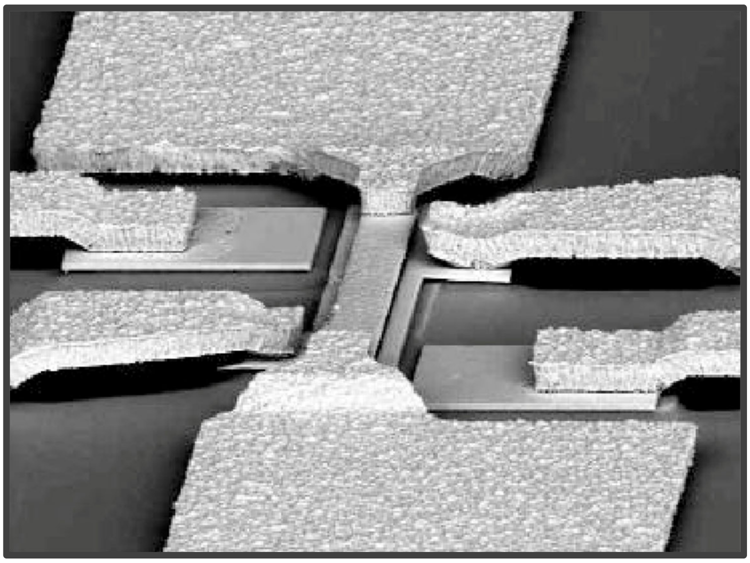
Selected problems in semiconductor physics and electronic devices



Ramon Alcubilla, Cristóbal Voz
Universitat Politecnica de Catalunya

PREFACE

This book presents a set of solved examples on semiconductor device physics. Semiconductor devices is a core subject in electrical engineering and physics curricula. The level of the proposed examples corresponds to a semester course at senior undergraduate or junior graduate level. Readers are expected to have a basic background on quantum and solid state physics, moreover a reasonable mathematical knowledge reaching differential equations is also assumed.

There are many excellent text books on semiconductor device physics, however very often the examples are mostly numerical, trying to fix the order of magnitude of the obtained results. In this book, problems with a certain level of complexity are solved and explained step by step presenting at the same time the involved physics. This work does not replace a text book and lecture notes, but it may definitely be a good complement.

At the Universitat Politècnica de Catalunya (BarcelonaTech) one of the authors, R. Alcubilla, has been lecturing on semiconductor devices at different levels for more than 20 years. In particular together with C. Voz started in 2013 a semester course for engineering physics students at senior undergraduate level. This book collects the exercises of the different written tests since 2013. The level of the students has been always rather high and the acceptation of the course has been also very good. Even if teachers tend to overestimate the motivation of the students, our feeling has been always very positive and the average result of the proposed tests good.

We cannot claim the authorship of all the proposed problems. As most professors know, the process for preparing a test is not easy, trying to question about the different parts of the course, equilibrating the difficulties, looking for interesting questions (the definition of interesting questions is subject of discussion between teacher and students). Some of the problems are roughly original, classical ones or fruit of the previous experience of the authors, others are adapted from material found in different books, web sites, etc…

R. Alcubilla wants to thank Universitat Politècnica de Catalunya (BarcelonaTech) for a 6 months sabbatical leave used for writing the biggest part of this book. Finally, we want to thank our students that during these years have "suffered" these exercises.

Barcelona, January 2020

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Chapter 1.

Semiconductor Physics

1. ***Radiative, band-to-band, recombination is proportional to the product of electron and hole concentrations. Consider a p-type semiconductor with an impurity concentration N_A :***

   a) Find an expression for its radiative lifetime under low-injection conditions.

   b) Find an expression for its radiative lifetime under high-injection conditions. Comment the results you obtain.

a) The total amount of recombination events should be proportional to both electron and hole concentrations. Furthermore, it should tend to zero along with any of the carrier concentrations. Thus, the number of electron-hole pairs that recombine per unit time and unit volume R ($cm^{-3}s^{-1}$) can be written as:

$$R = B\, n \cdot p$$

where B is a proportionality factor in units of cm^3s^{-1} and n and p are the electron and hole concentrations, respectively. We can write $n = n_0 + \Delta n$ and $p = p_0 + \Delta p$ where n_0 and p_0 are the equilibrium concentrations and Δn and Δp the differences from their actual concentration values. Thus, we call Δn and Δp the excess electron and hole concentrations. Of course, these excesses are null under equilibrium. Then:

$$R = B\,(n_0 + \Delta n)\,(p_0 + \Delta p) = Bn_0\,p_0 + Bn_0\Delta p + Bp_0\Delta n + B\Delta n\Delta p$$

Let's now proceed with the kind of tricks that students usually hate. Be careful however because there is usually a lot of physics inside those tricks:

i. The first term is the recombination rate under equilibrium, i.e., when both excesses are null. If there is some recombination in equilibrium but the concentrations remain constant, then there must be an equal generation rate. Consequently, it is better to think about the net recombination rate U ($cm^{-3}s^{-1}$). That is the recombination not compensated by thermal generation:

$$U = Bn_0\Delta p + Bp_0\Delta n + B\Delta n\Delta p$$

ii. If we assume a $p\text{-}type$ semiconductor then $p_o \gg n_o$ ($n_o \gg p_o$ in case of $n\text{-}type$)

iii. Additionally, $\Delta n \approx \Delta p$ due to quasineutrality. We start considering a semiconductor neutral in equilibrium, this means that positive and negative charges equilibrate for a zero-net charge. The hypothesis of quasineutrality means that even if we move away from equilibrium, the net charge will still be very small compared to the amount of positive and negative charges. More details will be given in the next problem.

iv. Finally, we can talk about low or high injection conditions depending on the magnitude of the excess carrier concentration. The most usual case is low injection regime, which means that excesses are much lower than the majority carrier concentration. On the contrary, in high injection the excesses are much higher than the majority carrier concentration.

Summarizing, taking (iii) into account the net recombination rate is given by:

$$U = B\,n_0\Delta n + Bp_0\Delta n + B\Delta n\Delta n$$

Particularizing for a $p\text{-}type$ semiconductor (ii):

$$U = Bp_0\Delta n + B\Delta n\Delta n$$

Then, in low injection (iv):

$$U = Bp_0\Delta n$$

We can now rewrite this expression in another way:

$$U = Bp_0\Delta n = \Delta n/\tau_n \text{ , where we identify } \tau_n = \frac{1}{Bp_0}$$

We call τ_n the electron (minority carrier) lifetime, which can be interpreted as the mean time for electrons to recombine (symmetrically for holes in an $n\text{-}type$ semiconductor). Note that the lifetime is inversely proportional to the doping concentration. Minority carriers will recombine faster as there are more majority carriers available to complete the process.

b) Under high injection everything is the same but with $\Delta n\Delta p \gg p_0\Delta n$ we arrive to:

$$U = B\Delta n\Delta n = \Delta n/\tau_n \text{ , where } \tau_n = \frac{1}{B\Delta n}$$

Now the electron lifetime depends on the injection level and it cannot be taken as a constant parameter. The lifetime decreases, that is recombination increases, with the injection level.

2. ***Analyzing the behavior of semiconductors under permanent regime, the hypothesis of quasi-neutrality is often invoked. Consider a uniform region of a semiconductor with negligible recombination where somehow quasi-neutrality is perturbed. Then, a volumetric density of charge ρ would appear and consequently an electric field. The drift current produced by this electric field will re-establish the neutrality in a brief lapse of time. This relaxation process, nearly instantaneous in a doped semiconductor has associated a characteristic time t_d.***

   Using well-known equations, find the expression for the temporal evolution of the volumetric charge density during the relaxation process. Identify the characteristic time t_d and evaluate its value for a silicon sample of conductivity $\sigma = 1\ \Omega^{-1} \cdot cm^{-1}$

   Data: $\varepsilon_r = 11.9$, $\varepsilon_o = 8.85 \times 10^{-14}\ F/cm$

   Imagine that somehow you are injecting electrons from the left side of a semiconductor. Consequently, you should have an excess of electrons $\Delta n(x)$ over the equilibrium concentration decreasing from left to right. You can find in textbooks that because of quasi-neutrality there will exist also a hole excess $\Delta p(x) = \Delta n(x)$. You can take it for granted if necessary, but to my view the reason has been rather a mystery for a long time. Finally, I understood why it is the case. I will try to explain it in this problem.

   Let's consider the electron concentration. For simplicity, consider that there is not any generation and that recombination can be neglected in that region. Then, the continuity equation writes:

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx}$$

   where n is the electron concentration (cm^{-3}), J_n the electron current density (A/cm^2) and q the elementary charge (C).

   We also have $J_n = \sigma E$, being σ the electrical conductivity (Ω^{-1}·cm^{-1}) and E the electric field (V/cm).

   Besides, there is the Gauss equation:

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$$

   with ρ the charge density (C/cm^{-3}) and ε the dielectric constant (F/cm).

   Combining these equations, we arrive to:

$$\frac{dn}{dt} = -\frac{1}{q}\frac{d\rho}{dt} = \frac{1}{q}\frac{dJ_n}{dx} = \frac{1}{q}\sigma\frac{dE}{dx} = \frac{1}{q}\frac{\sigma}{\varepsilon}\rho$$

   Then:

$$\frac{d\rho}{dt} = -\frac{\sigma}{\varepsilon}\rho$$

and this differential equation can be solved to obtain the temporal evolution of the charge density. Considering that at $t = 0$ the charge density is $\rho(0)$ (initial condition):

$$\rho(t) = \rho(0) exp\left(-\frac{\sigma}{\varepsilon}t\right)$$

the ratio $\frac{\varepsilon}{\sigma}$ can be identified as a relaxation time t_d and write:

$$\rho(t) = \rho(0) exp\left(-\frac{t}{t_d}\right)$$

The value of t_d for silicon with a rather typical resistivity of 1 Ω·cm is:

$$t_d = \frac{\varepsilon}{\sigma} = \frac{8.85 \times 10^{-14} \times 11.9}{1} \approx 10^{-12} s$$

This time is short (very short) compared to the usual carrier lifetimes in silicon. That is the reason why we can consider that any disequilibrium in the charge density comes back to zero nearly instantaneously. In other words, if any reason (usually injection) causes an excess carrier concentration (either Δn or Δp) at a given point x at time t_1, shortly after (in practice we assume instantaneously) the semiconductor re-arranges itself for having equal excesses at this point. In other words, an unbalanced charge produces an electric field, which is the origin of a drift current that re-arranges the carrier distribution cancelling the charge.

It's important to be aware that in low conductivity semiconductors (amorphous or organic materials where the conductivity may be many orders of magnitude lower than in inorganic crystalline semiconductors) this is no longer the case. The relaxation time may be comparable to the carrier lifetime and quasi-neutrality would be questionable or simply no longer valid.

3. ***A silicon sample is doped with donor impurities, phosphorous, whose energy level is located $\boldsymbol{45\ meV}$ below the conduction band, the concentration of impurity atoms is $\boldsymbol{N_D = 10^{15}\ cm^{-3}}$.***

   a) Discuss qualitatively the evolution of the Fermi level from $\boldsymbol{0\ K}$ until high temperatures above $\boldsymbol{600\ K}$. Sketch also qualitatively the Fermi level position and the fraction of ionized impurities in such a wide temperature range.

   b) Calculate the temperature for a 50% ionization of the phosphorous impurities.

   Hint: Solve first for the room temperature effective density of states in the conduction band $\boldsymbol{N_C(300\ K)}$. Then, iterate for a more accurate result.

   Data: $\boldsymbol{k = 8.62 \times 10^{-5}\ eV/K}$, $\boldsymbol{N_C(300\ K) = 2.86 \times 10^{19}\ cm^{-3}}$,

   $$\boldsymbol{N_C(T) = N_C(300\ K)\left(\frac{T}{300\ K}\right)^{3/2}}$$

a) Since at $0\ K$ the lowest energy states are occupied, electrons will stay at the impurity level (0% ionization) and the conduction band empty. Then, E_F will be located above the impurity level and below the conduction band edge E_C. An increase in the temperature starts the ionization of the impurities and E_F will shift towards the impurity level. Eventually, when E_F crosses the impurity level they become 50% ionized. As the temperature is increased the fraction of ionized impurities rapidly goes to 100% with E_F already below the impurity level. At even higher temperatures E_F will continue moving down towards mid gap.

b) There is a temperature T_x for a 50% ionization of the impurities. It is expected to be a relatively low temperature. Thus, the Boltzmann approximation should apply:

$$\frac{N_D}{2} = N_C(T_x)exp\left(-\frac{E_C - E_F}{kT_x}\right)$$

As it has been discussed before, this happens when E_F crosses the impurity level.
Then, $E_F = E_D$ and we can write:

$$\frac{N_D}{2} = N_C(T_x)exp\left(-\frac{E_C - E_D}{kT_x}\right)$$

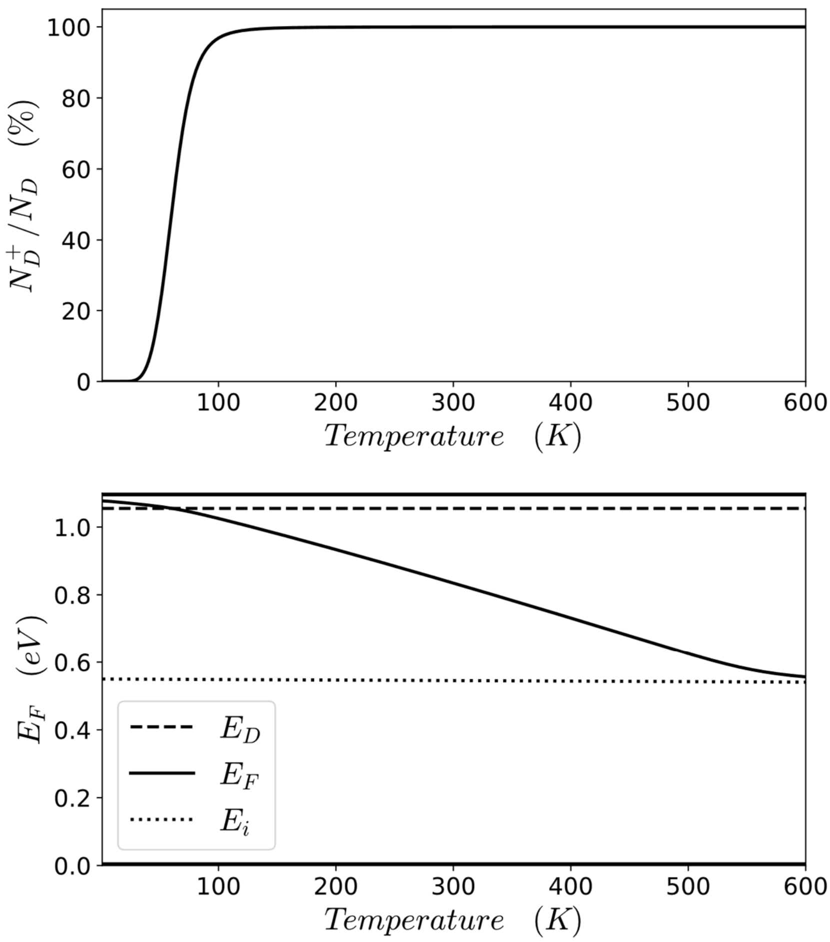
which leads to:

$$T_x = \frac{E_C - E_D}{k\ ln\left[\frac{N_C(T_x)}{N_D/2}\right]} = \frac{E_C - E_D}{k\ ln\left[\frac{N_C(300)}{N_D/2}\left(\frac{T_x}{300}\right)^{3/2}\right]}$$

Now we start to iterate at 300 K obtaining a first value of $T_x = 47\ K$. A few more iterations rapidly converge to the final solution:

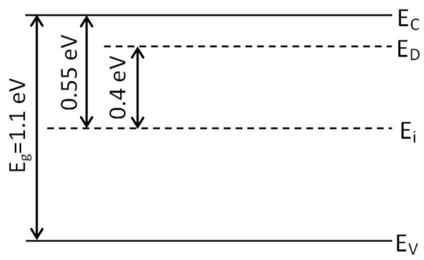
$$T_x = 47\ K \rightarrow 63\ K \rightarrow 60\ K \rightarrow 61\ K \rightarrow 61\ K$$

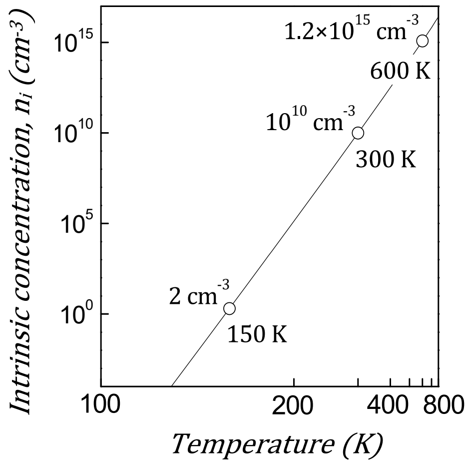
Then, $T_x = 61\ K$ is the temperature for a 50% ionization of the impurities.



100
80
60
40
20
0
$N_D^+ / N_D \quad (\%)$
100 200 300 400 500 600
$Temperature \quad (K)$
1.0
0.8
0.6
0.4
0.2
0.0
$E_F \quad (eV)$
E_D
E_F
E_i
100 200 300 400 500 600
$Temperature \quad (K)$

4. ***A Silicon sample is doped by donor impurities with a concentration $N_D = 10^{15}\ cm^{-3}$. The energy level E_D of the donor impurity is located $0.4\ eV$ above the intrinsic Fermi level E_i. Suppose that neither the silicon bandgap neither the energy levels shown in the figure vary noticeably with temperature. Additionally, the figure below shows the intrinsic carrier concentration n_i of silicon as a function of the temperature.***





   a) ***Calculate the position of the Fermi level E_F referred to E_i at $300\ K$. You can do those approximations that you find reasonable.***

   b) ***Repeat this calculation for a temperature of $600\ K$. Discuss the main differences with the case of room temperature.***

   c) ***We know that at $150\ K$ the Fermi level is located at $E_F - E_i = 0.4173\ eV$. Calculate the corresponding concentrations of electrons and holes and the fraction of ionized impurities.***

   d) ***Write down the transcendental equation that would allow to calculate numerically the location of E_F at any temperature.***

   Data: $k_B = 8.62 \times 10^{-5}\ eV/K$

a) At 300 K we can consider that all the impurities are already ionized, $N_D^+ \approx N_D$. Besides, it is clear that $n_i \ll N_D$ and $p \ll n$. Then:

$$n \approx N_D = n_i exp\left(\frac{E_F - E_i}{kT}\right) \Longrightarrow E_F - E_i = kT\ ln\frac{N_D}{n_i} = 0.2977eV$$

b) At 600 K we know from the plot that $n_i = 1.2 \times 10^{15}\ cm^{-3}$ and for sure $N_D^+ = N_D$.

We write the neutrality equation, i.e., total negative charges equal to total positive charges:

$$n = p + N_D^+ = p + N_D = \frac{n_i^2}{n} + N_D$$

$$n^2 - N_D n - n_i^2 = 0$$

$$n = \frac{N_D \pm \sqrt{N_D^2 + 4n_i^2}}{2} = 1.8 \times 10^{15} cm^{-3}$$

The negative value has no physical meaning.

c) At $150\ K$ we know from the plot the value of n_i, whereas $E_F - E_i$ is also known. Then, we can calculate the carrier concentrations:

$$n = n_i exp\left(\frac{E_F - E_i}{kT}\right) = 2.07 \times 10^{14}\ cm^{-3}\ ,\ p = \frac{n_i^2}{n} = 1.93 \times 10^{-14}\ cm^{-3} \approx 0$$

$$N_D^+ = N_D \frac{1}{1 + exp\left(-\frac{E_D - E_F}{kT}\right)}$$

$$E_D - E_F = (E_D - E_i) - (E_F - E_i) = 0.4 - 0.4173 = -0.0173\ eV$$

Note that $N_D^+ = n = 2.07 \times 10^{14}\ cm^{-3}$. Thus, the percentage of ionized impurities is 20.7 %.

d) According to the condition of charge neutrality:

$$p + N_D^+ = n$$

On the other hand, we have:

$n = n_i exp\left(\frac{E_F - E_i}{kT}\right)$ and $p = n_i exp\left(\frac{E_i - E_F}{kT}\right)$

and the concentration of ionized impurities is given by:

$$N_D^+ = N_D[1 - f(E_D)]$$

$$N_D^+ = N_D \left[1 - \frac{1}{1 + exp\left(\frac{E_D - E_F}{kT}\right)}\right] = N_D \frac{1}{1 + exp\left(-\frac{E_D - E_F}{kT}\right)}$$

We can now re-write the charge neutrality equation as:

$n_i \frac{1}{x} + N_D \frac{1}{1 + \frac{x}{exp\left(\frac{E_D - E_i}{kT}\right)}} = n_i x$, where $x = exp\left(\frac{E_F - E_i}{kT}\right)$

This last equation can be solved for x to determine $E_F - E_i$ at any arbitrary temperature.

5. ***The Fermi level $\boldsymbol{E_F}$ determines the electron and hole concentrations of a semiconductor in equilibrium. Out of equilibrium we can define the quasi-Fermi levels $\boldsymbol{E_{Fn}}$ and $\boldsymbol{E_{Fp}}$ to calculate the electron and hole concentrations, respectively. For example, the electron concentration would be:***

$$\boldsymbol{n = N_C exp\left(-\frac{E_C - E_{Fn}}{kT}\right)}$$

Then, prove that the total electron current can be written as:

$$\boldsymbol{J_n = \mu_n n \frac{dE_{Fn}}{dx}} \qquad \textbf{\textit{(1)}}$$

The electron current is usually written as the addition of its drift and diffusion terms:

$$J_n = qn\mu_n E + qD_n \frac{dn}{dx}$$

With q the elementary charge, μ_n the electron mobility ($cm^2 \cdot V^{-1} \cdot s^{-1}$), E the electric field (V/cm) and D_n the electron diffusion constant (cm^2/s)

By introducing the electron quasi-Fermi level (1), we can write the derivative $\frac{dn}{dx}$ as:

$$\frac{dn}{dx} = N_C\left(-\frac{1}{kT}\right)\left(\frac{dE_C}{dx} - \frac{dE_{Fn}}{dx}\right)\exp\left(-\frac{E_C - E_{Fn}}{kT}\right)$$

$$\frac{dn}{dx} = -\frac{n}{kT}\left(-q\frac{dV}{dx} - \frac{dE_{Fn}}{dx}\right) = -\frac{n}{kT}\left(qE - \frac{dE_{Fn}}{dx}\right)$$

where we have used that $dE_C = -q\,dV$ (the band energy changes with the electrostatic potential but the electron charge is $-q$). We have also used that $E = -\frac{dV}{dx}$
Now, we use this last expression of $\frac{dn}{dx}$ in the equation of the total current density:

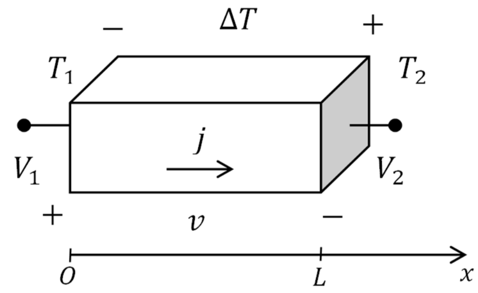
$$J_n = qn\mu_n E - qD_n \frac{n}{kT}\left(qE - \frac{dE_{Fn}}{dx}\right)$$

and considering the Einstein relation $D_n = \frac{kT}{q}\mu_n$, we finally arrive to:

$$J_n = \mu_n n \frac{dE_{Fn}}{dx}$$

6. Between the extremes 1 and 2 of a *p*-type semiconductor there is a temperature difference $\Delta T = T_2 - T_1 > 0$. In this situation, if the terminals 1 and 2 are in short-circuit we measure a current j_{sc} in the negative direction of x. This current is due to a net flow of majority carriers from the hot side towards the cold side. In the open-circuit condition, a voltage difference $v_{oc} = V_1 - V_2 > 0$ is also measured.

a) Explain the reason of this behavior, which is known as Seebeck effect. Under exactly the same circumstances, what would be different about the measured j_{sc} and v_{oc} values if the semiconductor were n type?



