following expression for the thermoelectric field E_T:

$$E_T + \frac{1}{e}\nabla_z\zeta(\vartheta_e) = \alpha_e(\vartheta_e)\nabla_z T_e + \alpha_p(\vartheta_e)\nabla_z T_p;\ \alpha_e = -\frac{\beta^e(\vartheta_e)}{\sigma(\vartheta_e)};\ \alpha_p = -\frac{\beta^p(\vartheta_e)}{\sigma(\theta_e)}, \qquad (7)$$

where α_e and α_p are the electron (e) and phonon (p) parts of the thermoelectric power,

$$\sigma(\vartheta_e) = \frac{e^2}{3\pi^2\hbar^3 m}\exp\left[\frac{\zeta(\vartheta_e)}{T\vartheta_e}\right]\int_0^\infty \frac{p^3(x)e^{-x}}{\nu(x)}\left[1 + \frac{\gamma(x)}{(1-\gamma(1))}\frac{\nu(x)}{\nu(1)}\right]dx;\ x = \frac{\varepsilon}{T_e},\ \vartheta_e = \frac{T_e}{T},\ \vartheta_p = \frac{T_p}{T}, \qquad (8)$$

$$\beta^e(\vartheta_e) = \frac{e}{3\pi^2\hbar^3 m}\exp\left[\frac{\zeta(\vartheta_e)}{T\vartheta_e}\right]\int_0^\infty \frac{p^3(x)e^{-x}}{\nu(x)}\left\{x - \frac{\zeta(\vartheta_e)}{T\vartheta_e} + \left[1 - \frac{\zeta(\vartheta_e)}{T\vartheta_e}\right]\frac{\gamma(x)}{(1-\gamma(1))}\frac{\nu(x)}{\nu(1)}\right\}dx, \qquad (9)$$

$$\beta^p(\vartheta_e) = \frac{e}{3\pi^2\hbar^3 m}\exp\left[\frac{\zeta(\vartheta_e)}{T\vartheta_e}\right]\int_0^\infty \frac{p^3(x)e^{-x}}{\nu(x)}\left[\lambda(x) + \lambda(1)\frac{\gamma(x)}{(1-\gamma(1))}\frac{\nu(x)}{\nu(1)}\right]dx. \qquad (10)$$

Here $\zeta(\vartheta_e)$ is the chemical potential of hot electrons, the coefficient $\lambda(x)$ characterizes the efficiency of the thermal drag, whereas coefficient $\gamma(x)$ describes the same for the mutual drag:

$$\lambda(x) = \frac{1}{4p^4}\frac{ms_0^2}{T_p}\nu_p(x)\int_o^{2p}\frac{1}{\beta(q)}q^3 dq, \qquad (11)$$

$$\gamma(x) = \frac{1}{4p^4}\frac{\nu_p(x)}{\nu(x)}\int_o^{2p}\frac{\beta_e(q)}{\beta(q)}q^3 dq, \qquad (12)$$

$\nu_p(x)$ is the electron scattering frequency by phonons, and $\beta_e(q)$ is the phonon scattering frequency by electrons. In the parabolic case the chemical potential of hot non-degenerate electrons with concentration N takes the form:

$$\zeta(\vartheta_e) = T\vartheta_e \ln \frac{4\pi^{\frac{3}{2}}\hbar^3 N}{(2mT)^{3/2}} \vartheta_e^{-\frac{3}{2}}. \quad (13)$$

Under the conditions of strong electron-phonon mutual drag, i.e. when the electrons and phonons are scattered mainly by each other, as it seen from (12) $\gamma(x) \to 1$. In these conditions for the electron and phonon parts of thermoelectric power we obtain from (7)-(12):

$$\alpha_e = -\frac{1}{e}\left[1 + \frac{3}{2}\ln \frac{2mT}{\pi\left(4\hbar^3 N\right)^{3/2}} \vartheta_e\right], \quad (14)$$

$$\alpha_p = -\frac{1}{e}\frac{4\sqrt{2}}{3\pi^{\frac{3}{2}}}\frac{(mT)^{\frac{3}{2}}}{\hbar^3 N}\vartheta_e^{\frac{3}{2}}. \quad (15)$$

As it follows from (13) for the non-degenerate electrons $\frac{(mT)^{\frac{3}{2}}}{\hbar^3 N} \approx \exp\left[-\frac{\varsigma(T)}{T}\right] >> 1$, and from the comparison (14) with (15) it is seen that under the conditions of strong mutual drag $|\alpha_p| >> |\alpha_e|$, i.e. the thermopower mainly consists of the phonon part.