The Seebeck effect can be described by the following equation:

$$j = \sigma\left(E - S\frac{dT}{dx}\right) \qquad (1)$$

where σ is the electrical conductivity of the sample, E the electric field and S is called the Seebeck coefficient.

b) Integrate the equation (1) between both extremes of the semiconductor, assuming a uniform electric field, to obtain the j-v characteristic of this thermoelectric device.

c) How would you determine the value of S with the sample in open-circuit conditions?

From now on consider that the semiconductor is a p-type silicon sample of length $L = 200\ \mu m$. Its electrical conductivity is $\sigma = 0.2\ \Omega^{-1}\cdot cm^{-1}$ and the Seebeck coefficient $S = 400\ \mu V/K$.

d) Calculate the j_{sc} and v_{oc} values for a temperature difference $\Delta T = 50\ K$.

e) Calculate the maximum electrical power that this device (called thermopile) may deliver assuming a unit area.

a) The thermal velocity of the charge carriers in 2 is greater than in 1. Consequently, in the short-circuit condition there is a net flow of holes (majority carriers) from 2 to 1. This is observed as a net current j_{sc} in the negative direction of the x axis. In open-circuit an electric field from 1 to 2 appears to stop the flow of holes and cancel this current. This electric field leads to a positive voltage difference between the two extremes of the sample, $v_{oc} = V_1 - V_2 > 0$.

If the semiconductor were n-type, the measured j_{sc} current would be positive. This current would be explained by a net flow of electrons in short-circuit from 2 to 1. The voltage in open-circuit would be negative, $v_{oc} = V_1 - V_2 < 0$. Now, an electric field from 2 to 1 is needed to stop the electron flow.

b) The Seebeck effect may be described by:

$$j = \sigma\left(E - S\frac{dT}{dx}\right) = \sigma\left(-\frac{dV}{dx} - S\frac{dT}{dx}\right)$$

We can integrate this equation assuming a uniform electric field:

$$j\,dx = -\sigma\,dV - \sigma\,S\,dT$$

$$\int_1^2 j\,dx = -\int_1^2 \sigma\,dV - \int_1^2 \sigma\,S\,dT \implies jL = \sigma(V_1 - V_2) - \sigma(T_2 - T_1)$$

$$j = \frac{\sigma}{L}v - \frac{\sigma}{L}S\Delta T$$

Then, for a given ΔT the j-v characteristic is a straight line not intersecting the origin.

c) In the open-circuit condition $j = 0$. Thus:

$$0 = \frac{\sigma}{L}v_{oc} - \frac{\sigma}{L}S\Delta T \implies S = \frac{v_{oc}}{\Delta T}$$

Then, by measuring the v_{oc} value you can easily calculate the Seebeck coefficient if you already know the temperature difference between both extremes of the sample.

d) If $\Delta T = 50\ K$ and taking $S = 400\ \mu V/K$:

$$v_{oc} = S\,\Delta T = 0.02\ V$$

j_{sc} is the short-circuit current that flows through the device for $v = 0$:

$$j_{sc} = -\frac{\sigma}{L}S\Delta T = -0.2\ A/cm^2$$

e) The electrical power P is calculated as:

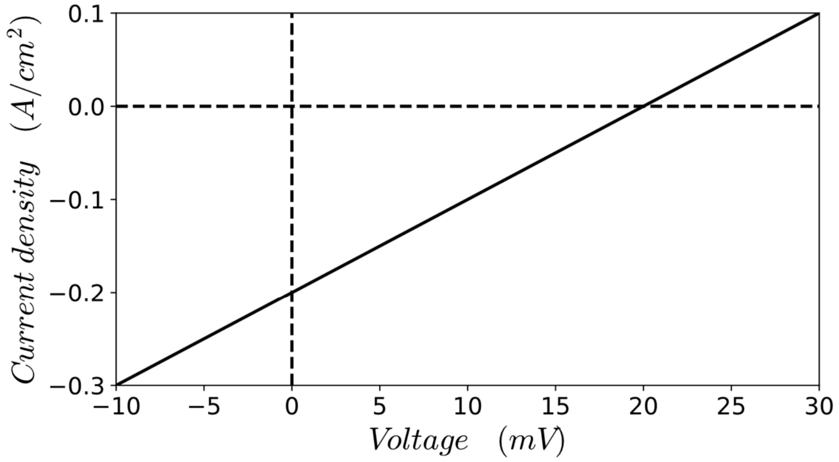
$$P = jv = \frac{\sigma}{L}v^2 - \frac{\sigma}{L}S\Delta T v$$

where P takes a negative value, meaning that the device is delivering power to a hypothetical external load. We can now look for the maximum power point:

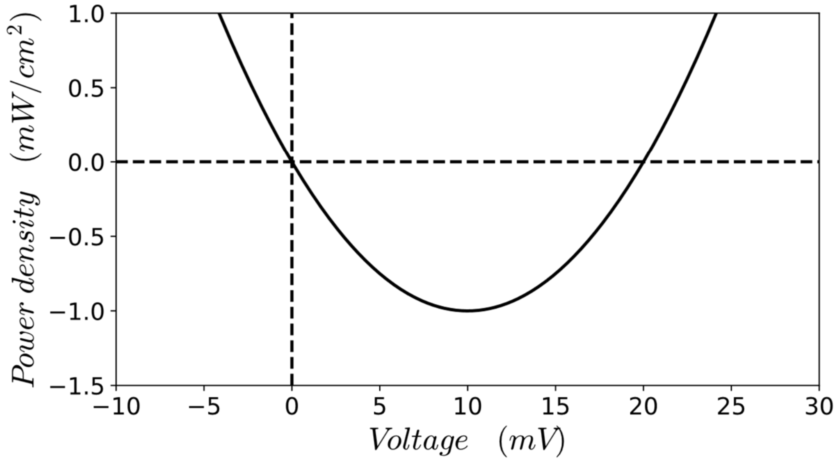
$$\frac{dP}{dV} = 0 \implies v = 0.01\ V\ ,\ j = -0.1\,A/cm^2$$

$$P_{max} = 1\ mW/cm^2$$

It has been shown that some electrical power can be obtained from a temperature difference by the Seebeck effect. This is a very active research field, where nanostructured materials may be adequate to obtain high values of the Seebeck coefficient. Note that for a high conversion efficiency the electrical conductivity must be also good. On the other hand, to maintain the temperature difference between both extremes of the device a low thermal conductivity (κ) is preferred. In fact, a figure of merit $Z = S^2\sigma/\kappa$ has been defined for these thermoelectric devices. Materials combining good electrical conductivity with low thermal conductivity are quite unusual, which makes this topic an exciting research field in materials science.

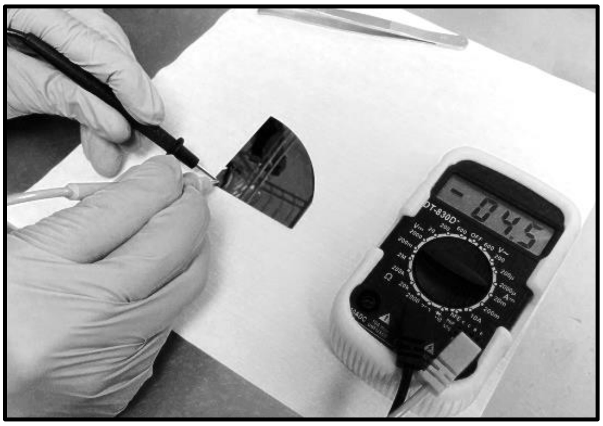


0.1
0.0
−0.1
−0.2
−0.3
Current density (A/cm^2)
−10
−5
0
5
10
15
20
25
30
Voltage (mV)



1.0
0.5
0.0
−0.5
−1.0
−1.5
Power density (mW/cm^2)
−10
−5
0
5
10
15
20
25
30
Voltage (mV)

7. ***We take a multimeter measuring in the voltmeter DC mode. The red probe is heated externally (with a lighter), while the black probe remains at room temperature. Then, we contact on a silicon sample with the two probes close to each other (as shown in the figure). The heat transfer establishes a temperature gradient in the semiconductor. The reading on the voltmeter (voltage at the red probe minus voltage at the black probe) is negative. With this information, could you determine if the sample is p or n?***



Let's consider first an n-type semiconductor. Because of the temperature gradient there will be a net flow of the hotter electrons towers the cold side, i.e., from near the red probe towards the black one. In open-circuit (voltmeter DC mode) an electric field appears to cancel this current, which is oriented from the red probe towards the black one. Therefore, as the multimeter measures the voltage at the red probe minus the voltage at the black probe, the reading would be positive. However, the image shows a negative value and we conclude that the semiconductor is actually p-type.

8. ***Consider a n-type semiconductor in equilibrium with a doping profile given by $N_D(x) = N_o exp(-x^2/L^2)$.***

a) Find the expression of the electric field E in the semiconductor as a function of x.

b) Find also the expression of the charge density ρ.

The general situation can be explained as follows. We have a doping profile that decreases towards the right side (positive direction of the x axis). Intuitively, the electron concentration will also decrease in the x direction. This profile results in a net flow of electrons towards the right side (diffusion current in the opposite direction). Since in equilibrium the current must be null (both for electrons and holes), there must be a drift current of electrons to compensate its diffusion term. If a drift current appears it is because of an electric field E. This will result from a charge density ρ that can be calculated by the Gauss theorem.
We can proceed now with the equations:

a) If the temperature is not very low all the impurities are ionized, $N_D^+(x) = N_D(x)$. Besides, if it is not very high the hole concentration can be neglected. Then, at usual operation temperatures $qN_D^+(x)$ and $qn(x)$ are respectively the positive and negative charge densities. As a first approach we can assume $n(x) = N_D(x)$, which implies a zero-charge density. On the other hand, the total electron current in equilibrium must be zero:

$$J_n = qn\mu_n E + qD_n \frac{dn}{dx} = 0$$

Now, we use that $n(x) = N_D(x)$:

$$qN_o \exp\left(-\frac{x^2}{L^2}\right)\mu_n E - qD_n N_o \frac{2x}{L^2}\exp\left(-\frac{x^2}{L^2}\right) = 0$$

and using the Einstein relation $D_n = \frac{kT}{q}\mu_n$, we arrive to:

$$E = 2\frac{kT}{q}\frac{x}{L^2}$$

b) According to the Gauss law:

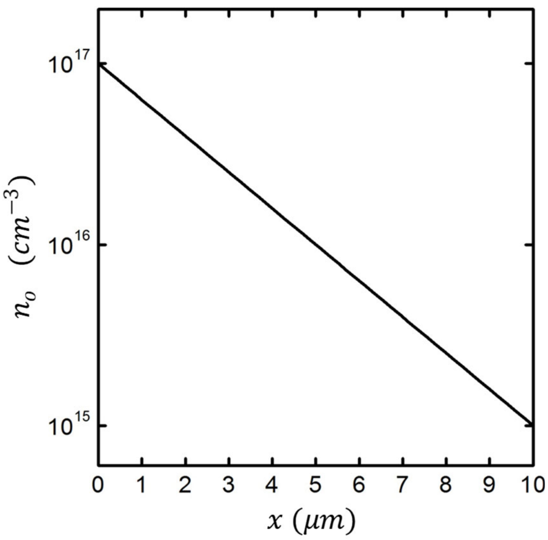
$$\frac{dE}{dx} = \frac{\rho}{\varepsilon} = \frac{\rho}{\varepsilon_r \varepsilon_0}$$

$$\rho(x) = \varepsilon_r \varepsilon_0 2\frac{kT}{q}\frac{1}{L^2}$$

We have finally obtained that a small charge density actually exists instead of a zero charge density assumed above. This charge density could be used to calculate a more accurate second order solution. Nevertheless, for usual values of N_o and L this correction is very small. Consequently, in most cases the first approximation becomes a very accurate solution. This means that quasi-neutrality can be generally assumed.
If the doping profile changes very abruptly, this could not be the case. Then, a numerical or iterative solution could be more precise.

9. Consider a n-type silicon sample under thermodynamical equilibrium at room temperature. In a diffused region $(0 \leq x \leq 10\ \mu m)$ with a non-uniform doping, the electron concentration is given by:

$$\boldsymbol{n_o(x) = 10^{17} \times 10^{-2000\,x}}\ \textit{\textbf{(cm}}^{-3}\textit{\textbf{)}},\quad \textit{\textbf{(x in cm)}}$$



a) Obtain an analytical expression for the hole concentration in the same region. Plot this concentration profile and give the numerical values at each extreme.

b) Calculate the electric field. Discuss the result and its sign.

c) Obtain the expression of the electrostatic potential in the same region. Plot it taking the reference at $x = 0$. Calculate the potential difference between both extremes of the diffused region.

d) Finally, obtain the charge density in the diffused region. Can this region be considered neutral? Would it be neutral independently of the doping profile?

Data: $\boldsymbol{V_T = 25\ mV, n_i = 10^{10}\ cm^{-3}, \varepsilon_{r_{Si}} = 11.9, \varepsilon_o = 8.85 \times 10^{-14}\ F/cm}$

a) Since the semiconductor is in equilibrium, $n(x)\,p(x) = n_i^2$. Then:

$$p(x) = \frac{n_i^2}{n(x)} = 10^3 \cdot 10^{2000\,x}, \quad (x \text{ in cm})$$

$$p(0) = 10^3\ cm^{-3},\ p(10\ \mu m) = 10^5\ cm^{-3}$$

b) Besides, in equilibrium $j_n = qn\mu_n E + qD_n \frac{dn}{dx} = 0$. Thus:

$$E(x) = -\frac{D_n}{\mu_n}\frac{\frac{dn}{dx}}{n} = -V_T\,(-2000)\,ln(10) = 115\ V/cm$$

The decreasing profile in the electron concentration causes a net flow of electrons to the right (diffusion current to the left). Then, there must be an electric field pulling the electrons to the left in order to cancel the diffusion term (drift current to the right). Yes, indeed the electric field is positive in the direction of the x axis.

c) We calculate the voltage difference between both extremes of the doped region:

$$E(x) = -\frac{dV}{dx} \Longrightarrow dV = -E(x)dx$$

In our case the electric field is a constant:

$$\int_0^x dV = V(x) - V(0) = -\int_0^x E\, dx = -E\, x$$

Now, taking the potential at $x = 0$ as a reference:

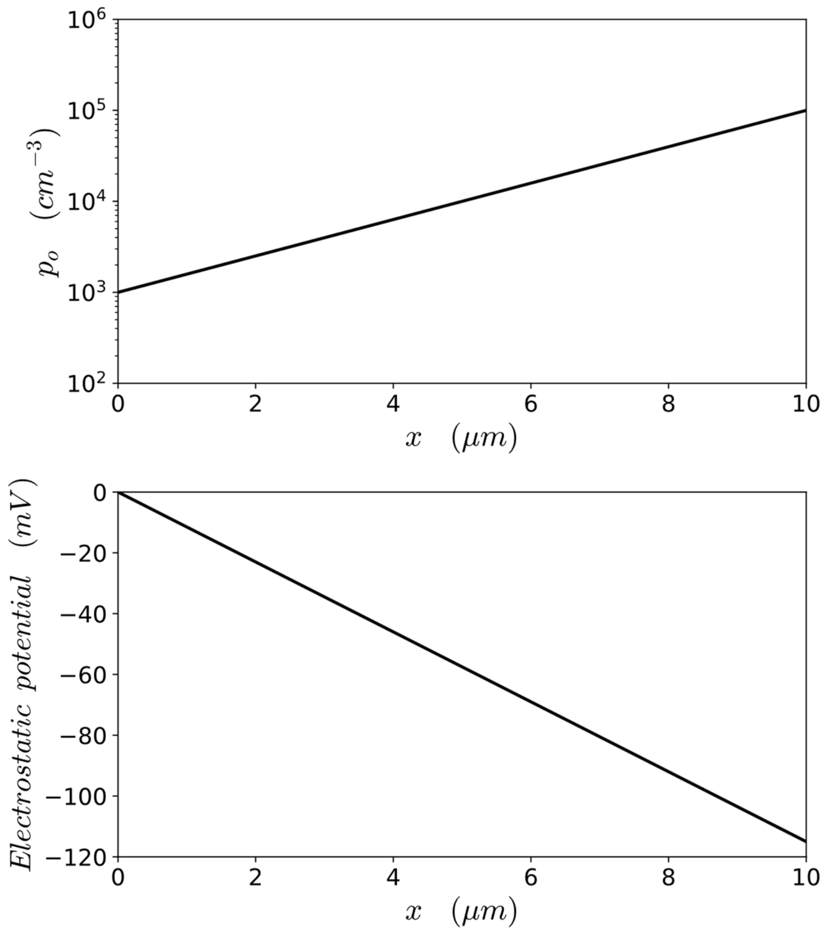
$$V(x) = -E\, x\,, \quad (x \text{ in cm})$$

$$V(10\ \mu m) = -115\ mV$$

d) By considering the Gauss equation:

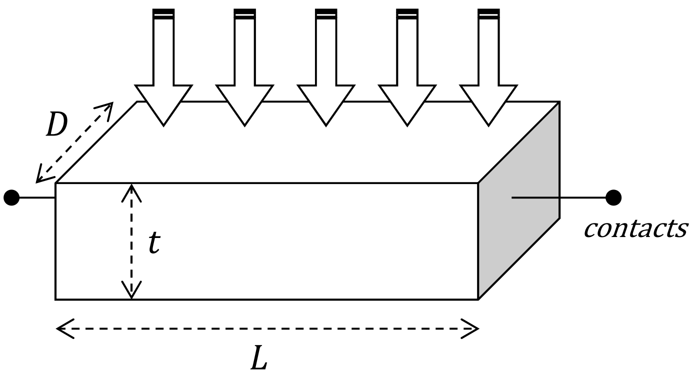
$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_r \varepsilon_0} = 0$$

Since E is a constant, $\rho = 0$ and the sample is neutral. However, this is not a general case and it could be different for another doping profile. Nevertheless, if it does not change very fast with distance, the sample can be typically assumed to be quasi-neutral.



10. ***Consider a photoconductor of length L (distance between the contacts), width D and thickness $t = 5\ \mu m$. The semiconductor material is intrinsic Si ($n_i = 10^{10}\ cm^{-3}$) and the lifetime for both electrons and holes is $\tau = 10^{-6}\ s$. A monochromatic radiation of wavelength $\lambda = 0.83\ \mu m$ falls on the photoconductor with a power density of $0.1\ W/cm^2$. The absorption coefficient of silicon is $\alpha = 10^3\ cm^{-1}$ at this wavelength. You can assume the generation rate constant with depth and equal to its average value. Calculate the ratio R_{dark}/R_{light}, being R_{dark} the resistance of the semiconductor in dark and R_{light} its value under illumination***

Data: $h = 6.62 \times 10^{-34}\ J \cdot s$, $c = 3 \times 10^{10}\ cm/s$, $q = 1.6 \times 10^{-19} C$



The incident power density of 0.1 W/cm^2 consists of photons with $\lambda = 0.83\ \mu m$. The energy of each photon is given by:

$$E_{ph} = \frac{hc}{\lambda} = 6.62 \times 10^{-34}\ J \cdot s \frac{3 \times 10^{10}\ cm/s}{0.83 \times 10^{-4}\ cm} = 2.39 \times 10^{-19}\ J = 1.48\ eV > E_g$$

and the energy of each photon is able to generate an electron-hole pair. If the power density that falls on the photoconductor is 0.1 W/cm^2, the corresponding number of incident photons at the front surface can be calculated as:

$$\emptyset_0 = 0.1\ \frac{J}{s \cdot cm^2} \cdot \frac{1\ photon}{2.39 \times\ 10^{-19} J} \approx 4 \times 10^{17}\ \frac{photons}{cm^2 \cdot s}$$

On the other hand, the absorption of photons in a semiconductor follows the Lambert's law:

$$\emptyset(x) = \emptyset_0 e^{-\alpha x}$$

being $\emptyset(x)$ the remaining (non-absorbed) photons at a depth x from the surface. Note that photons disappear when they are absorbed. Then, the generation rate of electron-hole pairs is related to the decrease of the photon flux as:

$$G(x) = -\frac{d\emptyset}{dx} = \emptyset_0\ \alpha\ e^{-\alpha x}$$

The problem suggests that we can assume the generation rate constant and equal to its average value, $\bar{g}$:

$$\bar{g} = \frac{1}{t}\int_0^t \emptyset_0 \alpha e^{-\alpha x} dx = -\frac{1}{t}\emptyset_0 e^{-\alpha x}|_0^t = \frac{\emptyset_0}{t}(1 - e^{-\alpha t}) = 3.1 \times 10^{20} cm^{-3} s^{-1}$$

The resistance of the photoconductor in dark is:

$$R_{dark} = \rho_{dark}\frac{L}{D\ t} = \frac{1}{\sigma_{dark}}\frac{L}{D\ t}$$

where the dark conductivity is $\sigma_{dark} = q(\mu_p + \mu_n)n_i$. Thus:

$$R_{dark} = \frac{1}{q(\mu_p + \mu_n)n_i}\frac{L}{D\ t}$$

On the other hand, under illumination the conductivity is $\sigma_{light} = q(\mu_p + \mu_n)(n_i + \Delta n)$ and the resistance becomes:

$$R_{light} = \frac{1}{q(\mu_p + \mu_n)(n_i + \Delta n)}\frac{L}{D\ t}$$

The ratio of the dark to the light resistance values is:

$$\frac{R_{dark}}{R_{light}} = \frac{{}^{1}/_{n_i}}{{}^{1}/_{(n_i + \Delta n)}} = 1 + \frac{\Delta n}{n_i}$$

The excess carrier concentration Δn can be calculated from the continuity equation under permanent regime. We also consider a homogeneous sample without any gradient in the current density in the x direction:

$$\frac{dn}{dt} = 0 = \bar{g} - \frac{\Delta n}{\tau_n} \Longrightarrow \Delta n = \bar{g}\,\tau$$

Finally, we obtain:

$$\frac{R_{dark}}{R_{light}} = 1 + \frac{\Delta n}{n_i} = 1 + \frac{\bar{g}\,\tau}{n_i} \approx \frac{\bar{g}\,\tau}{n_i} = 3.1 \times 10^4$$

11. ***Consider an intrinsic silicon sample with length $L = 1\ cm$, width $W = 0.2\ cm$ and thickness $d = 50\ \mu m$. A very low dark current I_o flows between its terminals when a voltage of $100\ V$ is applied. Then, we illuminate the sample with a monochromatic radiation of wavelength $\lambda = 830\ nm$ and power density of $0.1\ \ W/cm^2$. Due to the impinging light, the current in the photoconductor increases by an amount ΔI. The lifetime for both electrons and holes is $\tau = 10\ \mu s$. You can assume that the sample is thick enough for absorbing all the incident photons. Besides, it is also quite long to neglect the influence of the contacts in the carrier distribution.***

   a) Calculate the average value for the carrier generation rate in the semiconductor. From now on, assume that the generation rate is uniform with depth and equal to the calculated average value.

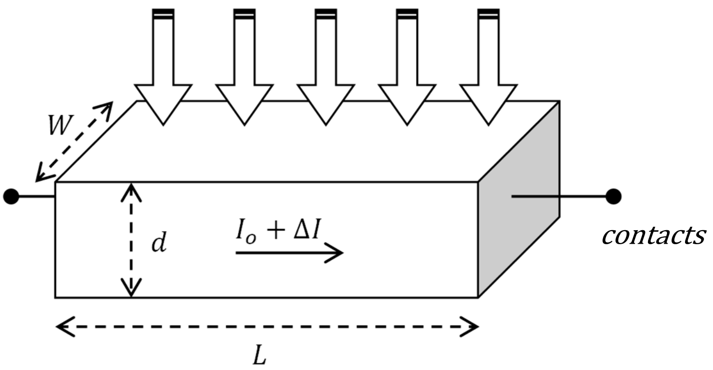
   b) Calculate the increase of the current ΔI in the photoconductor due to lighting. Compare with the current I_o flowing in darkness.

   c) Considering the drift of the charge-carriers due to the applied voltage, calculate the transit times for electrons (t_n) and holes (t_p). Those are the times taken by the respective charge-carrier to traverse the length L of the photoconductor.

   d) Write the increase in the current ΔI in terms of the transit times for both charge-carriers.

   e) Find an expression for the ratio between the increase in the number of charge-carriers and the total number of photons incident on the sample per unit time.

   f) Could be the result found in e) greater than 1? Could you give a physical interpretation for this result?



Data: $h = 6.62 \times 10^{-34}\ J \cdot s$, $c = 3 \times 10^{10}\ cm/s$, $q = 1.6 \times 10^{-19}\ C$,
$n_i = 10^{10}\ cm^{-3}$, $\mu_n = 1500\ \ cm^2/V \cdot s$, $\mu_p = 500\ \ cm^2/V \cdot s$

As it has been calculated in the previous exercise, 0.1 W/cm^2 of radiation with wavelength 830 nm means that $4 \times 10^{17}\ \ photons/cm^2 \cdot s$ are falling on the surface of the sample.

a) Compared to the previous exercise, now the sample is thick enough to absorb almost all the incident photons:

$$G(x) = -\frac{d\emptyset}{dx} = \alpha\emptyset_0 e^{-\alpha x}$$

$$\bar{g} = \frac{1}{d}\int_0^d \alpha\emptyset_0 e^{-\alpha x}dx = \frac{\emptyset_0}{d}[1-e^{-\alpha d}] = \frac{\emptyset_0}{d} \approx \frac{4\times 10^{17} cm^{-2}s^{-1}}{50\times 10^{-4} cm} = 8\times 10^{19}\ cm^{-3}s^{-1}$$

where we have used that $e^{-\alpha d} \ll 1$.

b) In darkness, as the sample is intrinsic, $n_o = p_o = n_i$:

$$J_0 = q(\mu_n n_i + \mu_p n_i)\frac{V}{L} = 0.32\ \ mA/cm^2$$

$$I_0 = J_0\, Wd = 0.32\ \mu A$$

Under illumination, both the electron and hole concentrations will increase by:

$$\Delta n = \Delta p = \bar{g}\tau = 8\times 10^{14}\ cm^{-3}$$

These excesses lead to a higher conductivity and the consequent increase in the current flowing through the sample, $J_0 + \Delta J$ where:

$$\Delta J = q(\mu_n + \mu_p)\Delta n\frac{V}{L} = q(\mu_n + \mu_p)\bar{g}\tau\frac{V}{L} = 25.6\ \ A/cm^2$$

$$\Delta I = \Delta J\, Wd = 25.6\, mA$$

c) The transit time of electrons (t_n) and holes (t_p) are the times they need to cross the photoconductor of length L. The transport mechanism is drift by the electric field, then:

$$t_n = \frac{L}{v_n} = \frac{L}{\mu_n E} = \frac{L}{\mu_n\frac{V}{L}} = \frac{L^2}{\mu_n V} = 6.67\ \mu s$$

$$t_p = \frac{L^2}{\mu_p V} = 20\ \mu s$$

d) The increase in the current can be rewritten as:

$$\Delta J = q(\mu_n + \mu_p)\bar{g}\tau\frac{V}{L} = q\bar{g}\tau L\left(\mu_n\frac{V}{L^2} + \mu_p\frac{V}{L^2}\right) = q\bar{g}\tau L\left(\frac{1}{t_n} + \frac{1}{t_p}\right)$$

$$\Delta I = \Delta J\, Wd = q\bar{g}\tau LWd\left(\frac{1}{t_n} + \frac{1}{t_p}\right)$$

e) The increase of charge-carriers crossing the semiconductor per unit time is:

$$\Delta I/_q = \bar{g}\tau LWd\left(\frac{1}{t_n} + \frac{1}{t_p}\right) = \frac{\emptyset_0}{d}\tau LWd\left(\frac{1}{t_n} + \frac{1}{t_p}\right) = \emptyset_0\tau LW\left(\frac{1}{t_n} + \frac{1}{t_p}\right)$$

while the total number of photons incident on the sample per unit time is $\emptyset_0 LW$.

Then, the ratio of the increase of charge-carriers to the number of incident photons per unit time is:

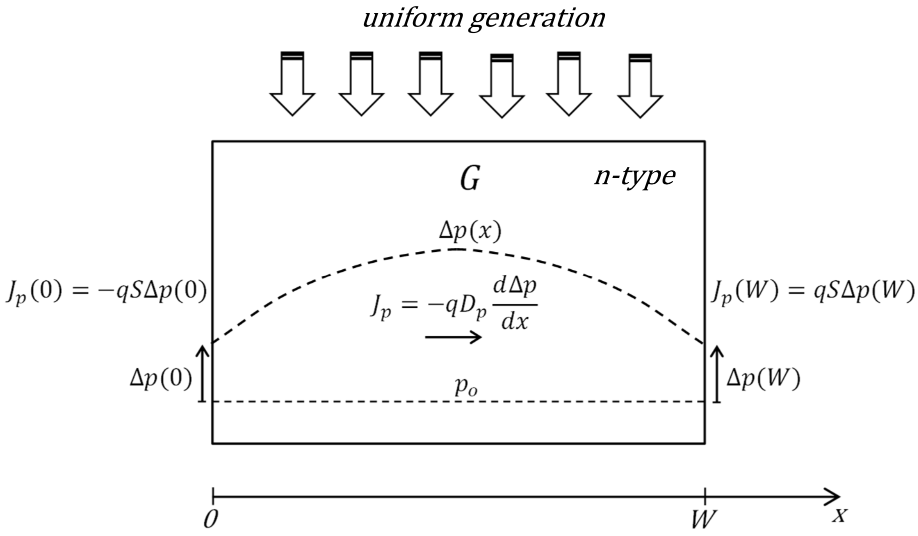
$$\frac{\emptyset_0 \tau LW\left(\frac{1}{t_n}+\frac{1}{t_p}\right)}{\emptyset_0 LW} = \tau\left(\frac{1}{t_n}+\frac{1}{t_p}\right) = \frac{\tau}{t_{eff}} \approx 2$$

f) The calculated ratio is indeed greater than 1. This means that a single photon generating one electron-hole pair may result in more than one charge-carrier circulating between the contacts. Note that the gain is given by the ratio of the lifetime to an effective transit time. This ratio can be greater than one if we have a long lifetime together with short transit times. The generated charge-carrier could circulate more than once before it recombines.

12. *An n-type silicon sample of width W is illuminated to obtain a uniform generation rate G. At both extremes the surface recombination velocity takes the same value, S. The minority carrier diffusion length in the bulk is much longer than the sample width.*

a) Find the expression for the excess of minority carriers $\Delta p(x)$.

b) Compare the excess of minority carriers at the extremes $\Delta p(0) = \Delta p(W)$ with the value at the center of the sample $\Delta p(W/2)$. Discuss the result depending on the relative values of S and D_p/W, where D_p is the diffusion coefficient for holes.



c) Assuming that the bulk of the sample is quasi-neutral, explain how the different mobility of electrons and holes originates a small internal electric field. Find its expression $E(x)$ and plot it graphically.

a) Since the minority carrier diffusion length is much longer than the sample width, bulk recombination can be neglected compared to recombination at both surfaces. Then, we can take the diffusion equation of minority carriers (holes) without the recombination term:

$$D_p \frac{d^2\Delta p}{dx^2} + G = 0 \implies \frac{d^2\Delta p}{dx^2} = -\frac{G}{D_p}$$

which can integrate directly to obtain:

$$\Delta p(x) = -\frac{G}{D_p}\frac{x^2}{2} + Ax + B$$

Now, we must find A and B by imposing the boundary conditions at $x = 0$ and $x = W$. Note that the generation rate is constant, there is no bulk recombination, and we have equal surface recombination velocities at both extremes. Intuitively, the solution should be a parabola symmetric with respect to the center of the sample. The boundary conditions at both extremes are given by the surface recombination velocity S:

$$J_p(0) = -qS\Delta p(0)\ ,\ J_p(W) = qS\Delta p(W)$$

Observe that both hole currents should be directed towards the surface and consequently with opposite directions.

$$J_p(x) = -qD_p \frac{d\Delta p}{dx} = -qD_p\left(-\frac{G}{D_p}x + A\right)$$

At the extreme $x = 0$:

$$-qD_pA = -qSB \;\Rightarrow\; B = \frac{D_p}{S}A$$

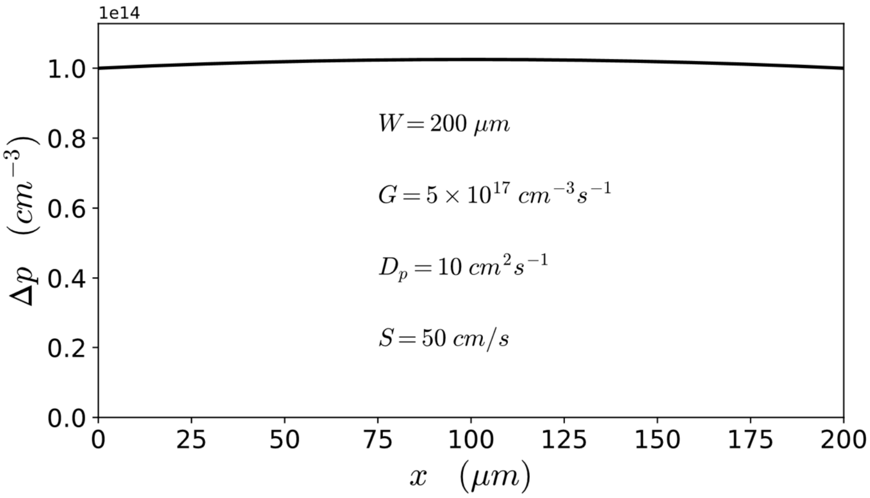
while at $x = W$:

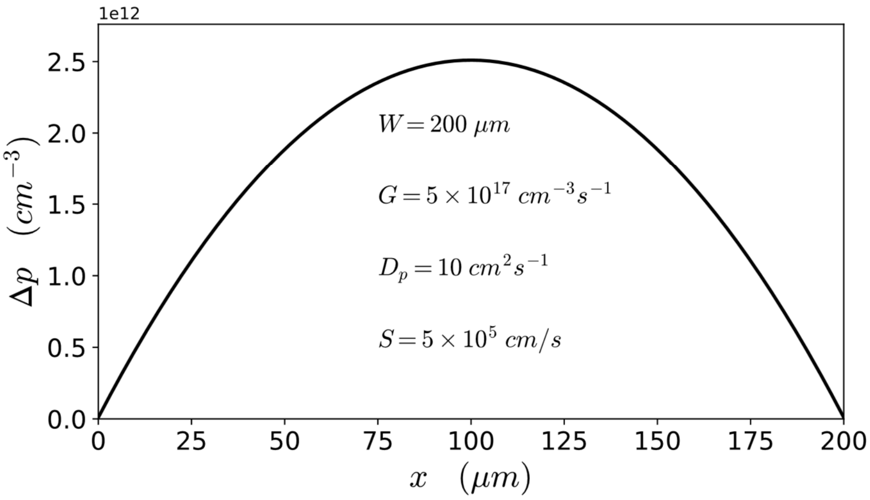
$$-qD_p\left(-\frac{G}{D_p}W + A\right) = qS\left(-\frac{G}{D_p}\frac{W^2}{2} + AW + B\right)$$

$$A = \frac{GW + S\frac{G}{D_p}\frac{W^2}{2}}{SW + 2D_p} = \frac{GW}{SW + 2D_p}\left(1 + \frac{SW}{2D_p}\right) = \frac{GW}{2D_p}$$

Finally, by introducing A and B in the expression of $\Delta p(x)$ we arrive to:

$$\Delta p(x) = -\frac{G}{D_p}\frac{x^2}{2} + \frac{GW}{2D_p}x + \frac{D_p}{S}\frac{GW}{2D_p} = \frac{GW}{2}\left(\frac{1}{S} + \frac{x}{D_p} - \frac{1}{W}\frac{x^2}{D_p}\right) = \frac{GW}{2}\left(\frac{1}{S} + \frac{x}{D_p}\left(1 - \frac{x}{W}\right)\right)$$





b) The excesses of minority carriers at both extremes are:

$$\Delta p(0) = \frac{GW}{2S} \ , \Delta p(W) = \frac{GW}{2S}$$

which confirms a symmetric profile, as we predicted intuitively.

On the other hand, at the center of the sample:

$$\Delta p\left(\frac{W}{2}\right) = \frac{GW}{2}\left(\frac{1}{S} + \frac{W/2}{D_p}\left(1 - \frac{W/2}{W}\right)\right) = \frac{GW}{2}\left(\frac{1}{S} + \frac{W}{4D_p}\right)$$

For very high values of S:

$$S \to \infty \implies \Delta p(0) \longrightarrow 0 \ , \Delta p(W) \longrightarrow 0 \ , \text{ while } \Delta p\left(\frac{W}{2}\right) \approx \frac{GW^2}{8D_p}$$

while for very low values of S:

$$S \ll \frac{D_p}{W} \implies \Delta p(0) \approx \Delta p\left(\frac{W}{2}\right) \approx \Delta p(W) = \frac{GW}{2S}$$

and we have a profile of minority carriers that tends to be flat across the sample.

c) Under the open-circuit condition:

$$J = J_p + J_n = 0$$

Because of quasi-neutrality the electric field is very small, we can neglect the drift current of holes (minority carriers):

$$J_p \approx J_{p,diff} = -qD_p \frac{d\Delta p}{dx}$$

On the other hand, for the electrons (majority carriers) we have:

$$J_n = J_{n,drift} + J_{n,diff} = qn\mu_n E(x) + qD_n \frac{d\Delta n}{dx}$$

In this case, even if the electric field is small, we cannot neglect the drift current because electrons are majority carriers. Additionally, because of the quasi-neutrality $\Delta n(x) = \Delta p(x)$. Considering all of this in the expression for the total current density we arrive to:

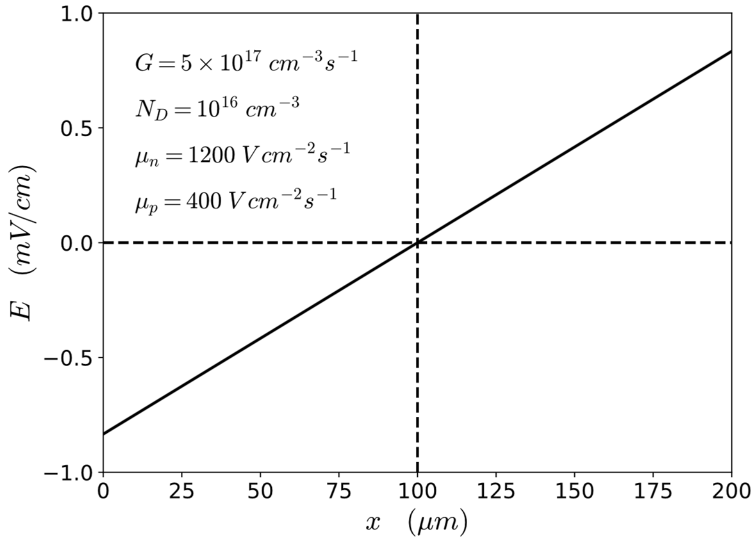
$$0 = -qD_p \frac{d\Delta p}{dx} + qn\mu_n E(x) + qD_n \frac{d\Delta p}{dx}$$

$$E(x) = \frac{D_p - D_n}{\mu_n} \frac{1}{n(x)} \frac{d\Delta p}{dx} = \frac{kT}{q} \frac{\mu_p - \mu_n}{\mu_n} \frac{1}{n(x)} \frac{d\Delta p}{dx}$$

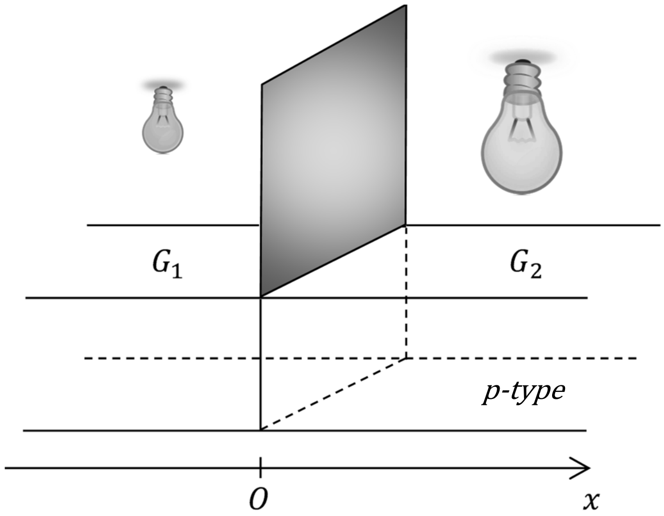
Under low injection, $\Delta p(x) = \Delta n(x) \ll N_D$ being N_D the doping concentration.
Then, $n(x) = N_D + \Delta n(x) \approx N_D$ and we obtain:

$$E(x) = \frac{kT}{q} \frac{\mu_p - \mu_n}{\mu_n} \frac{1}{N_D} \frac{d\Delta p}{dx} = \frac{kT}{q} \frac{\mu_p - \mu_n}{\mu_n} \frac{1}{N_D} \frac{G}{D_p} \left(\frac{W}{2} - x\right) = \frac{\mu_p - \mu_n}{\mu_p \, \mu_n} \frac{1}{N_D} G \left(\frac{W}{2} - x\right)$$

Because of the different mobilities of electrons and holes a small electric filed (note the majority carrier concentration in the denominator) appears to cancel the current in open-circuit.



13. Consider a p-type semiconductor sample with infinite length in both directions. An opaque screen splits this sample in two regions: right ($x > 0$) and left ($x < 0$). Light at the left side produces a generation rate G_1 uniform for $x < 0$. At the right side a stronger illumination produces a generation rate G_2, also uniform for $x > 0$.



a) Find an expression for $\Delta n(x)$ within the sample and plot this profile qualitatively. Discuss the continuity conditions imposed at $x = 0$. Comment on the values for Δn at $x = 0$ and $x = \pm\infty$.

b) Find the electron and hole currents $j_n(x)$ and $j_p(x)$, considering that the sample is in open-circuit. Distinguish between the drift and diffusion components, justify if you neglect any of them. Sketch qualitatively the corresponding profiles as a function of x.

Data:
τ_n (electron lifetime)
L_n (electron diffusion length)
μ_p (hole mobility) $= \mu_n/3$
N_A (impurity concentration)
$V_T = kT/q$ (thermal voltage)

c) Find the electric field $E(x)$ self-established for maintaining the quasi-neutrality. Plot its profile qualitatively. Calculate the voltage difference between the extremes of the sample: $\Delta V = V(+\infty) - V(-\infty)$.

a) At a first glance, in the semi-infinite left side far enough from the boundary ($x = 0$) we should find a constant excess minority carrier concentration $\Delta n = G_1\tau_n$. Similarly, far enough in the right side we would expect $\Delta n = G_2\tau_n$. Additionally, the electron concentration should be continuous everywhere, in particular at $x = 0$. Otherwise, the diffusion current would be infinite and that does not seem reasonable. We expect also the derivative to be continuous. In fact, it is reasonable to expect at the boundary an average carrier concentration between the values $G_1\tau_n$ and $G_2\tau_n$. Let's try to translate this into equations and see if we are right.