Under the conditions of the strong mutual drag ($\vartheta_e = \vartheta_p, \gamma_0 \to 1$) the electron temperature is determined from the energy balance equation

$$\sigma(\vartheta_e)E^2 = W_{pp}(\vartheta_e), \quad (16)$$

where $W_{pp}(\vartheta_e)$ is the power transferred by the long-wavelength phonons to the thermal reservoir of the short-wavelength phonons. Under the conditions of strong mutual drag according to (8), $\sigma(\vartheta_e) \sim (1-\gamma)^{-1}$, i.e. it grows with raising γ and tends to infinity at $\gamma \to 1$. However, though the electrons and phonons are scattered mainly by each other, they are partly scattered by some other scattering centre as well. Thus to calculate γ from (12) one should take into account as well some other non-basic mechanisms of electron scattering (by impurity ions ν_i) and phonon scattering (by phonons β_p and crystal boundaries β_b). We consider the following limiting cases:

1) The relaxation channel of the electron momentum by impurities becomes wider than that of the phonon momentum scattering by the crystal boundaries or by the phonons ($\frac{\beta_p + \beta_b}{\beta_e} << \frac{\nu_i}{\nu_p}$). In this case the calculation of the expression $\vartheta_e(E)$ from (16) at $\vartheta_p = \vartheta_e >> 1$ gives $\vartheta_e \sim E^{\frac{4}{3}}$.

2) The phonons are scattered by phonons or by crystal boundaries more intensively than electrons are scattered by impurities ($\frac{\beta_p + \beta_b}{\beta_e} >> \frac{\nu_i}{\nu_p}$). In this case from (16) at $\vartheta_p = \vartheta_e >> 1$ we obtain:

2a) $\vartheta_e \sim E^{\frac{1}{3}}$, if $\beta_p >> \beta_b$;

2b) $\vartheta_e \sim E^{\frac{4}{11}}$, if $\beta_p << \beta_b$.

Using these expressions one can easily obtain from (14) and (15) the dependences of thermopower upon the heated electric field in the considered cases.

Experimentally interesting is the total thermopower given by

$$V = \int_0^{L_z} \left(\alpha_e \nabla_z T_e + \alpha_p \nabla_z T_p\right) dz \,, \tag{I7}$$

where L_z is the linear dimension of the specimen in the z-direction. Let us consider dependences of V on the heating electric field and lattice temperature under the following conditions: at one end of the specimen the electrons are in a state characterized by the lattice temperature T, whereas at the other end they are heated $(\vartheta_e > 1)$ by the external electric field (for instance, by placing one end of the specimen into a waveguide with a heating microwave radiation).

In the region of weak-heated $(\vartheta_e - 1 << 1)$ electric field $\vartheta_e = 1 + \left(\frac{E}{E_{ch}}\right)^2$, where E_{ch} is the characteristic field. In this region the total thermopower V is proportional to E_0^2, with E_0 being the heating electric field intensity at the end of the specimen where electrons are heated. In the region of strongly-heated $(\vartheta_e >> 1)$ electric field V is proportional to $E_0^{\frac{10}{3}}$ in the case 1), $V \sim E_0^{\frac{5}{6}}$ in the case 2a), and $V \sim E_0^{\frac{10}{11}}$ in the case 3b).

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GÜCLÜ ELEKTRON-FONON QARŞILIQLI SÖVQÜ ŞƏRAİTİNDƏ QIZDIRICI ELEKTRİK SAHƏSİNDƏKİ CIRLAŞMAMIŞ YARIMKEÇİRİCİLƏRDƏ TERMO-EHQ

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Elektron və fononların elektrik sahəsində qızmasını və qarşılıqlı sövqünü nəzərə almaqla cırlaşmamış yarımkeçiricilərdə termo-ehq tədqiq edilmişdir. Elektron və fononların əsasən bir-birindən səpildiyi güclü qarşılıqlı sövq şəraitinə baxılmışdır. Göstərilmişdir ki, elektronların ikinci səpilmə mexanizmi olan aşqar ionlardan səpilmə kanalı fononların impulsunun kristalın sərhədindən və fononlardan səpilmə kanalından daha geniş olduqda tam termo-ehq (V) qızdırıcı elektrik sahəsinin artması ilə kəskin artır: $V \sim E^{10/3}$..

ТЕРМО-ЭДС НЕВЫРОЖДЕННЫХ ПОЛУПРОВОДНИКОВ В СИЛЬНОМ ЭЛЕКТРИЧЕСКОМ ПОЛЕ В УСЛОВИЯХ СИЛЬНОГО ВЗАИМНОГО УВЛЕЧЕНИЯ ЭЛЕКТРОНОВ И ФОНОНОВ

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Исследована термо-эдс в невырожденных полупроводниках с учетом разогрева электронов и фононов и их взаимного увлечения. Рассмотрено условие сильного взаимного увлечения, когда электроны рассеиваются на фононах, а фононы - на электронах. Показано, что если второй (дополнительный) канал рассеяния электронов (рассеяние на ионах примеси) шире, чем канал рассеяния фононов на фононах и на границах кристалла, то полная термо-эдс (V) сильно растет с ростом греющего электрического поля: $V \sim E^{10/3}$.

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