In each region the excess electron concentration is found by solving the diffusion equation for an infinite sample under constant illumination, either G_1 or G_2:

$$\Delta n(x) = A\, exp\left(\frac{x}{L_n}\right) + G_1\tau_n \qquad x < 0$$

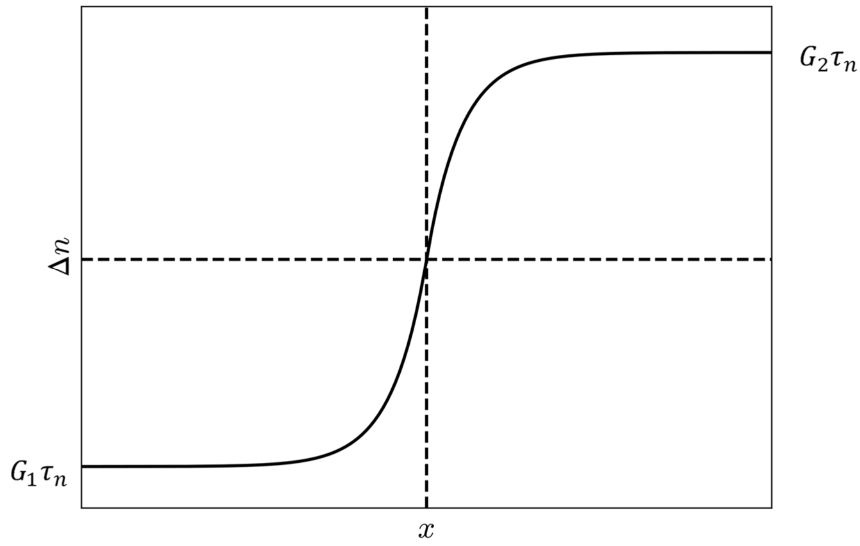
$$\Delta n(x) = B\, exp\left(-\frac{x}{L_n}\right) + G_2\tau_n \qquad x > 0$$

$\Delta n(x)$ must also be continuous at $x = 0$:

$$\Delta n(0) = A + G_1\tau_n = B + G_2\tau_n \Rightarrow A - B = (G_2 - G_1)\tau_n$$

Furthermore, as the sample is infinite in both directions whatever that happens at $x = 0$ will disappear as you go far away from the boundary.

Then, a qualitative sketch of the excess electron concentration is shown here:



Intuitively, at $x = 0$ we expect $\Delta n(0) = \left(\frac{G_1+G_2}{2}\right) \tau_n$

Let's find this result analytically. Not just the carrier concentration, but its derivative must be continuous:

$$\left.\frac{d\Delta n}{dx}\right|_{x=0} = \frac{A}{L_n} = -\frac{B}{L_n} \Longrightarrow A = -B$$

From $A - B = (G_2 - G_1)\tau_n$ and $A + B = 0$, we obtain:

$$A = \frac{(G_2-G_1)}{2}\tau_n\ ,\ B = -\frac{(G_2-G_1)}{2}\tau_n$$

Thus, the final expressions are:

$$\Delta n(x) = \frac{(G_2-G_1)\tau_n}{2}\exp\left(\frac{x}{L_n}\right) + G_1\tau_n \qquad x < 0$$

$$\Delta n(x) = -\frac{(G_2-G_1)\tau_n}{2}\exp\left(-\frac{x}{L_n}\right) + G_2\tau_n \qquad x > 0$$

and as we expected from the very beginning, $\Delta n(0) = \left(\frac{G_1+G_2}{2}\right) \tau_n$

b) We distinguish the drift and diffusion components of the current densities:

$$J_n = J_{n,drift} + J_{n,diff}$$

Assuming quasi-neutrality the electric field will be very small and we neglect the drift component of the minority carrier current density:

$$J_n = J_{n,diff} = qD_n\frac{d\Delta n}{dx}$$

which leads to:

$$J_n(x) = \ qD_n \frac{(G_2-G_1)}{2} \tau_n \frac{1}{L_n} \exp\left(\frac{x}{L_n}\right) = \ q \frac{(G_2-G_1)}{2} L_n \exp\left(\frac{x}{L_n}\right) \qquad x < 0$$

$$J_n(x) == \ q \frac{(G_2-G_1)}{2} L_n \exp\left(-\frac{x}{L_n}\right) \qquad x > 0$$

In addition, because of the quasi-neutrality $\Delta n(x) = \Delta p(x)$. Thus:

$$J_{p,diff} = -qD_p \frac{d\Delta p}{dx} = -\frac{1}{3} J_n \text{, remember that } \mu_p = \mu_n/3$$

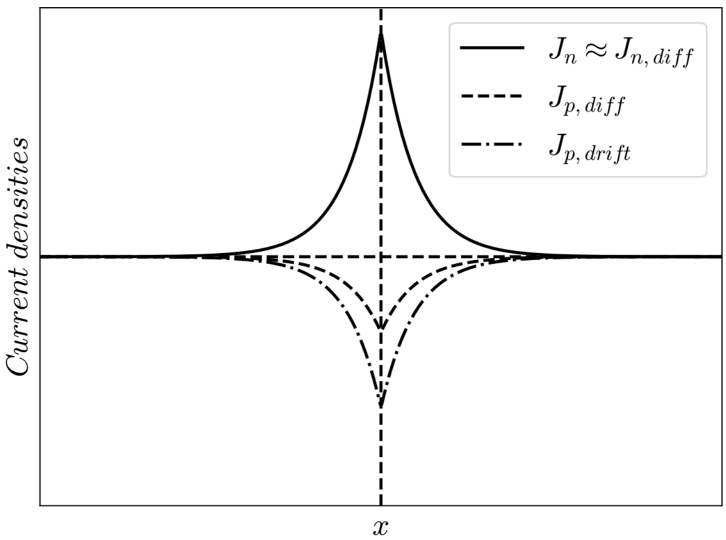
Finally, for the drift current density of the majority carrier, since the sample is in open-circuit:

$$J = J_n + J_p = \left(1 - \frac{1}{3}\right) J_{n,diff} + J_{p,drift} = 0 \ \Rightarrow J_{p,drift} = -\frac{2}{3} J_n$$

that is:

$$J_{pdrift}(x) = \ -q \frac{(G_2-G_1)}{3} L_n \exp\left(\frac{x}{L_n}\right) \qquad x < 0$$

$$J_{pdrift}(x) = -\ q \frac{(G_2-G_1)}{3} L_n \exp\left(-\frac{x}{L_n}\right) \qquad x > 0$$



c) Once we know the drift component of the hole current density, we can find the self-established electric field:

$$J_{p,drift}(x) = qp\mu_p E$$

and assuming that the sample is in low injection $p \approx N_A$, which leads to $E(x) = \frac{J_{p,drift}(x)}{qN_A\mu_p}$. Then:

$$E = -\frac{(G_2-G_1)}{3N_A\mu_p} L_n \exp\left(\frac{x}{L_n}\right) \qquad x < 0$$

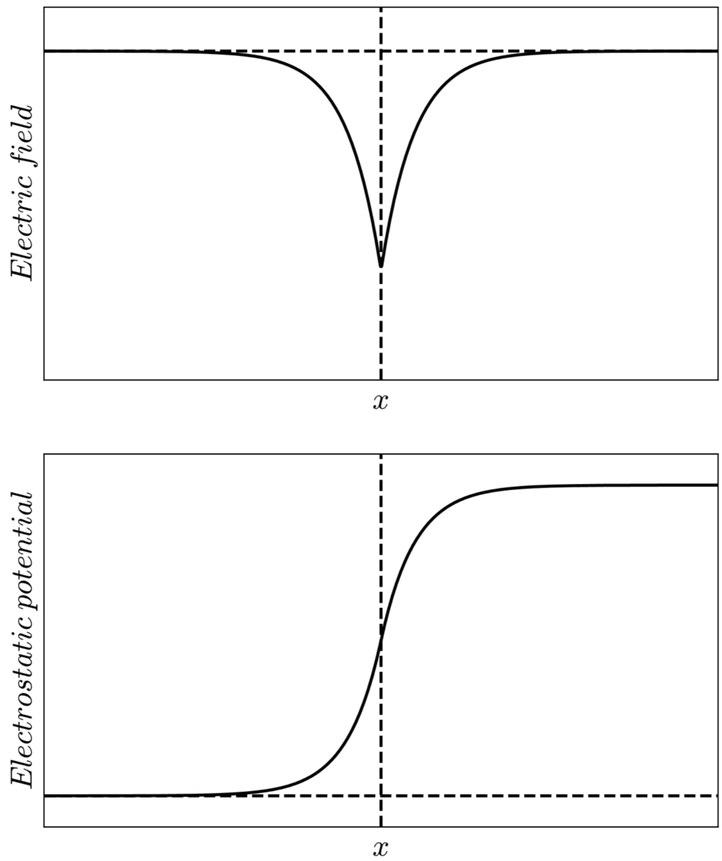
$$E = -\frac{(G_2-G_1)}{3N_A\mu_p} L_n \exp\left(-\frac{x}{L_n}\right) \qquad x > 0$$

Finally, the internal voltage established across the whole sample is:

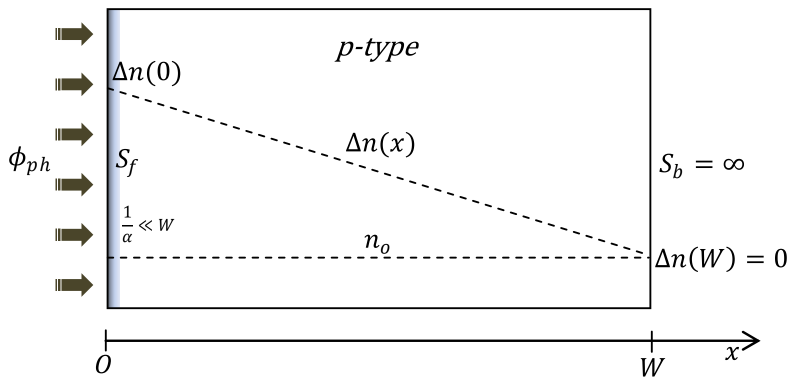
$$V(\infty) - V(-\infty) = \int_{-\infty}^{\infty} -E dx = \int_{-\infty}^{0} -E dx + \int_{0}^{\infty} -E dx$$

$$\Delta V = \frac{(G_2 - G_1)}{3N_A\mu_p} L_n \left(\int_{-\infty}^{0} \exp\left(\frac{x}{L_n}\right) dx + \int_{0}^{\infty} \exp\left(-\frac{x}{L_n}\right) dx \right)$$

$$\Delta V = 2\frac{(G_2 - G_1)}{3N_A\mu_p} D_n\tau_n = 2\frac{(G_2 - G_1)\tau_n}{N_A} V_T$$



14. Consider a p-type silicon sample in open-circuit. Its thickness (W) is short compared to the diffusion length of the minority carrier. We illuminate the front side with a short-wavelength monochromatic radiation, which is absorbed within the first nanometers of the sample. The front side is characterized by a surface recombination velocity S_f, while at the back surface it tends to infinity.



Under these conditions the electron excess at the front surface can be approximatively calculated using the following expression:

$$\Delta n(0) = \frac{\phi_{ph}}{\frac{D_n}{W} + S_f}$$

where ϕ_{ph} is the number of incident photons per unit area and unit time and D_n is the electron diffusion coefficient. Assuming quasi-neutrality in the bulk of the semiconductor:

a) Explain why a different mobility between electrons and holes produces an internal electric field, which is needed for a total current equal to zero in open-circuit. Find an expression for this electric field.

b) Justify that in this particular case, under low injection conditions, this electric field can be considered approximatively uniform. Calculate its value using the data of the problem.

c) This electric field causes a voltage difference between the extremes (Dember effect). Evaluate this voltage considering also its sign.

Data:

$\boldsymbol{\phi_{ph} = 1.5 \times 10^{18} cm^{-2} s^{-1}}$, $\boldsymbol{W = 60\ \mu m}$, $\boldsymbol{S_f = 10^4\ \ cm/s}$, $\boldsymbol{q = 1.6 \times 10^{-19}\ C}$, $\boldsymbol{k_B = 1.38 \times 10^{-23}\ J/K}$, $\boldsymbol{T = 300\ K}$, $\boldsymbol{N_A = 2 \times 10^{15}\ cm^{-3}}$, $\boldsymbol{n_i = 1.5 \times 10^{10}\ cm^{-3}}$, $\boldsymbol{\mu_n = 1200\ cm^2 V^{-1} s^{-1}}$, $\boldsymbol{\mu_p = 400\ cm^2 V^{-1} s^{-1}}$

a) Now that we already have some experience in this kind of problems, we can proceed straightforward:

- in open-circuit $J = J_n + J_p = 0$
- because of quasi-neutrality $\Delta n = \Delta p$
- the drift current of minority carriers can be neglected
- in low injection $p \approx N_A$

Then:

$$J = 0 = q\mu_p pE + q(D_n - D_p)\frac{d\Delta n}{dx}$$

$$E = -\frac{(D_n - D_p)}{\mu_p N_A}\frac{d\Delta n}{dx} = -\frac{kT}{q}\frac{(\mu_n - \mu_p)}{\mu_p N_A}\frac{d\Delta n}{dx}$$

As $\mu_n \neq \mu_p$ a majority carrier drift current appears to cancel the total current in open-circuit.

b) In this particular case

$$\Delta n(x) = \Delta n(0)\left(1 - \frac{x}{W}\right) \Rightarrow \frac{d\Delta n}{dx} = -\frac{\Delta n(0)}{W}$$

$$E = \frac{kT}{q}\frac{\left(\mu_n - \mu_p\right)}{\mu_p N_A}\frac{\Delta n(0)}{W} \approx 0.42\,V/cm$$

c) We integrate the electric field to obtain the voltage difference established between the extremes of the sample:

$$V(0) - V(W) = E\,W = \frac{kT}{q}\frac{\left(\mu_n - \mu_p\right)}{\mu_p N_A}\Delta n(0) \approx 2.5mV$$

Chapter 2.

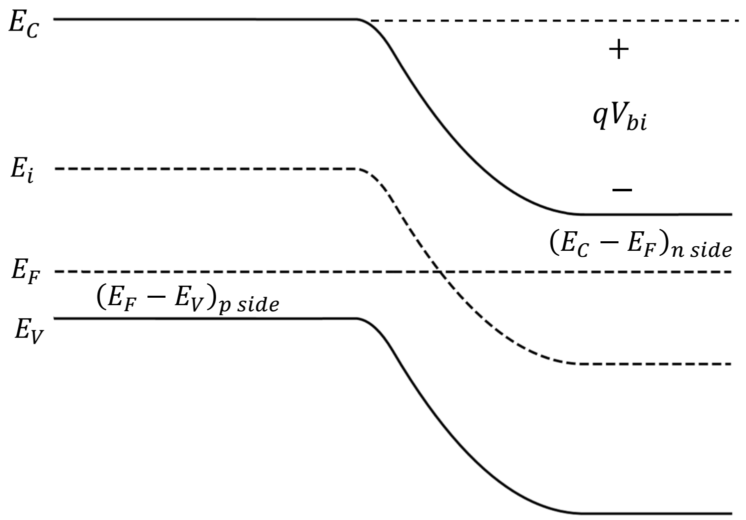
PN junction devices

1. ***In a pn junction, how does the built-in potential V_{bi} change when the temperature increases? You can start by the considering the band diagram and the expected evolution of the Fermi level with temperature. Assume also that the band gap does not change much with temperature.***

Following the suggestion of the problem, the built-in potential can be calculated as:

$$qV_{bi} = E_G - (E_F - E_V)_{p\ side} - (E_C - E_F)_{n\ side}$$

We know that the Fermi level shifts towards the center of the gap when the temperature increases. Then, both $(E_F - E_V)_{p\ side}$ and $(E_C - E_F)_{n\ side}$ will increase with the temperature. Consequently, the V_{bi} value of the junction will be reduced.



2. ***a) Show that the built-in potential V_{bi} of a pn junction can be calculated using (i).***

$$V_{bi} = \frac{kT}{q} ln\left(\frac{N_A N_D}{n_i^2}\right) \qquad \text{(i)}$$

A small temperature increase will change the V_{bi} value. If the increase from a reference temperature T_o is small enough, we can assume a linear variation (first-order Taylor development):

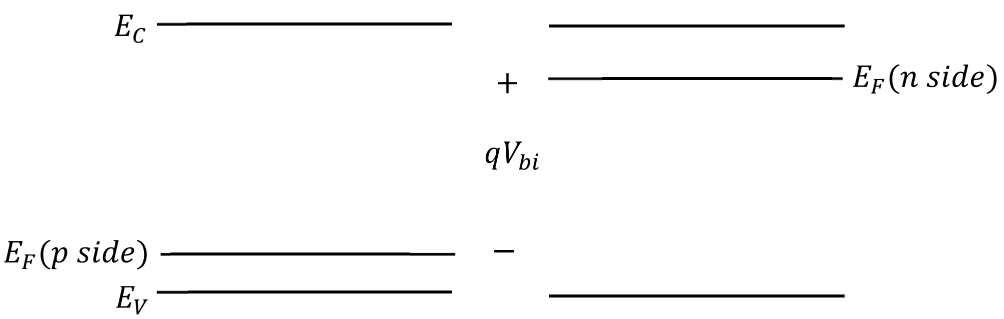
$$V_{bi}(T) = V_{bi}(T_o) + \alpha(T_o)\cdot(T - T_o)\text{, where } \alpha(T_o) = \left.\frac{dV_{bi}}{dT}\right|_{T_o} \qquad \text{(ii)}$$

b) Find the expression for the thermal coefficient $\alpha(T_o)$ in equation (ii). Calculate its value for the data of the problem, indicating clearly its sign.

Note: For small temperature variations you can assume that N_C, N_V and E_g do not change significatively compared with their values at the reference temperature T_o.

Data: $k = 1.38\times10^{-23}\ J/K$, $q = 1.6\times10^{-19}\ C$, $T_o = 300\ K$, $N_C(T_0) = 2.86\times10^{19}\ cm^{-3}$, $N_V(T_0) = 2.66\times10^{19}\ cm^{-3}$, $E_g(T_o) = 1.12\ eV$, $N_A = 10^{18}\ cm^{-3}$, $N_D = 10^{16}\ cm^{-3}$

a) There are several ways to arrive to the expression (i). Probably, the simplest one is by considering that $qV_{bi} = \emptyset_p - \emptyset_n$, where $\emptyset_p$ and $\emptyset_n$ are respectively the work functions of the p and n-type regions before the junction is formed.



$$N_D = N_C\ exp\left(-\frac{E_C - E_F\ (n\ side)}{kT}\right)$$

$$N_A = N_V\ exp\left(-\frac{E_F(p\ side) - E_V}{kT}\right)$$

$$N_A N_D = N_C N_V \exp\left(-\frac{E_C - E_V}{kT}\right)\exp\left(\frac{E_F(n\ side) - E_F(p\ side)}{kT}\right)$$

$$N_A N_D = N_C N_V \exp\left(-\frac{E_g}{kT}\right)\exp\left(\frac{qV_{bi}}{kT}\right)$$

and using that:

$$n_i^2 = N_C N_V \exp\left(-\frac{E_g}{kT}\right)$$

we finally arrive to:

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

b) The value of V_{bi} depends on the temperature explicitly and also through the change in n_i^2 :

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = \frac{E_g}{q} - \frac{kT}{q} \ln \frac{N_C N_V}{N_A N_D}$$

The thermal coefficient is given by:

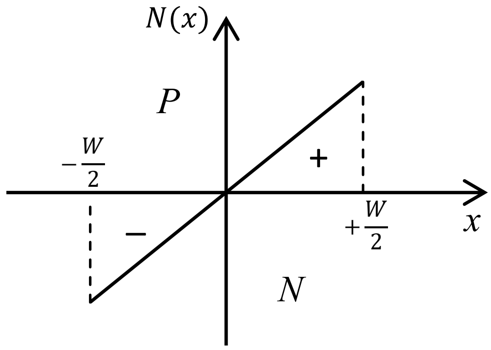
$$\alpha(T_o) = \left. \frac{dV_{bi}}{dT} \right|_{T_o}$$

and assuming that E_g, N_C and N_V do not change much for small temperature variations:

$$\alpha(T_o) = -\frac{k}{q} ln \frac{N_C N_V}{N_A N_D} = -0.97 \ mV/K$$

The negative sigh shows that the value of V_{bi} decreases with increasing temperature.

3. We have a pn junction with a gradual impurity profile. The net impurity concentration is $N(x) = ax$. For $x < 0$ the semiconductor is p-type, whereas for $x > 0$ it becomes n-type. Assume that the space charge region (SCR) has a total width W.



a) Find the expression for the electric field in the SCR. Remember that the electric field should be zero at both extremes of the SCR.

b) Integrate the electric field along the SCR and find an expression for the built-in potential V_{bi}.

Now, considering that an approximated V_{bi} value can be calculated from the impurity concentration at the extremes of the SCR:

$$\boldsymbol{V_{bi} \approx \frac{k_B T}{q} ln\left[\frac{\left(a\frac{W}{2}\right)^2}{n_i^2}\right]}$$

c) Calculate the values of V_{bi} and W comparing this last equation with the expression obtained in b).

Data: $\boldsymbol{q = 1.6 \times 10^{-19}\ C, k_B = 1.38 \times 10^{-23}\ J/K,\ T = 300\ K,}$
$\boldsymbol{\varepsilon_o = 8.85 \times 10^{-14}\ J/K, \varepsilon_r = 11.9, n_i = 1.5 \times 10^{10}\ cm^{-3},}$
$\boldsymbol{a = 5 \times 10^{20}\ cm^{-4}}$

a) We start with the Gauss's law:

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_r \varepsilon_o}$$

where E is the electric field, $\rho(x)$ is the charge density in (C/cm^3) and ε_r and ε_o are the relative and vacuum permittivity, respectively.

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_r \varepsilon_o} = \frac{qax}{\varepsilon_r \varepsilon_o}$$

$$dE = \frac{qax}{\varepsilon_r \varepsilon_o} dx \Longrightarrow E(x) = \frac{qa}{\varepsilon_r \varepsilon_o}\frac{x^2}{2} + C$$

Since the electric field should be 0 at $x = \pm\frac{W}{2}$, we obtain:

$$E\left(\pm\frac{W}{2}\right) = \frac{qa}{\varepsilon_r \varepsilon_o}\frac{\left(\pm\frac{W}{2}\right)^2}{2} + C = 0 \Longrightarrow C = \frac{qa}{\varepsilon_r \varepsilon_o}\frac{W^2}{8}$$

$$E(x) = \frac{qa}{\varepsilon_r \varepsilon_o}\left(\frac{x^2}{2} - \frac{W^2}{8}\right) = -\frac{qa}{\varepsilon_r \varepsilon_o}\frac{\left(\frac{W}{2}\right)^2 - x^2}{2}$$

b) Regarding the built-in potential V_{bi}, it is calculated by integrating the electric field across the SCR:

$$E = -\frac{dV}{dx} \Rightarrow dV = E(x)\, dx$$

$$V_{bi} = \int_{-\frac{W}{2}}^{\frac{W}{2}} \frac{qa}{\varepsilon_r \varepsilon_o} \frac{(W/2)^2 - x^2}{2} dx = \frac{qa}{\varepsilon_r \varepsilon_o} \frac{1}{2} \left[\left(\frac{W}{2}\right)^2 x - \frac{x^3}{3} \right]_{-\frac{W}{2}}^{\frac{W}{2}}$$

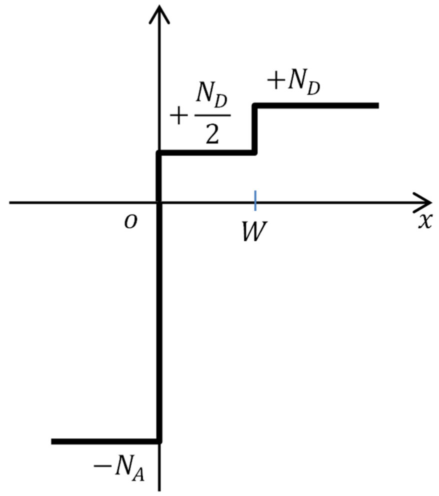
$$V_{bi} = \frac{qaW^3}{12\, \varepsilon_r \varepsilon_o} \qquad (i)$$

c) If we compare this expression for V_{bi} with that given in the problem:

$$V_{bi} \approx \frac{k_B T}{q} ln \left[\frac{\left(a \frac{W}{2}\right)^2}{n_i^2} \right] = 2 \frac{kT}{q} \ln \left(\frac{aW}{2n_i} \right) \qquad (ii)$$

We can start by taking somehow arbitrarily $V_{bi} = 0.5\ V$ in (i) to obtain $W = 0.43\ \mu m$. If we use this W value in (ii) we calculate $V_{bi} = 0.53\ V$. This value can be used again in (i) to obtain $W = 0.44\ \mu m$, which in (ii) leads to $V_{bi} = 0.7\ V$. We iterate in (i) obtaining $W = 0.48\ \mu m$, which in (ii) gives again $V_{bi} = 0.7\ V$. Thus, we can take $W = 0.48\ \mu m$ and $V_{bi} = 0.7\ V$ as the final solution.

4. ***Consider a pn junction with a doping profile as the one sketched in the figure. In equilibrium the space charge region extends throughout the intermediate zone of width W and impurity concentration $N_D/2$.***



a) Draw qualitatively the charge density and the electric field in the different zones of the junction.

b) Calculate the built-in potential V_{bi} between the extremes.

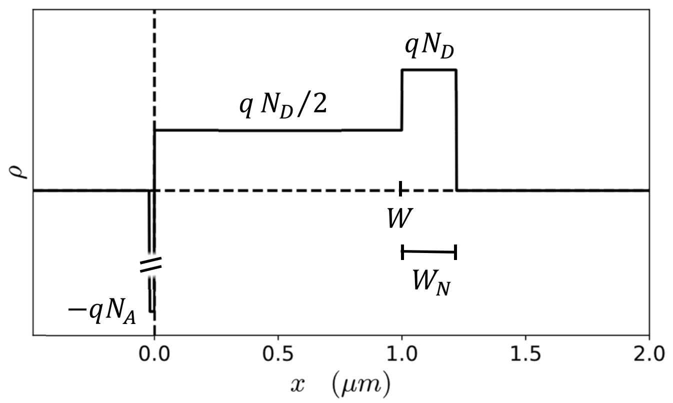
As you can see from the data, $N_A \gg N_D$. Consequently, you may suppose all the built-in potential dropping in the n zone. Using this approximation:

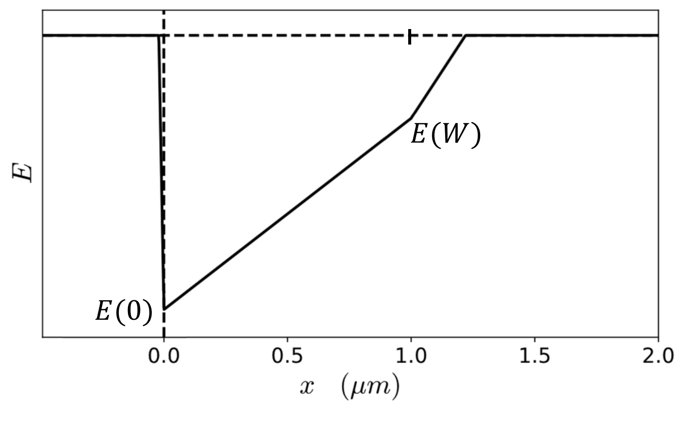
c) Calculate the width of the space charge region in the right-hand side zone with N_D doping.

d) Calculate the value of the electric field at $x = 0$ and $x = W$.

Data: $q = 1.6 \times 10^{-19}\ C$, $\varepsilon_o = 8.85 \times 10^{-14}\ F/cm$, $\varepsilon_r = 11.9$, $V_T = 25\ mV$, $n_i = 10^{10}\ cm^{-3}$, $W = 1\ \mu m$, $N_A = 10^{18}\ cm^{-3}$, $N_D = 10^{15}\ cm^{-3}$

a)





b) The built-in potential can be calculated from the doping levels at the extremes:

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} = 0.748\ V$$

c) In the region $W < x < W + W_N$:

$$E_2(x) = \frac{qN_D}{\varepsilon} x + A$$

Since the electric field must be null at the extreme of the space-charge-region:

$$E_2(W + W_N) = \frac{qN_D}{\varepsilon}(W + W_N) + A = 0$$

we arrive to:

$$E_2(x) = -\frac{qN_D}{\varepsilon}(W + W_N - x)$$

In the region $0 < x < W$:

$$E_1(x) = \frac{qN_D/2}{\varepsilon} x + B$$

but the electric field is continuous at $x = W$, thus:

$$E_1(W) = \frac{qN_D/2}{\varepsilon} W + B = E_2(W) = -\frac{qN_D}{\varepsilon} W_N$$

Therefore:

$$E_1(x) = -\frac{qN_D}{\varepsilon}\left(W_N + \frac{W}{2} - \frac{x}{2}\right)$$

If we integrate the electric field across the SCR we should obtain the same V_{bi} value calculated before.

$$V_{bi} = \frac{1}{2} E(W) W_N + E(W) W + \frac{1}{2}\big(E(0) - E(W)\big) W)$$

$$V_{bi} = \frac{1}{2}\frac{qN_D}{\varepsilon} W_N^2 + \frac{qN_D}{\varepsilon} W_N W + \frac{1}{4}\frac{qN_D}{\varepsilon} W^2$$

$$W_N^2 + 2W\ W_N + \frac{W^2}{2} - \frac{2\varepsilon}{qN_D} V_{bi} = 0$$

This last equation can be solved to obtain $W_N = 0.22\ \mu m$.

d) Finally, we calculate the electric field at the points $x = 0$ and $x = W$:

$$E(0) = -\frac{qN_D}{\varepsilon}\left(W_N + \frac{W}{2}\right) = -10.9 \ \ kV/cm$$

$$E(W) = -\frac{qN_D}{\varepsilon}W_N = -3.3 \ \ kV/cm$$

5. ***A diode goes into breakdown when the maximum electric field at the junction E_{max} reaches the critical value in the semiconductor E_c. First, consider a silicon abrupt p⁺n junction where the n side can be considered semi-infinite. The space charge region will extend a width W, mainly in the less doped n region. In this case:***

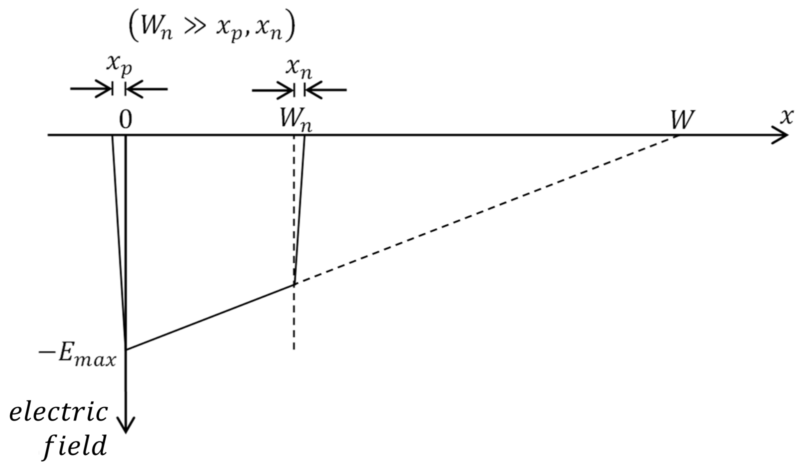
   a) ***Calculate the value of W when the diode goes into breakdown.***

   b) ***Calculate the corresponding breakdown voltage V_R.***

   Now, consider a modified p⁺nn⁺ junction. The width of the n region is W_n (figure). For this new diode:

   c) ***Calculate the intensity of the electric field at $x = W_n$ just at breakdown.***

   d) ***Calculate the new breakdown voltage and compare it with the value for a standard pn junction.***



Data: $\boldsymbol{N_D = 10^{16}\ cm^{-3}}$ ***(n zone doping concentration),*** $\boldsymbol{E_c = 4 \times 10^5\ V/cm}$,
$\boldsymbol{W_n = 0.5\ \mu m}$, $\boldsymbol{\varepsilon_{Si} = 10.62 \times 10^{-13}\ F/cm}$, $\boldsymbol{q = 1.6 \times 10^{-19}\ C}$
$\boldsymbol{k_B = 8.62 \times 10^{-5}\ eV/K}$, $\boldsymbol{T = 300\ K}$

a) In the n side of the space-charge-region the Gauss law is:

$$\frac{dE}{dx} = q\frac{N_D}{\varepsilon_{Si}} \Rightarrow E(x) = q\frac{N_D}{\varepsilon_{Si}}x + C$$

Since $E(W) = 0$, we can calculate the integration constant C and arrive to:

$$E(x) = -q\frac{N_D}{\varepsilon_{Si}}(W - x)$$

Therefore, the maximum electric field is:

$$E_{max} = |E(0)| = q\frac{N_D}{\varepsilon_{Si}}W$$

Finally, when this maximum electric field reaches the critical value E_c:

$$W = \frac{\varepsilon_{Si}}{qN_D}E_c = 2.6 \times 10^{-4}\ cm$$

b) The voltage across the junction will be:

$$V_{bi} - (-V_R) \approx V_R \approx \frac{1}{2}E_cW$$

Remember that the voltage is calculated by integrating the electric field, i.e., the area of the triangle:

$$V_R \approx \frac{1}{2}E_c W = \frac{1}{2}E_c \frac{\varepsilon_{Si}}{qN_D}E_C = \frac{\varepsilon_{Si}}{2qN_D}E_c^2 = 53\ V$$

c) Now, we consider the modified p⁺nn⁺ structure. In the region $0 < x < W_n$:

$$E(x) = q\frac{N_D}{\varepsilon_{Si}}x + C$$

When the diode just goes into breakdown $E(0) = -E_C$, thus:

$$E(x) = -E_C + q\frac{N_D}{\varepsilon_{Si}}x$$

$$E(W_n) = -E_C + q\frac{N_D}{\varepsilon_{Si}}W_n = -3.2 \times 10^5\ V/cm$$

d) The breakdown will occur at a lower voltage:

$$V_R = W_n|E(W_n)| + \frac{1}{2}|E(0) - E(W_n)|W_n = 18\ V$$

6. ***A pin structure consists of an intrinsic region between two doped regions. In this problem, both doped regions have the same concentration of impurities $(N_A = N_D = 10^{15}\ cm^{-3})$. The width of the intrinsic region is $d = 2\ \mu m$. The figure shows the profile of the charge density in the pin structure.***

   a) Draw qualitatively the profiles of the electric field and the electrostatic potential along the pin structure. Sketch also the band diagram in equilibrium.

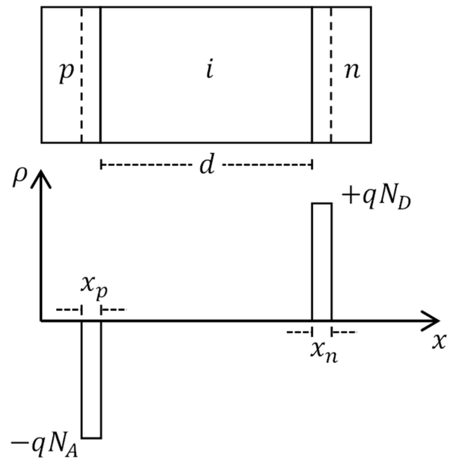
   b) Calculate the built-in potential V_{bi} between both extremes of the pin structure.

   c) Calculate the width $(x_p = x_n)$ of the space charge region in the doped zones.

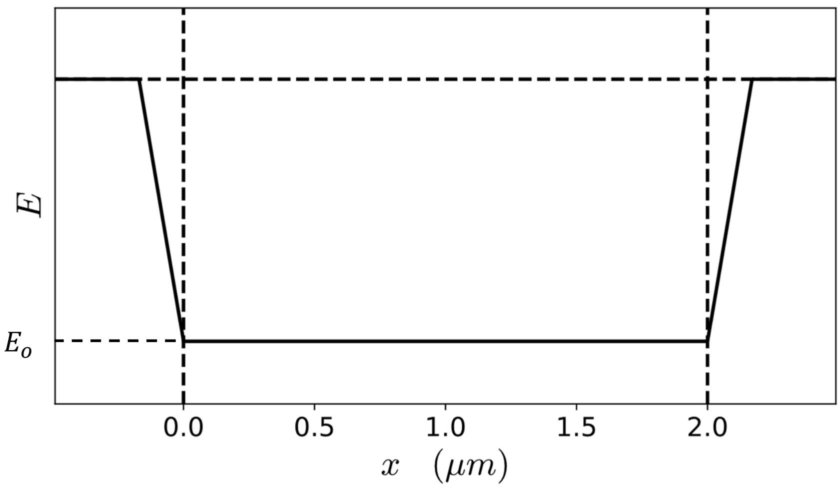
   d) Calculate the electric field in the intrinsic region.

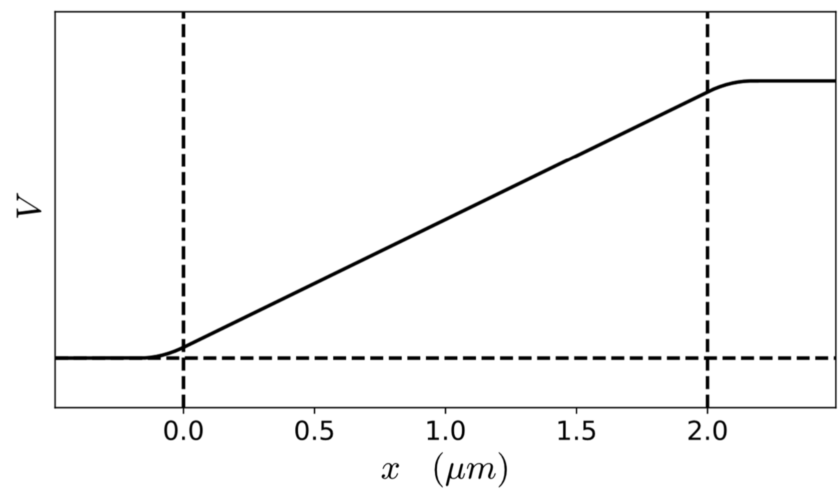
   e) Which is the fraction of the built-in voltage sustained by the intrinsic region?

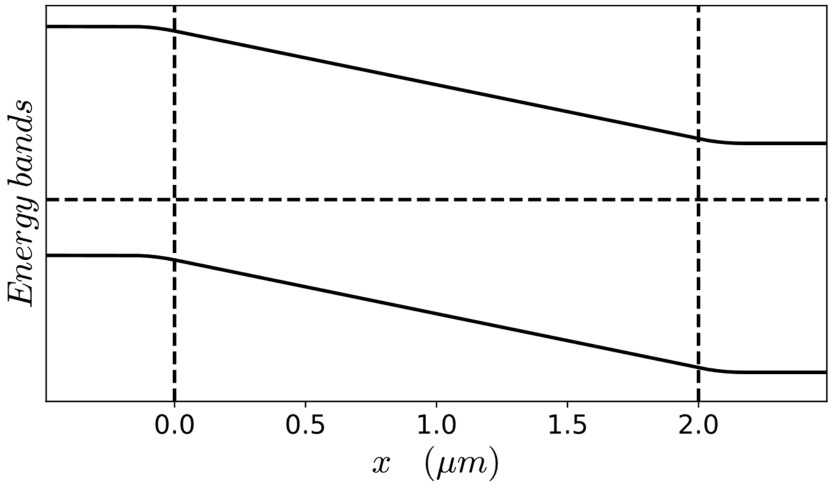
   Data: $q = 1.6 \times 10^{-19}\ C$, $n_i = 10^{10}\ cm^{-3}$, $V_T = 25\ mV$, $\varepsilon_r = 11.9$, $\varepsilon_o = 8.85 \times 10^{-14}\ F/cm$



a)







b) The built-in potential can be calculated with the equation:

$$V_{bi} = V_T \ln \frac{N_A N_D}{n_i^2} = 576\ mV$$

c) The built-in voltage is also given by the integral of the electric field, which can be calculated geometrically.

$$\frac{1}{2} x_p E_o + E_o d + \frac{1}{2} x_n E_o = V_{bi}$$

Now, since $x_p = x_n$ and $E_o = q \frac{N_A}{\varepsilon_{Si}} x_p$:

$$E_o(x_p + d) = V_{bi} \Rightarrow q \frac{N_A}{\varepsilon_{Si}} x_p^2 + q \frac{N_A}{\varepsilon_{Si}} d\, x_p - V_{bi} = 0$$

$$x_p^2 + d\, x_p - \frac{\varepsilon_{Si}}{q N_A} V_{bi} = 0 \Rightarrow x_p = \frac{-d \pm \sqrt{d^2 + 4 \frac{\varepsilon_{Si}}{q N_A} V_{bi}}}{2} = 0.17 \times 10^{-4}\ cm$$

d) The electric field in the intrinsic region is uniform and equal to:

$$E_0 = \frac{q N_A}{\varepsilon_{Si}} x_p = 2.6\ kV/cm$$

e) The built-in potential that drops in the p and n regions is only:

$$V_P = V_N = \frac{1}{2} x_p E_0 = 23\ mV = 4\%\ V_{bi}$$

whereas the intrinsic region sustains the greatest part:

$$V_i = E_0\, d = 530\ mV = 92\%\ V_{bi}$$

7. ***Consider an abrupt p⁺n junction. We measure its capacitance C as a function of the applied voltage V. If we plot $1/C^2$ vs. V we observe a linear behavior (figure).***

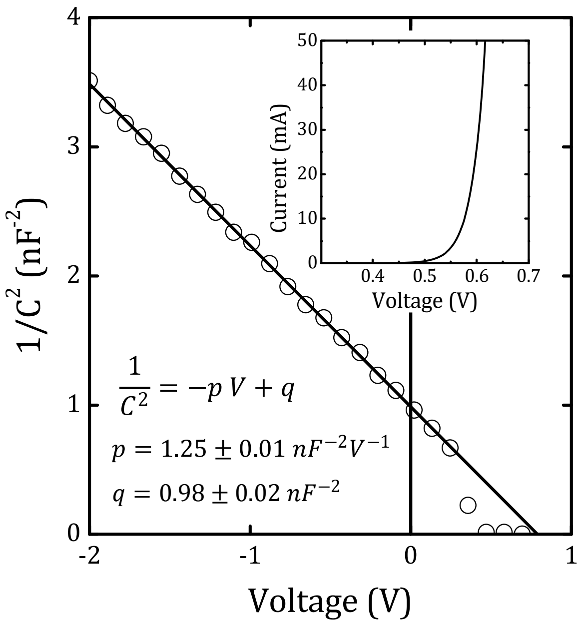
   a) ***From this fitting, find the built-in potential V_{bi} and its margin of error.***

   b) ***Takin into account that it is an asymmetric junction, calculate the doping level of the n region and its margin of error.***

   c) ***Finally, calculate the doping concentration of the highly-doped p region. Discuss its margin of error.***

   We have also measured the I-V characteristic of the diode (inset), reading $25\ mA$ at $0.6\ V$. This value is dominated by diffusion of minority carriers in the n region, which can be considered short with an ohmic contact at the end.

   d) ***Give an estimation of the hole diffusion coefficient D_p in the n region.***



Data: $\boldsymbol{A = 0.1\ cm^2}$, $\boldsymbol{q = 1.6 \times 10^{-19}\ C}$, $\boldsymbol{k_B = 1.38 \times 10^{-23}\ J/K}$, $\boldsymbol{T = 300\ K}$

$\boldsymbol{n_i = 10^{10}\ cm^{-3}}$, $\boldsymbol{\varepsilon_o = 8.85 \times 10^{-14}\ F/cm}$, $\boldsymbol{\varepsilon_{r_{Si}} = 11.9}$, $\boldsymbol{W = 200\ \mu m}$

a) V_{bi} is the point where the $1/C^2$ vs. V straight line intercepts the x axis. Remember that in general for an abrupt junction the junction capacitance writes as $C_j = \varepsilon_{Si}\frac{A}{W}$ where A is the junction area an W is the width of the space charge region. Developing the expression we get:

$$C_j = \varepsilon_{Si}\frac{A}{\sqrt{\frac{2\varepsilon_{Si}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)(V_{bi}-V)}}$$

Then, C_j goes to infinite when $V \to V_{bi}$. From the fit given in the figure:

$$V_{bi} = \frac{q}{p} = \frac{0.98}{1.25} = 0.784\ V$$

About the error margin on V_{bi}, $\varepsilon(V_{bi})$ will depend on the error we have when determining p and q. If we consider V_{bi} as a function of p and q, in fact we are looking how small variations on p and q modifies the value of V_{bi}. This is a core subject for the differential calculus. We write, being $\varepsilon(q)$and $\varepsilon(p)$ the margin errors in q and p respectively:

$$\varepsilon(V_{bi}) = \left|\frac{\partial V_{bi}}{\partial q}\right|\varepsilon(q) + \left|\frac{\partial V_{bi}}{\partial p}\right|\varepsilon(p) = \frac{1}{p}\varepsilon(q) + \frac{q}{p^2}\varepsilon(p) = \pm 0.02\ V\ (2.6\%)$$

We take all the derivatives in absolute value, errors in the different variables will not cancel each other.

b) If we start again from

$$C_j = \varepsilon_{Si}\frac{A}{\sqrt{\frac{2\varepsilon_{Si}}{q}(\frac{1}{N_A}+\frac{1}{N_D})(V_{bi}-V)}} \Longrightarrow C_j^2 = \varepsilon_{Si}\frac{A^2}{\frac{2}{q}(\frac{1}{N_A}+\frac{1}{N_D})(V_{bi}-V)}$$

$$\frac{1}{C_j^2} = \frac{2}{qA^2\varepsilon_{Si}}(\frac{1}{N_A}+\frac{1}{N_D})(V_{bi}-V)$$

as $N_A \gg N_D$ we get :

$$\frac{1}{C_j^2} = \frac{2}{qA^2\varepsilon_{Si}}\frac{1}{N_D}(V_{bi}-V)$$

Then, since $\frac{1}{C_j^2} = -pV + q$, we have:

$$p = \frac{2}{qA^2\varepsilon_{Si}}\frac{1}{N_D} \Longrightarrow N_D = \frac{2}{qA^2\varepsilon_{Si}}\frac{1}{p} \approx 10^{15}\ cm^{-3}$$

The margin of error in N_D is:

$$\varepsilon(N_D) = \left|\frac{\partial N_D}{\partial p}\right|\varepsilon(p) = \frac{2}{qA^2\varepsilon_{Si}}\frac{1}{p^2} = \pm 7\cdot 10^{12} cm^{-3}$$

which is only around a 0.7% of its value.

c) Finally, if we try to obtain the doping concentration of the highly-doped region N_A:

$$V_{bi} = \frac{kT}{q}ln\frac{N_A N_D}{n_i^2} \Longrightarrow N_A = \frac{n_i^2}{N_D}exp\left(\frac{qV_{bi}}{kT}\right) = 4.2\times 10^{18}\ cm^{-3}$$

Let's have a look to the margin of error:

$$\varepsilon(N_A) = \left|\frac{\partial N_A}{\partial N_D}\right|\varepsilon(N_D) + \left|\frac{\partial N_D}{\partial V_{bi}}\right|\varepsilon(V_{bi})$$

$$\varepsilon(N_A) = \frac{n_i^2}{N_D^2}e^{\frac{qV_{bi}}{kT}}\varepsilon(N_D) + \frac{q}{kT}\frac{n_i^2}{N_D}e^{\frac{qV_{bi}}{kT}}\varepsilon(V_{bi})$$

$$\varepsilon(N_A) = \pm 2.9\times 10^{16}\ cm^{-3} \pm 3.3\times 10^{18}\ cm^{-3} = \pm 4.2\times 10^{18}\ cm^{-3}$$

$$N_A = 4.2\times 10^{18} \pm 3.3\times 10^{18} cm^{-3}$$

The margin of error is comparable to the calculated value (78%). Thus, we can just roughly estimate the order of magnitude of the doping concentration in the p$^+$ region.

d) Finally, we are asked about the diffusion coefficient D_p. If the current is dominated by the minority carrier diffusion in the less doped zone and the zone can be considered short with ohmic contact at the end (probably the simplest case), the current can be written:

$$I = qA\frac{D_P}{W}\frac{n_i^2}{N_D}\left(exp\left(\frac{qV}{kT}\right)-1\right) \Longrightarrow D_p = \frac{I\,W\,N_D}{q\,A\,n_i^2\left(exp\left(\frac{qV}{kT}\right)-1\right)} = 11.8\ cm^2/s$$

8. ***In a pn junction, integrating the continuity equation in the n zone (with extremes 0 and W) gives:***

$$\boldsymbol{J_P(0) - J_P(W) = q\int_0^W \frac{\Delta p}{\tau}\,dx}$$

Compare the relative magnitudes of the three terms in the above equation if (a) the n zone is much longer, or (b) much shorter than the hole diffusion length.

If the zone is short $W \ll L_p$ the minority carrier distribution (solution of the diffusion equation) can be approximated by a straight line, consequently its derivative is constant and so is the hole diffusion current which is constant in the whole zone in particular $J_p(0) \approx J_p(W)$. Consequently we deduce that the recombination within the bulk will be negligible in front of both $J_p(0)$ and $J_p(W)$. Usually at the end of the zone we have some kind of contact (usually ohmic) and $J_p(W)$ is the recombination at the contact. So we can re-phrase saying that in a short zone the recombination within the bulk is negligible in front of the recombination at the contact.

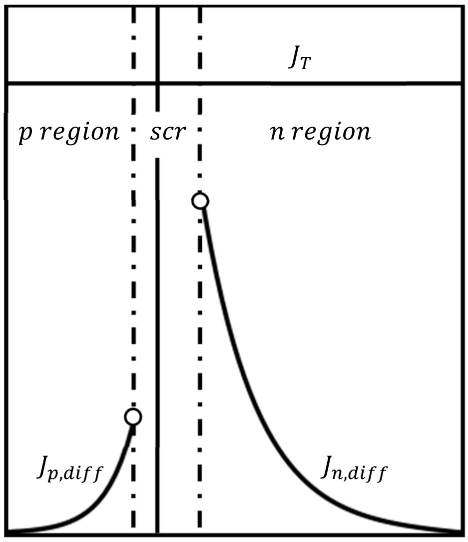
If the zone is long $W \gg L_p$the excess minority carrier distribution is an exponential and for high enough values of x tends to zero and so does its derivative, consequently the hole diffusion current is also zero and then the entering current $J_p(0)$ equals the recombination within the bulk of the zone:

$$J_P(0) = q\int_0^W \frac{\Delta p}{\tau}\,dx$$

9. ***Consider a pn junction biased at voltage V.***

   a) ***Show that the total current flowing through the device can be calculated by adding the minority carrier diffusion currents at the respective boundaries of the space charge region.***

   b) ***If the generation/recombination within the space charge region were not negligible how would be modified the J-V characteristic of the diode?***



a) Let's call x_n and x_p respectively the boundaries between the space charge region and the neutral n and p zones. Obvioulsy by Kirchoff's law:

$$J_T = J_p(x_i) + J_n(x_i);\ \forall x_i$$

As a point for calculating the current we may choose either x_n or x_p. Let's take for instance x_p:

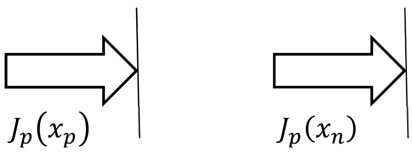
$$J_T = J_p(x_p) + J_n(x_p)$$

$$J_n(x_p) = J_{n,drift}(x_p) + J_{n,diff}(x_p)$$

At x_p electrons are minority carriers and x_p is boundary with the quasi neutral zone ($E \to 0$), then we can neglect the drift current (produced by the electric field E). Thus:

$$J_T = J_p(x_p) + J_{n,diff}(x_p)$$

Let's now consider the zone between x_p and x_n:



Writing the continuity equation between x_p and x_n we have:

$$\frac{1}{q}\frac{dJ_p}{dx} = G - \frac{\Delta p}{\tau_p}$$

If we neglect generation/recombination between x_p and x_n , the derivative of the hole current will be null, and the hole current constant within the space charge region i.e. $J_p(x_p) = J_p(x_n)$. We can then write:

$$J_T = J_p(x_p) + J_{n,diff}(x_p) = J_p(x_n) + J_{n,diff}(x_p)$$

Now:

$$J_T = J_p(x_n) + J_{n,diff}(x_p) = J_{p,drift}(x_n) + J_{p,diff}(x_n) + J_{n,diff}(x_p)$$

Again in x_n , boundary of the quasi neutral zone, holes are minority carriers and the electric field $E \to 0$. We can neglect the hole drift current. Finally:

$$J_T = J_{p,diff}(x_n) + J_{n,diff}(x_p)$$

b) If we do not neglect generation/recombination within the space charge region:

$$\frac{1}{q}\frac{dJ_p}{dx} = G - \frac{\Delta p}{\tau_p}$$

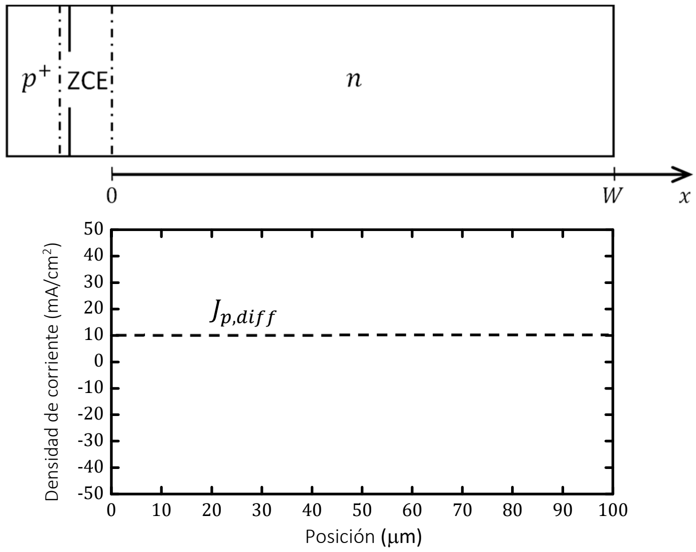
Integrating between x_p and x_n we obtain:

$$J_p(x_n) - J_p(x_p) = q\int_{x_p}^{x_n}\left(G - \frac{\Delta p}{\tau_p}\right)dx \Rightarrow J_p(x_p) = J_p(x_n) - q\int_{x_p}^{x_n}\left(G - \frac{\Delta p}{\tau_p}\right)dx$$

Finally, we arrive to:

$$J_T = J_{p,diff}(x_n) + J_{n,diff}(x_p) - q\int_{x_p}^{x_n}\left(G - \frac{\Delta p}{\tau_p}\right)dx$$

10. A p⁺n junction is directly biased. The quasi-neutral n region can be considered short when compared with the minority carrier diffusion length. In the figure we show the hole diffusion current density $\left(J_{p,diff}\right)$ in the quasi-neutral n region. (dotted line) The back contact is ohmic.



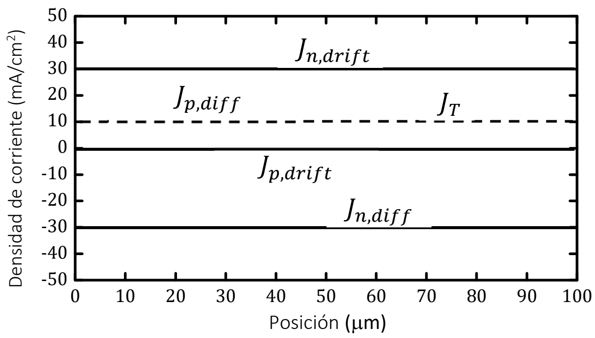
a) Show on the figure, for the same region, the electron diffusion current density $\left(J_{n,diff}\right)$.

b) Show, also on the figure, the drift currents densities for electrons $\left(J_{n,drift}\right)$ and holes $\left(J_{p,drift}\right)$.

c) Indicate the value of the total current density flowing through the device.

d) Calculate, with good approximation, the reverse saturation current J_o of the junction. Finally calculate the polarization voltage.

Data: $q = 1.6 \times 10^{-19}\ C$, $V_T = 25\ mV$,
$W = 100\ \mu m$, $n_i = 10^{10}\ cm^{-3}$, $N_D = 10^{15}\ cm^{-3}$
$\mu_n = 1500\ cm^2/Vs$, $\mu_p = 500\ cm^2/Vs$,



a) The electron diffusion current is by definition $J_{n,diff} = qD_n \frac{d\Delta n}{dx}$.

May be you are puzzled, because sometimes we write $J_{n,diff} = qD_n \frac{dn}{dx}$.

In fact, $n = n_o + \Delta n$. The concentration of electrons (or holes) can be written as the addition of the concentration under equilibrium n_o (or p_o) and the excess over the equilibrium value Δn (or Δp). It's clear that if the doping is uniform n_o (or p_o) is constant and the derivative is zero. However, what happens if the doping concentration is not constant? In equilibrium the electron (hole) current is null. That means that somehow the existing diffusion due to a non-uniform doping is compensated by a drift also in equilibrium. All together, we can neglect both the diffusion and the drift existing in equilibrium because they compensate each other and focus only on the net currents. In the case of diffusion, concentrate on the derivatives of the excesses.

Let's come back to the beginning:

$$J_{n,diff} = qD_n \frac{d\Delta n}{dx} = -\frac{D_n}{D_p}\left(-qD_p \frac{d\Delta p}{dx}\right) = -\frac{D_n}{D_p} J_{p,diff} = -\frac{\mu_n}{\mu_p} J_{p,diff} = -30\ mA/cm^2$$

Remember that because of quasi-neutrality, $\Delta n(x) = \Delta p(x)$. You can come back to exercise 2 in chapter 1 for an explanation about the physical origin of quasi-neutrality.

b, c) We go now for the drift currents. First, the hole drift current can be neglected. Holes are minority carriers in the quasi-neutral n zone where the electric field $E \rightarrow 0$. Thus:

$J_{p\ ,drift}$=0

What about the electron (majority carrier) drift current? We know that the total current can be calculated by adding the minority carrier diffusion currents at the edges of the space charge region (see the previous problem). If the material is uniform (no heterojunctions are involved) and the junction is asymmetric (either p^+n or n^+p), the diffusion current of the minority carrier at the boundary of the less doped zone is much greater than the other one. In our case:

$$J_T \approx J_{p,diff}(0) = 10\ mA/cm^2$$

In the figure we see that $J_{p,diff}$ is constant along the whole n zone.

$$J_T = J_{p,diff} + \cancel{J_{p,drift}} + J_{n,diff} + J_{n,drift}$$

Since $J_T = J_{p,diff}$, we have that $J_{n,drift} = -J_{n,diff} = 30\ mA/cm^2$

d) Finally, the reverse saturation current for an asymmetric junction with the lowly-doped zone being short can be written as:

$$J_o \approx q\frac{D_p}{W}p_o = q\frac{D_p}{W}\frac{n_i^2}{N_D} = 20\ pA/cm^2$$

If you are not sure about the above result, remember that if the diffusion current is constant that means that the excess hole distribution has to be a straight line. The current is going to be proportional to the slope of this straight line. The excess at $x = 0$ is the boundary condition:

$$\Delta p(0) = \frac{n_i^2}{N_D}\left(exp\left(\frac{qV}{kT}\right) - 1\right)$$

The excess of minority carriers at $x = W$ is 0 because of the ohmic contact.
The applied voltage can be now calculated:

$$J = J_o\left(exp\left(\frac{qV}{kT}\right) - 1\right) \Rightarrow V = \frac{kT}{q} ln\left(\frac{J}{J_o} + 1\right) = 0.5\ V$$

11. *In a p⁺n junction we focus in the quasi-neutral n zone out of the space charge region. A voltage V is applied injecting an excess of minority carriers (holes) in the quasi-neutral zone of width W. Holes are going to diffuse in this region (with diffusion coefficient D_p). We may consider that the diffusion length L_p is much longer than the width of the quasi-neutral zone (short-zone approach). The other extreme of the n region has a contact extracting the excess of minority carriers with a recombination velocity S. In the permanent regime:*

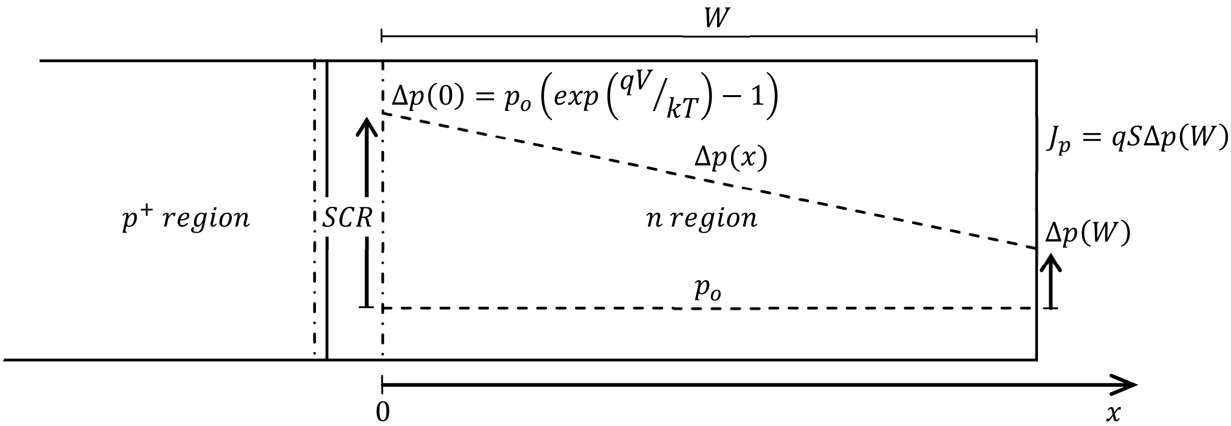
a) Find the expression for the minority carrier excess $\Delta p(x)$ in the quasi neutral zone n. In particular, find the hole excess $\Delta p(x)$ precisely at the extreme of the n region.

b) Find the minority carrier diffusion current and compare with the total hole current.

c) Find the expression for the total charge produced by the excess carriers in the n zone.

d) Find the expression for the diffusion capacity related to the n zone.

e) Finally, find the transit time for minority carriers (holes) in the n zone. Discuss the result when $S \gg D_p/W$ and when $S \ll D_p/W$.



a) We start writing the diffusion equation for holes under permanent regime and without external generation. In addition as the zone is electrically short $W << L_p$ we can neglect the recombination:

$$\frac{d^2p}{dx^2} = 0$$

The solution for this differential equation is straightforward:

$$p(x) = Ax + B$$

Boundary condition at $x = 0$ is $p(0) = p_o exp\left({}^{qV}/_{kT}\right)$ with $p_o = \frac{n_i^2}{N_D}$ and N_D the doping concentration of the n zone.

At $x = W$ the boundary condition is referred to a recombination velocity S. We write:

$$J_p = qS\Delta p(W) \Rightarrow -qD_p \left.\frac{dp}{dx}\right|_W = qS\Delta p(W)$$

Then, we have:

$$B = p_o exp\left({}^{qV}/_{kT}\right)$$

and

$$-D_p A = S(AW + B - p_o)$$

where we have used that $\Delta p(W) = p(W) - p_o$. We obtain A:

$$-\left(D_p + SW\right)A = Sp_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)$$

$$A = -\frac{S}{D_p + SW}p_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)$$

Therefore:

$$p(x) = -\frac{S}{D_p + SW}p_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)x + p_o exp\left({}^{qV}/_{kT}\right)$$

$$\Delta p(x) = p(x) - p_o = p_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)\left(1 - \frac{S}{\frac{D_p}{W} + S}\frac{x}{W}\right)$$

$$\Delta p(W) = p_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)\frac{\frac{D_p}{W}}{\frac{D_p}{W} + S}$$

Note that for $S \to \infty$ as in an ohmic contact $\Delta p(W) \to 0$.

b) The total current equals the current of the minority carrier at $x = 0$ because the junction is asymmetric.

$$J_T \approx J_{p\,diff}(x = 0)$$

$$J_T = -qD_p\left.\frac{dp}{dx}\right|_0 = q\frac{D_p}{W}\frac{S}{\frac{D_p}{W} + S}p_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)$$

c) Find now the total charge produced by the excess carrier in the n zone. Is only matter of a bit of calculus. The charge will be:

$$Q_p = \int_0^W q\Delta p\,(x)dx = \int_0^W qp_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)\left(1 - \frac{S}{\frac{D_p}{W}+S}\frac{x}{W}\right)dx$$

$$Q_p = qp_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)\left(W - \frac{S}{\frac{D_p}{W} + S}\frac{W}{2}\right)$$

$$Q_p = qp_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)\frac{W}{2}\left(2 - \frac{S}{\frac{D_p}{W} + S}\right) \quad \text{in units of } C/cm^3$$

d) The diffusion capacity is the derivative of the charge with respect the voltage

$$C_D = \frac{dQ_D}{dV} = qp_o \frac{W}{2}\left(2 - \frac{S}{\frac{D_p}{W} + S}\right)\frac{q}{kT} exp\left({}^{qV}/_{kT}\right)$$

e) Finally, the transit time is by definition:

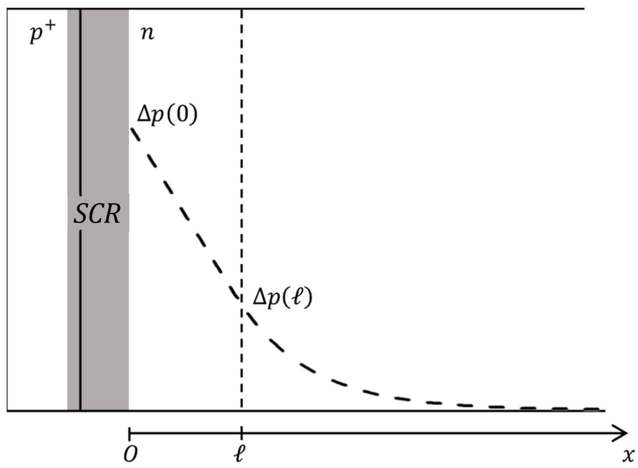
$$\tau_{tr} = \frac{Q_p}{J_p} = \frac{\frac{W}{2}\left(2 - \frac{S}{\frac{D_p}{W}+S}\right)}{\frac{D_p}{W}\frac{S}{\frac{D_p}{W}+S}} = \frac{W^2}{2D_p}\left(2\frac{\frac{D_p}{W}+S}{S} - 1\right)$$

In the limiting case $S \gg \frac{D_p}{W} \implies \tau_{tr} \approx \frac{W^2}{2D_p}$

whereas in the case $S \ll \frac{D_p}{W} \implies \tau_{tr} \approx \frac{W}{S}$

If the contact extracts the excess of carriers very slowly, then the diffusion constant does not play any role.

12. Consider a p^+n junction for which we may calculate the current regarding only the quasi-neutral n zone. Due a particular fabrication process the n region has two distinct zones. Until a distance $\boldsymbol{\ell}$ from the junction the recombination is negligible because the lifetime is extremely long. After $\boldsymbol{\ell}$ the recombination is high and the corresponding diffusion length $\boldsymbol{L_p}$, is short compared with the remaining width of the zone n. For simplicity consider the diffusion coefficient $\boldsymbol{D_p}$ constant all along the zone. The bias voltage results in a hole excess $\boldsymbol{\Delta p(0) = p_o(exp(V/V_T) - 1)}$ at the edge of the space charge region.



a) Find the expression for $\boldsymbol{\Delta p(\ell)}$ as a function of $\boldsymbol{\Delta p(0)}$, $\boldsymbol{\ell}$ and $\boldsymbol{L_p}$.

b) As the electrical characteristic can be written as $\boldsymbol{J = J_o(exp(V/V_T) - 1)}$, find the expression for the reversed saturation current $\boldsymbol{J_o}$.

a) In the first zone i.e. for $0 < x < \ell$ the recombination is negligible and consequently we can assume that the excess minority carrier distribution will be a straight line. We can write:

$$0 < x < \ell; \quad \Delta p(x) = \Delta p(0) - \left(\frac{\Delta p(0) - \Delta p(\ell)}{\ell}\right)x$$

The second zone is electrically long, then the excess carrier distribution is an exponential starting at $x = \ell$

$$\ell < x < \infty; \quad \Delta p(x) = \Delta p(\ell) exp\left(-\frac{x-\ell}{L_p}\right)$$

We know that at $x = 0$ we have:

$$\Delta p(0) = \frac{n_i^2}{N_D}\left(exp\left({}^{qV}/_{kT}\right) - 1\right)$$

On the other hand the excess carrier distribution at $x = \ell$ should be continuous and also the derivative (proportional to the diffusion current).

$$\Delta p(\ell^-) = \Delta p(\ell^+)$$

$$\left.\frac{d\Delta p}{dx}\right|_{x=\ell^-} = \left.\frac{d\Delta p}{dx}\right|_{x=\ell^+}$$

And now some calculus:

$$-\left(\frac{\Delta p(0) - \Delta p(\ell)}{\ell}\right) = \frac{\Delta p(\ell)}{L_p}$$

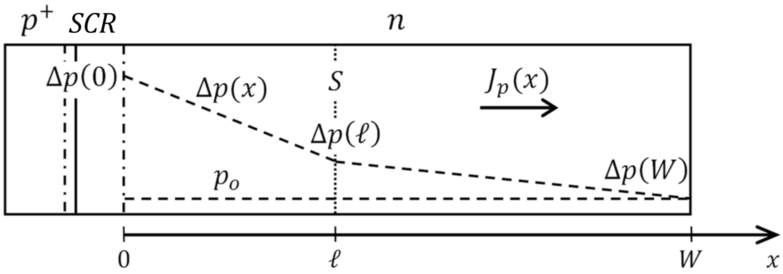
$$\Delta p(\ell) = \frac{\Delta p(0) L_p}{(L_p + \ell)}$$

b) Now, for the current density:

$$J = -qD_p \frac{d\Delta p}{dx}\bigg|_{x=0} = qD_p \frac{\Delta p(0) - \Delta p(\ell)}{\ell} = q\Delta p(0)\frac{D_p}{\ell}\left(\frac{\ell}{L_p + \ell}\right)$$

$$J = q\frac{n_i^2}{N_D}\frac{D_p}{\ell}\left(\frac{\ell}{L_p + \ell}\right)\left(exp\left({}^{qV}/_{kT}\right) - 1\right) = J_o\left(exp\left({}^{qV}/_{kT}\right) - 1\right)$$

13. *Multicrystalline silicon consists of small crystals with different dimensions and crystalline orientation. These crystalline domains are separated by grain boundaries where the high concentration of structural defects results in a high recombination. Consider a p^+n junction with a grain boundary in the volume of the n region. Its effect can be considered including a surface recombination S at a distance ℓ of the junction. Within the crystalline domains the hole diffusion length is long compared with the crystal dimensions. In the extreme of the n region we assume an ideal ohmic contact.*



a) Briefly justify the following boundary conditions:

$$\Delta p(0) = p_o(exp(V/V_T) - 1)$$
$$\Delta p(W) = 0$$
$$\Delta p \text{ continuous en } x = \ell$$
$$J_p(\ell^-) - J_p(\ell^+) = qS\Delta p(\ell)$$

b) Show that the current circulating through the diode can be calculated as:

$$J = \underbrace{q\frac{D_p}{W}p_o\frac{\frac{S}{D_p/W}+\frac{W}{W-\ell}}{\frac{S}{D_p/\ell}+\frac{W}{W-\ell}}}_{J_o}\left(exp\left(\frac{V}{V_T}\right)-1\right)$$

c) Comment on the J_o expression analysing the limiting cases depending on the S value and the location of the grain boundary.

A careful reading shows that actually we have a neutral zone divided in two regions, both electrically short where recombination can be neglected and the excess carrier distribution approximated by a straight line, and a plane with a certain amount of recombination determined by a surface recombination velocity S at the boundary between them.

a)

- At $x = 0$ we have the usual boundary condition at the edge of the space charge region
- At $x = W$ we assume an ideal ohmic contact and then $\Delta p(W) = 0$
- Δp has to be continuous at $x = \ell$ (and anywhere else) if the material is homogeneous
- We have a surface recombination at $x = \ell$. The current reaching the plane is equal to the current leaving the plane plus the recombination current at the plane

b) The circulating current will be $J \approx J_{p\,diff}(0) = -qD_p\frac{d\Delta p}{dx}\Big|_{x=0}$

Thus, we must find $\Delta p(x)$ between $x = 0$ and $x = \ell$.

$$x < \ell\,; \qquad \Delta p(x) = Ax + B$$

$$\ell < x < W; \qquad \Delta p(x) = Cx + D$$

Now we have to play wisely with the boundary conditions in order to find A that is what at the end we need.

$$B = \Delta p(0)$$

$$WC + D = 0$$

$$\ell A - \ell C - D = -\Delta p(0) \Longrightarrow \ell A + (W - \ell)C = -\Delta p(0)$$

$$-D_p A + D_p C = S\ell A + S\Delta p(0) \Longrightarrow \left(D_p + S\ell\right)A - D_p C = -S\Delta p(0)$$

Finally:

$$\left(D_p + S\ell\right)A + D_p \frac{\Delta p(0) + \ell A}{W - \ell} = -S\Delta p(0)$$

$$A = -\frac{S + \frac{D_p}{W - \ell}}{S\ell + D_p \frac{W}{W - \ell}} \Delta p(0)$$

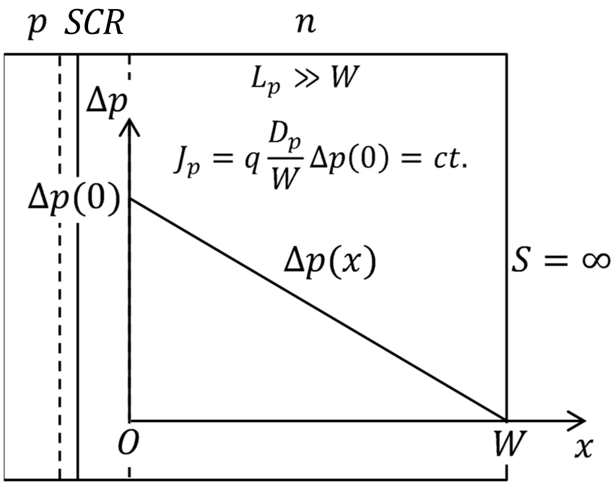
and:

$$J \approx J_{p\,diff}\,(0) = -qD_p A = qD_p \frac{S + \frac{D_p}{W - \ell}}{S\ell + D_p \frac{W}{W - \ell}} \Delta p(0)$$

$$J = q \frac{D_p}{W} p_o \frac{\frac{S}{D_p/W} + \frac{W}{W - \ell}}{\frac{S}{D_p/\ell} + \frac{W}{W - \ell}} \left(exp\left(\frac{V}{V_T}\right) - 1\right)$$

c) If $S \to \infty$ then $J_0 \approx q\frac{D_p}{\ell}p_o$ as you could expect. The infinite recombination at $x = \ell$ hides the right zone and the current is only fixed by the zone at the left of the boundary.
If $S \to 0$ then $J_0 \approx q\frac{D_p}{W}p_o$, the boundary between both zones does not affect anymore.

14. We focus in the quasi-neutral n zone of a semiconductor junction. The hole diffusion length is much longer than its width . As it is well known in this case the recombination is negligible and we can consider that all injected holes at $x = 0$ will be extracted at the contact (at $x = W$) and consequently the hole current will be constant.



a) Starting from the fact that at each point x we can express the current as a function of the carrier concentration and their velocity, find the expression for the hole velocity $v(x)$ at each point of the n zone.

b) Verify that the transit time obtained from a cinematic definition, that is:

$$\tau_{tr} = \int_0^W \frac{dx}{v(x)}$$

Coincides with the statistical definition $\tau_{tr} = Q_p/J_p$ more common in semiconductor devices textbooks.

a) The current can be written as the charge (moving) times the velocity i.e.

$$J = q\Delta p(x)v(x) = q\Delta p(0)\left(1 - \frac{x}{W}\right)v(x)$$

Simultaneously, we can write also $J = q\frac{D_p}{W}\Delta p(0)$. Then:

$$v(x) = \frac{q\frac{D_p}{W}\Delta p(0)}{q\Delta p(0)\left(1-\frac{x}{W}\right)} = \frac{\frac{D_p}{W}}{\left(1-\frac{x}{W}\right)} = \frac{D_p}{W-x}$$

b) The transit time according to a cinematic interpretation:

$$v(x) = \frac{dx}{dt} \Longrightarrow dt = \frac{dx}{v(x)}$$

$$\tau_{tr} = \int_0^W \frac{dx}{v(x)} = \int_0^W \frac{W-x}{D_p}dx = \frac{W^2}{2D_p}$$

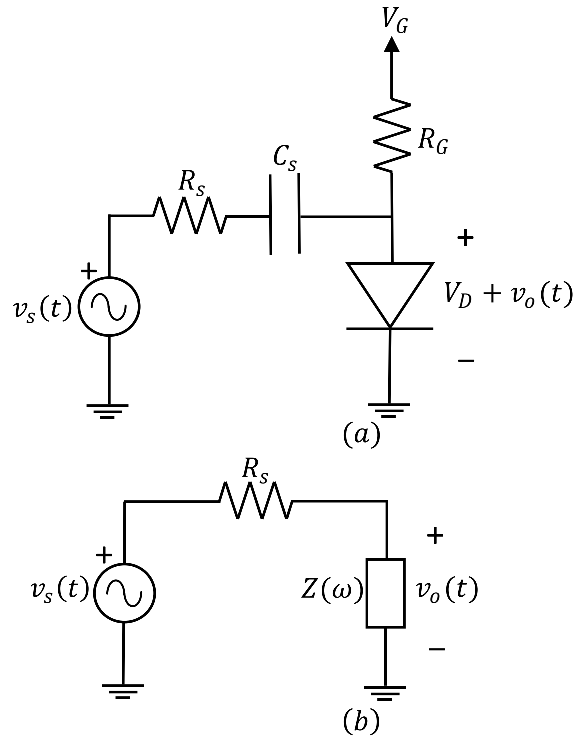
which is the usual definition of the transit time for a short zone (where the excess carrier distribution can be approximated by a straight line). We can further verify this by evaluating $\tau_{tr} = Q_p/J_p$:

$$Q_p = q\int_0^W \Delta p(x)dx = q\frac{\Delta p(0)}{2}W$$

$$J_p = q\frac{D_p}{W}\Delta p(0)$$

$$\tau_{tr} = \frac{Q_p}{J_p} = \frac{q\frac{\Delta p(0)}{2}W}{q\frac{D_p}{W}\Delta p(0)} = \frac{W^2}{2D_p}$$

15. A p⁺n diode is directly biased through a voltage source with $V_G = 30\,V$ and a resistance $R_G = 15\,k\Omega$, resulting in a $V_D = 0.56\,V$ (figure a). In the small signal circuit (figure b), the capacity C_s is so large than it can be considered a short circuit in the frequency range of v_s.



a) Identify the components of the impedance $Z(\omega)$ in the small signal circuit. In particular calculate the relevant parameters in the diode model. Cross section is $A = 0.01\,cm^2$ and the n region is semi-infinite.

b) Calculate the amplitude of the output voltage v_o at intermediate frequencies, in this range of frequencies the impedance $Z(\omega)$ is dominated by its resistive part. The output resistance of the signal generator is $R_s = 50\,\Omega$ and the amplitude of v_s is $V_p = 100\,mV$.

c) Calculate the frequency f_c for an amplitude of the output voltage reduced in a factor $\sqrt{2}$.

Data: $q = 1.6\times10^{-19}\,C$, $k_B = 1.38\times10^{-23}\,J/K$, $T = 300\,K$, $N_D = 10^{16}\,cm^{-3}$, $n_i = 1.5\times10^{10}\,cm^{-3}$, $\mu_p = 400\,cm^2V^{-1}s^{-1}$, $\tau_p = 0.1\,\mu s$

a) The impedance $Z(w)$ has a resistive part r_d (dynamic resistance) in parallel with a capacitance C_d. Under direct bias the capacitive part of the small signal equivalent circuit of the diode is the diffusion capacitance which models the fact that carrier concentration and consequently the associated charge have to be modified when the applied voltage varies.

Considering a p⁺n junction with semi-infinite n region the saturation current writes:

$$I_S \approx Aq\frac{D_p}{L_p}p_o = Aq\frac{D_p}{L_p}\frac{n_i^2}{N_D} = 3.6\times10^{-12}\,A$$

The current flowing through the diode will be:

$$I_D = I_S\left(exp\left(\tfrac{qV_D}{kT}\right) - 1\right) \approx I_S exp\left(\tfrac{qV_D}{kT}\right) = 1.9\,mA$$

On the one hand:

$$g_d = \frac{dI_D}{dV_D} = \frac{I_S\,exp\left(\frac{V_D}{V_T}\right)}{V_T} \approx \frac{I_D}{V_T} \Longrightarrow r_d = \frac{1}{g_d} = 13\,\Omega$$

where we have used that $V_T = kT/q$.

On the other hand:

$$C_D = \frac{dQ_D}{dV_D}$$

where Q_D is the charge accumulated by the excess minority carriers in the neutral zones. In this case, because the junction is asymmetric, we consider only the n zone (less doped):

$$Q_D = Aq\int_0^\infty \Delta p\,(x)dx = Aq\int_0^\infty \frac{n_i^2}{N_D}\left(exp\left(\frac{qV_D}{kT}\right) - 1\right)exp\left(-\frac{x}{L_p}\right)$$

We consider an exponential distribution because the n zone is long ($W \gg L_p$).

$$Q_D = Aq\frac{n_i^2}{N_D}L_p\left(exp\left(\frac{qV_D}{kT}\right) - 1\right)$$

and

$$C_D = \frac{dQ_D}{dV_D} = Aq\frac{n_i^2}{N_D}\frac{L_p}{V_T}exp\left(\frac{qV_D}{kT}\right)$$

As the circulating current is:

$$I_D = Aq\frac{n_i^2}{N_D}\frac{D_p}{L_p}\left(exp\left(\frac{qV_D}{kT}\right) - 1\right) \approx Aq\frac{n_i^2}{N_D}\frac{D_p}{L_p}exp\left(\frac{qV_D}{kT}\right)$$

and using that $L_p^2 = D_p\tau_p$, we can re-write the expression for C_D as:

$$C_D = \frac{I_D\tau_p}{V_T} = 7.6\,nF$$

b) At intermediate frequencies $Z \approx r_d$:

$$v_o = \frac{r_d}{R_S + r_d}v_s$$

if the input signal amplitude is $100\ mV$, then the output amplitude will be $21\ mV$.

c) At high frequencies, everything becomes more convolute:

$$\frac{1}{Z} = \frac{1}{r_d} + jC_d\omega = g_d + jC_d\omega \Longrightarrow Z = \frac{1}{g_d + jC_d\omega}$$

$$\left|\frac{V_0}{V_S}\right|^2 = \frac{1}{(1 + R_s g_d)^2 + (R_s C_d\omega)^2}$$

We are asked to find the frequency at which the output voltage reduces its low frequency value by a factor of $\sqrt{2}$ that is

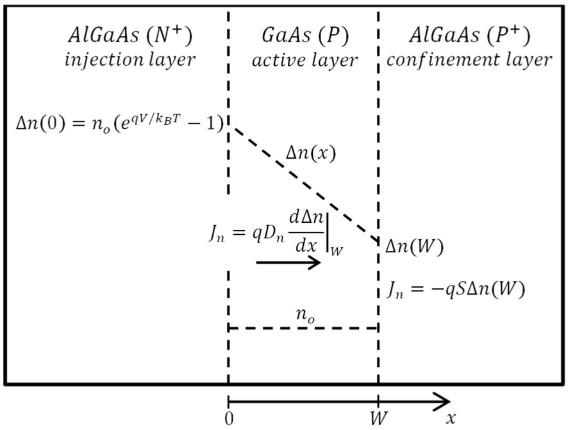
$$\left|\frac{V_0}{V_S}\right|^2 = \left(\frac{\frac{r_d}{R_s + r_d}}{\sqrt{2}}\right)^2 = \frac{1}{2}\frac{1}{(1 + R_s g_d)^2}$$

Comparing both equations we see that $(1 + R_s g_d)^2 = (R_s C_d\omega_c)^2$:

$$R_s C_d\omega_c = 1 + R_s g_d = \frac{R_s + r_d}{r_d}$$

$$\omega_c = 2\pi f_c = \frac{1}{\frac{r_d R_s}{R_s + r_d}C_d} = \frac{1}{(r_d||R_s)C_d} \rightarrow f_c = 2\ MHz$$

16. *Consider a heterojunction LED with an $AlGaAs\ (n^+)$ injection layer and an $GaAs\ (p)$ active layer whose width W is short compared with the minority carrier diffusion length. The role of the $AlGaAs\ (p^+)$ is to confine the electrons injected in the active zone increasing the radiative recombination. The effect of this confinement can be described through a recombination velocity S (or electron extraction velocity if you prefer) at the isotype boundary. The LED is directly polarized with a voltage V.*



a) Taking into account the electron affinity, the energy gap E_g and the doping type of each layer, sketch qualitatively the band diagram.

b) Find as a function of S and V the expression for the minority carrier excess $\Delta n(x)$ in the $GaAs$ active layer. Use the boundary conditions shown in the figure.

c) Find also the expression for the electron current density J_n. Discuss the reason why the total current circulating through the LED is mainly the electron component.

d) In the $GaAs$ active layer takes place a radiative recombination with characteristic time τ_n. Find as a function of S and V the expression for the total recombination in the active layer (width W).

In what follows calculate the values for the particular situation with $S = 10^4\ cm/s$ and V =1 V. For the $GaAs$ active layer take $n_i = 2 \times 10^6\ cm^{-3}$, $N_A = 10^{17}\ cm^{-3}$, $D_n = 20\ cm^2/s$, $W = 0.5\ \mu m$ y $\tau_n = 10\ ns$.

e) What is the LED emission wavelength? Calculate the emitted light power by unit area?

f) Calculate also the electric power consumed by the device and the energetic efficiency defined as the ratio emitted light power/consumed electric power.

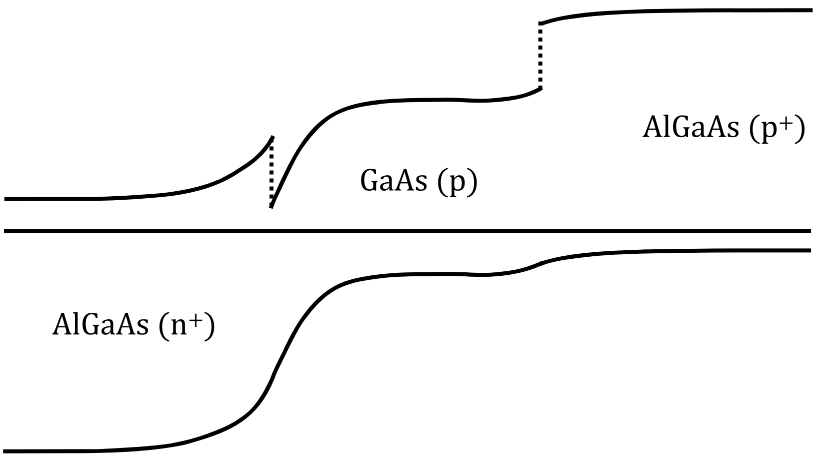
Data: In the active layer of $GaAs$: $\chi = 4.1\ eV$ and $E_g = 1.4\ eV$

In the $AlGaAs$: $\chi = 3.6\ eV$ and $E_g = 1.9\ eV$

Other data: $q = 1.6 \times 10^{-19}\ C$, $k_B = 8.62 \times 10^{-5}\ eV/K$, $T = 300\ K$,

$h = 6.62 \times 10^{-34}\ J/s$, $c = 3 \times 10^8\ m/s$

a)



b) As the active layer is short (compare L_n with W), the minority carrier distribution $\Delta n(x)$ can be, with good approximation, taken as a straight line.

$$\Delta n(x) = Ax + B$$

where

$$\Delta n(0) = B = n_o\left(exp\left(\frac{V}{V_T}\right) - 1\right)$$

The boundary condition at W writes:

$$qD_n \frac{d\Delta n}{dx}\bigg|_{x=W} = -qS\Delta n(W)$$

$$D_n A = -S\big(AW + \Delta n(0)\big)$$

$$A = -\frac{S}{S + \frac{D_n}{W}} \frac{1}{W} \Delta n(0)$$

$$\Delta n(x) = n_o\left(exp\left(\frac{V}{V_T}\right) - 1\right)\left(1 - \frac{S}{S + \frac{D_n}{W}} \frac{x}{W}\right)$$

c) The minority carrier diffusion currents are proportional to n_i^2, the square of the intrinsic concentration in the corresponding layer, which is on its turn proportional to $exp\big(-E_g/kT\big)$.

The net result is that a difference of 0.5 eV in the gap makes a huge difference in n_i^2 and consequently in the minority carrier diffusion length.

$$J = -J_{n,\ diff} = -qD_n \frac{d\Delta n}{dx} = q\frac{D_n}{W} \frac{S}{S + \frac{D_n}{W}} n_o\left(exp\left(\frac{V}{V_T}\right) - 1\right)$$

d) The total number of electron-hole pairs recombined per unit time and unit surface in the active layer will writes as:

$$\int_0^W \frac{\Delta n(x)}{\tau_n} dx = \frac{\Delta n(0)}{\tau_n} \frac{W}{2}\left(2 - \frac{S}{S + \frac{D_n}{W}}\right)$$

e) The LED emission wavelength is determined by the bandgap of the active layer $E_g = hc/\lambda$.

If we want to calculate the emitted light power by unit area we should multiply the number of photons emitted per second (obtained previously) by the energy of each photon:

$$P_{light} = h\nu\, n_o \left(exp\left(\frac{V}{V_T}\right) - 1\right) \frac{1}{\tau_n} \frac{W}{2} \left(2 - \frac{S}{S + \frac{D_n}{W}} \right) = 2.7\, mW/cm^2$$

f) The electric power consumed will be:

$$J \cdot V = V\, q\, \frac{D_n}{W} \frac{S}{S+\frac{D_n}{W}} n_o \left(exp\left(\frac{V}{V_T}\right) - 1\right) = 3.88\, mW/cm^2$$

and the LED efficiency:

$$\eta = \frac{P_{light}}{P_{electrical}} = \frac{h\nu\, n_o \left(exp\left(\frac{V}{V_T}\right) - 1\right) \frac{1}{\tau_n} \frac{W}{2} \left(2 - \frac{S}{S + \frac{D_n}{W}} \right)}{V\, q\, \frac{D_n}{W} \frac{S}{S+\frac{D_n}{W}} n_o \left(exp\left(\frac{V}{V_T}\right) - 1\right)} = \frac{h\nu}{qV} \frac{W^2}{2L_n^2} \left(1 + 2\frac{\frac{D_n}{W}}{S} \right)$$

Obviously, the expression is not valid for $S \to 0$, because several approximations that we make implicitly are not longer valid. On the one hand when we have assumed short zone and negligible recombination…, is it negligible in front of what? In fact we are saying that is negligible in front of recombination at the extreme (contact). If $S \to 0$ we have not recombination to compare with. On the other hand the flowing current also tends to zero. The flowing current is the addition of diffusion minority carrier current at both sides of the junction. The hole current has been neglected because the greater gap (neglected in front of hole current which is going to zero as S does). Finally if we look at the diffusion current in the active zone (thinking in terms of the continuity equation) is the addition of the recombination current within the active zone, which is neglected because we assume short zone, and the recombination at $x = W$ that is proportional to S and then goes to zero also. The consequence of all together is that we should be careful when we perform approximations. Neglect recombination terms in front of larger recombinations may be a good idea but neglect them in front of terms that have been already neglected may produce a disaster.

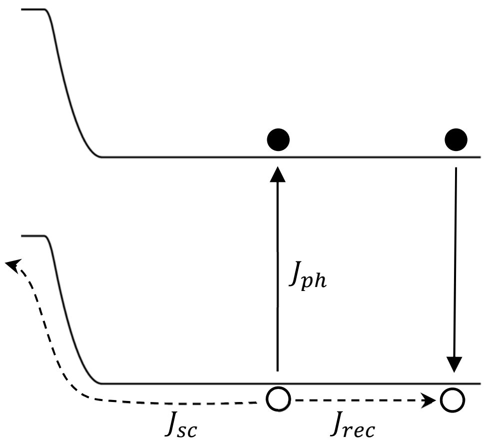
17. Consider a solar cell fabricated from a n type wafer with thickness $W = 200\ \mu m$. A p⁺ region, with negligible thickness in front of W, is diffused in only one face. The minority carrier diffusion length in the n zone is larger than the wafer thickness. The solar cell is illuminated with a radiation $(hc/\lambda \gtrsim E_g)$ leading to a uniform generation rate $G = 10^{19}\ cm^{-3}s^{-1}$ in all the device. The back contact is ohmic.

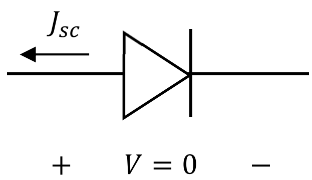
a) Explain why under short circuit a current J_{sc} flows due to the illumination. Show the direction of this current within the device and through the external circuit.

b) Calculate (without solving the diffusion equation) the J_{sc} value. Justify the obtained result and comment why it is a good approximation.

Data: $q = 1.6 \times 10^{-19}\ C$

a) The generation will take place in the n zone. The holes generated will diffuse in all directions. Those reaching the back contact will recombine while those diffusing towards the p zone will be collected. Holes are going to move from n to p (reverse direction).

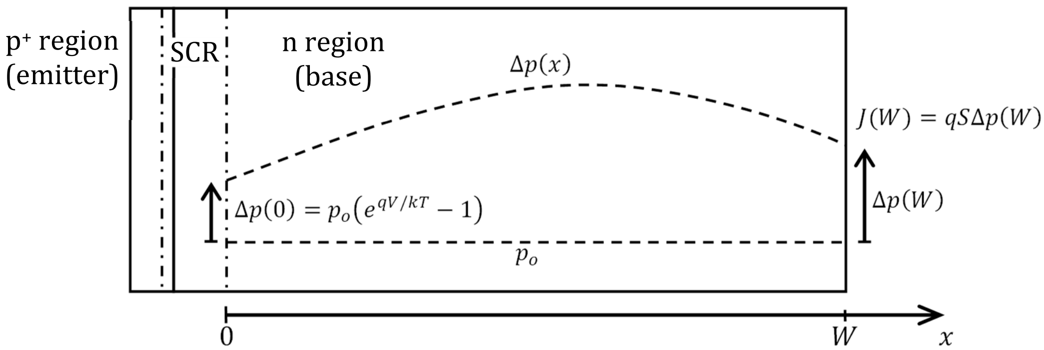




b) For the excess carrier distribution under short-circuit we have $\Delta p(0) = 0$ and $\Delta p(W) = 0$ because of the ohmic contact. If the generation is constant with x and the boundary conditions are symmetrical, the shape of $\Delta p(x)$should be also symmetrical. We are going to collect 50% of photogenerated carriers, losing the other 50% at the contact.

$$J_{sc} = \frac{1}{2} qGW = 16\ mA/cm^2$$

18. ***Consider a solar cell fabricated using an abrupt p^+n silicon junction where the p^+ region (which although there is not any physical reason is usually called emitter) is much thinner than the n zone (called "base" due to unclear historical reasons). Consequently the current flowing through the device can be reasonably calculated regarding only the minority carrier profile in the n zone. Suppose also that the quasineutral n zone is short compared with the minority carrier diffusion length. ($W \ll L_p$). The back contact presents a surface recombination velocity S. The device is illuminated with a wavelength long enough to get a carrier generation G uniform in the whole device.***



a) Find the expression for the excess minority carrier profile in the n zone for a given value of the applied voltage V.

b) Find the expression for the current density flowing through the device. Identify the current density J_{sc} circulating under short circuit conditions.

c) Comment about the variation of J_{sc} depending on the S value compared with D_p/W. Using the given data values calculate the S value allowing to collect the 90% of the photogenerated current within the device.

d) Find the expression for V_{oc} (voltage under open circuit conditions). Using the value for S previously found, calculate the reached value for V_{oc}.

Data: $N_D = 10^{16}\ cm^{-3}$, $n_i = 10^{10}\ cm^{-3}, D_p = 10\ cm^2 s^{-1}, W = 200\ \mu m, G = 5 \times 10^{18}\ cm^{-3} s^{-1} q = 1.6 \times 10^{-19}\ C, V_T = 25\ mV$

a) We solve the diffusion equation that gives the excess minority carrier distribution in the neutral zone:

$$D_p \frac{d^2 \Delta p}{dx^2} + G - \frac{\Delta p}{\tau_p} = 0$$

As the n zone is short compared with the minority carrier diffusion length we can neglect the recombination, then:

$$D_p \frac{d^2 \Delta p}{dx^2} = -G \ \rightarrow \frac{d^2 \Delta p}{dx^2} = -\frac{G}{D_p}$$

We can integrate directly:

$$\frac{d\Delta p}{dx} = -\frac{G}{D_p}x + A$$

and integrating again

$$\Delta p(x) = -\frac{G}{D_p}\frac{x^2}{2} + Ax + B$$

Now we impose the boundary conditions:

$$\Delta p(0) = B = p_o\left(exp\left(\frac{V}{V_T}\right) - 1\right)$$

$$-qD_p \left.\frac{d\Delta p}{dx}\right|_{x=W} = qS\Delta p(W)$$

Now a bit of algebraic work:

$$-D_p\left(-\frac{G}{D_p}W + A\right) = S\left(-\frac{G}{D_p}\frac{W^2}{2} + AW + \Delta p(0)\right)$$

$$A = GW\frac{1+\frac{SW}{2D_p}}{D_p + SW} - \frac{S}{D_p + SW}\Delta p(0)$$

and

$$\Delta p(x) = -\frac{G}{D_p}\frac{x^2}{2} + \left(GW\frac{1+\frac{SW}{2D_p}}{D_p + SW} - \frac{S}{D_p + SW}\Delta p(0)\right)x + \Delta p(0)$$

b) Remember that because the $p^+$is very thin no significant light absorption will occur, moreover the junction is asymmetric. All together allows us to calculate the total current as:

$$J = J_{p\,diff} = -qD_p \left.\frac{d\Delta p}{dx}\right|_{x=0} = -qD_pGW\frac{1+\frac{SW}{2D_p}}{D_p+SW} + qD_p\frac{S}{D_p+SW}\Delta p(0)$$

$$J = q\frac{D_p}{W}\frac{S}{\frac{D_p}{W}+S}\frac{n_i^2}{N_D}\left(exp\left(\frac{V}{V_T}\right) - 1\right) - qGW\frac{1+\frac{SW}{2D_p}}{1+\frac{SW}{D_p}}$$

Under short circuit conditions ($V = 0$) we have only the second term meaning that a current proportional to the generation produced by light is circulating from N to P. This current, called short circuit current, writes as:

$$J_{sc} = -qGW\frac{1+\frac{1}{2}\frac{S}{D_p/W}}{1+\frac{S}{D_p/W}}$$

c) If S is high, in particular $S \gg D_p/W$, then:

$$J_{sc} \approx -\frac{1}{2} qGW$$

which means that from the total photogenerated carriers GW (cm^2/s) per unit surface and unit time we are going to lose 50% due to recombination at the back contact.

If S is low, $S \ll D_p/W$, then:

$$J_{sc} = -qGW$$

We have a factor 2 between both extreme cases. It's not surprising that solar cell designers concentrate in reducing back recombination while maintaining a good electric contact (not an easy task).

The factor reducing the effectively collected J_{sc} below its maximum value is:

$$x = \frac{1 + \frac{1}{2} \frac{S}{D_p/W}}{1 + \frac{S}{D_p/W}}$$

If we replace D_p and W by its values, we have $D_p/W = 500\ cm/s$. If we want $x = 0.99$ we can solve for S obtaining $S = 125\ cm/s$ as a maximum value for S at the back surface.

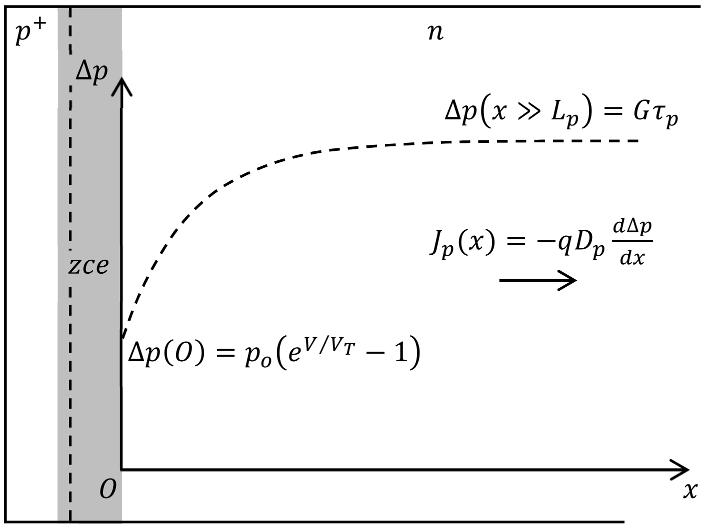
d) The open circuit voltage, V_{oc},is the applied voltage for which the circulating current is zero.

$$J = J_o \left(e^{V/V_T} - 1\right) - J_{sc}$$

For $V = V_{oc} \Longrightarrow J = 0$, then:

$$V_{oc} = V_T \ln\left(\frac{J_{sc}}{J_o} + 1\right) \approx 623\ mV;\ J_{sc} = 16\ mA/cm^2\ ;\ J_o = 0.16 \times 10^{-12}\ A/cm^2$$

19. *Consider a solar cell based on a p^+n junction with a very thin p^+ region. So, as we have seen in previous problems, the circulating current can be calculated considering only the n region. The cell is illuminated and we get an uniform generation profile G. Simultaneously we apply an external voltage V. The hole diffusion length is much shorter than the n zone width, i.e. we can consider the n zone long.*



a) Find the expression $\Delta p(x)$ for the excess minority carriers in the n region.

b) Calculate the total current J flowing through the device and the electric power generated by the cell.

c) Find the expression for the small electric field in the n region. Calculate its value for $x \gg L_p$ and comment about the result

$\boldsymbol{Data}$: $N_D = 10^{16}\ cm^{-3}$, $n_i = 10^{10}\ cm^{-3}$, $G = 2 \times 10^{19}\ cm^{-3}s^{-1}$, $kT/q = 25\ mV$
$q = 1.6 \times 10^{-19}\ C$, $V = 0.5\ V$, $D_p = 12\ cm^2 s^{-1}$, $\tau_p = 3\ \mu s$,
$L_p = \sqrt{D_p \tau_p} = 60\ \mu m$, $\frac{D_n}{D_p} = 3$

a) In order to find $\Delta p(x)$ we need to solve again the diffusion equation, this time with a constant excitation G:

$$\frac{d^2\Delta p}{dx^2} - \frac{\Delta p}{L_p^2} = -\frac{G}{D_p}$$

It's a second order differential equation with constant coefficients and constant excitation. The solution is the solution of the homogeneous equation (without excitation) plus a particular solution of the complete equation. For finding the particular solution of the complete equation a common possibility is to try in the equation a function of the same kind of the excitation in this case constant. They are some common sense reasons for doing so, far enough from the boundaries it's quite reasonable that the system response follows somehow the excitation. Let's try then $\Delta p(x) = K$. If we substitute in the equation we have

$$-\frac{K}{L_p^2} = -\frac{G}{D_p} \quad \rightarrow K = G\tau_p$$

The result could have been guessed intuitively. Far away from the junction what we have is a semiconductor with lifetime τ_p and constant generation G. It's easy to see that then:

$$\Delta p(x) = G\tau_p.$$

Together with the solution of the homogeneous equation we obtain the complete solution:

$$\Delta p(x) = A\ exp\left(-\frac{x}{L_p}\right) + B\ exp\left(\frac{x}{L_p}\right) + G\tau_p.$$

As the zone is long ($x \gg L_p$), B has to be zero:

$$\Delta p(x) = A\ exp\left(-\frac{x}{L_p}\right) + G\tau_p$$

$$\Delta p(0) = A + \ G\tau_p\ \rightarrow A = \Delta p(0) - G\tau_p$$

$$\Delta p(x) = \left(\Delta p(0) - G\tau_p\right) exp\left(-\frac{x}{L_p}\right) + G\tau_p$$

$$\Delta p(x) = \Delta p(0)\ exp\left(-\frac{x}{L_p}\right) + G\tau_p\left(1 + exp\left(-\frac{x}{L_p}\right)\right)$$

where:

$$\Delta p(0) = p_o\left(exp\left(\frac{V}{V_T}\right) - 1\right) = \frac{n_i^2}{N_D}\left(exp\left(\frac{V}{V_T}\right) - 1\right) = 4.85 \times 10^{12}\ cm^{-3}$$

and

$$G\tau_p\ = 6 \times 10^{13} cm^{-3}$$

b) The total current J approximatively equals the minority carrier diffusion current at the boundary of the less doped zone with the space charge region (remember that the junction is asymmetric)

$$J \approx J_{p\ diff}(0) = -qD_p\left.\frac{d\Delta p}{dx}\right|_{x=0}$$

$$J \approx J_{p\ diff}(0) = q\frac{D_p}{L_p}\Delta p(0) - qGL_p$$

$$J = 1.55\, mA/cm^2 - 19.2\ \ mA/cm^2\ = -17.65\ \ mA/cm^2$$

$$P = VJ = 0.5(-17.65)\, mW/cm^2\ = -8.8\ \ mW/cm^2$$

c) Now we are asked about the small electric field appearing in the n zone. First of all, let's write the total current as $J = J_p(x) + J_n(x)\ \forall x$. First take a x in the n zone. We can write:

$$J = J_{p,diff}(x) + J_{p,drift}(x) + J_{n,diff}(x) + J_{n,drift}(x)$$

We can neglect the hole drift current. On the one hand the electric field is very small (in fact we are looking for it) and on the other hand holes are minority carriers and are few (at least compared with majority carriers). We have then:

$$J = J_{p,diff}(x) + J_{n,diff}(x) + J_{n,drift}(x)$$

Because of quasi-neutrality $\Delta n(x) = \Delta p(x)$ so:

$$J_{n,diff}(x) = qD_n\frac{d\Delta n(x)}{dx} = -\frac{D_n}{D_p}\left(-D_p\frac{d\Delta p(x)}{dx}\right) = -3J_{p,diff}(x)$$

$$J_{p,diff}(x) = \left(qD_p\Delta p(0)\frac{1}{L_p} - qD_pG\tau_p\frac{1}{L_p}\right)exp\left(-\frac{x}{L_p}\right) = J_p(0)exp\left(-\frac{x}{L_p}\right)$$

The total current can be calculated as the minority carrier current at the boundary of the space charge region i.e. $J_p(0) = J$. We write then $J_{p,diff}(x) = J\ exp\left(-\frac{x}{L_p}\right)$.

The total current:

$$J = J_{p,diff}(x) + J_{n,diff}(x) + J_{n,drift}(x)$$

Will be written now:

$$J = Jexp\left(-\frac{x}{L_p}\right) - 3Jexp\left(-\frac{x}{L_p}\right) + J_{n\ drift}(x)$$

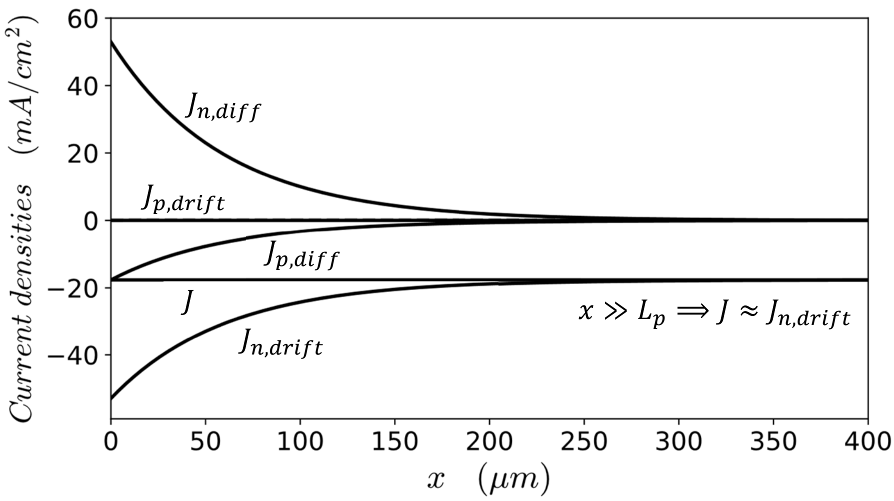
$$J = Jexp\left(-\frac{x}{L_p}\right) - 3Jexp\left(-\frac{x}{L_p}\right) + qN_D\mu_n E(x)$$

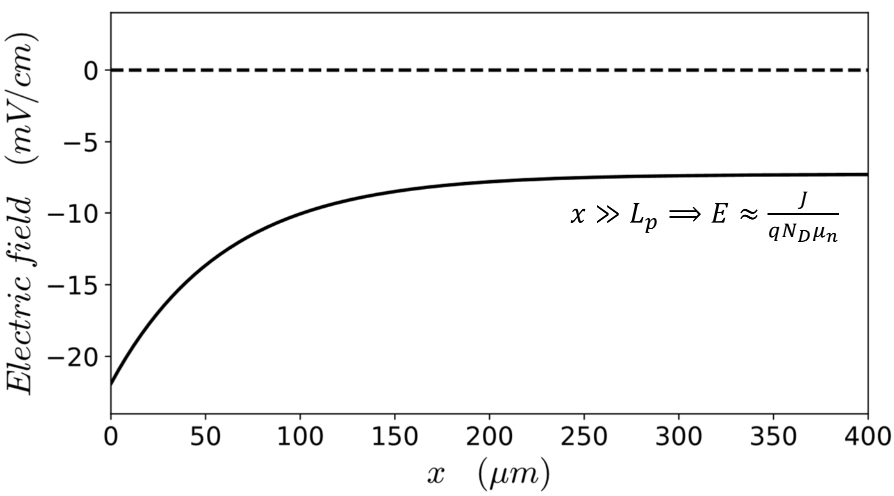
$$E(x) = \frac{J}{qN_D\mu_n}\left(1 + 2\ exp\left(-\frac{x}{L_p}\right)\right)$$

For $x \gg L_p$:

$$E \approx \frac{J}{qN_D\mu_n} = -7.3\ mV/cm$$

Both excesses $\Delta p(x)$ and $\Delta n(x)$ go to zero when x increases, also the diffusion currents go to zero. At the end, for x large enough the whole current is majority carrier drift current, drifted by this very small electric field.





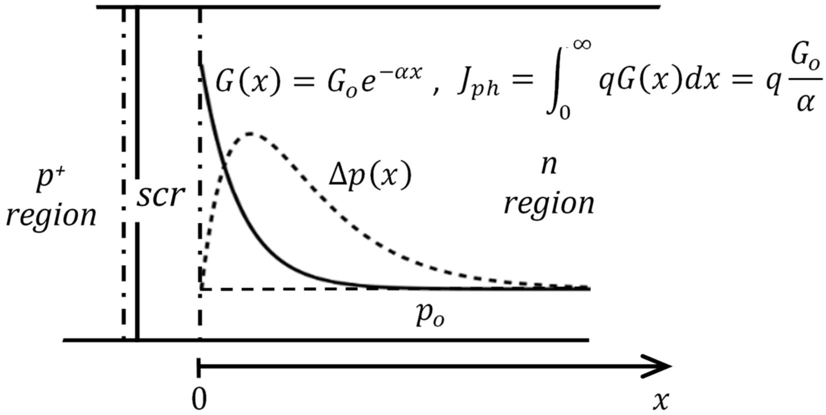
20. Consider an abrupt silicon p⁺n junction used as a solar cell. The p⁺ region is much thinner than the n zone, as a consequence the circulating current can be calculated with good approximation regarding only to the minority carrier profile in the n zone. The solar cell is illuminated with a monochromatic radiation producing an exponential generation profile $G(x)$. The light penetration length $(1/\alpha)$ is much shorter than the n region width, resulting in a complete absorption. Moreover, the n region can be considered long (semi-infinite) compared with the hole diffusion length (L_p). The solar cell is under short-circuit ($0\ V$ applied voltage).

a) Find the hole excess profile in the n region $\Delta p(x)$*.

b) Obtain an expression for the current density in short circuit conditions, J_{sc}.

The internal quantum efficiency (IQE) is defined as the fraction of the photogenerated current which is effectively collected:

$$IQE = \frac{J_{sc}}{J_{ph}}$$



c) Obtain an expression for IQE, depending only of α and L_p.

*** In the corresponding non-homogeneous differential equation try a particular solution $\Delta p(x) = Ke^{-\alpha x}$, with K a coefficient to determine.**

a) The situation resembles that of the previous problem, the difference is that now the excitation is exponential. The procedure is the same, for solving:

$$\frac{d^2\Delta p}{dx^2} - \frac{\Delta p}{L_p^2} = -\frac{G(x)}{D_p}$$

We have to add the solution of the homogeneous equation and a particular solution of the complete equation. This particular solution will have the shape of the excitation.

Let's try a particular solution as $\Delta p(x) = C\ exp(-\alpha x)$. Substituting this expression for $\Delta p(x)$ in the diffusion equation we get :

$$C\alpha^2 exp(-\alpha x) - \frac{Cexp(-\alpha x)}{L_p^2} = -\frac{G_o exp(-\alpha x)}{D_p} \Rightarrow C = -\frac{\frac{G_o}{D_p}}{\alpha^2 - \frac{1}{L_p^2}}$$

Then we can write for $\Delta p(x)$

$$\Delta p(x) = A\ exp\left(\frac{x}{L_p}\right) + B\ exp\left(-\frac{x}{L_p}\right) - \frac{\frac{G_o}{D_p}}{\alpha^2 - \frac{1}{L_p^2}} exp(-\alpha x)$$

The constant A should be zero because the region is long and $x \gg L_p$

$$\Delta p(x) = B\ exp\left(-\frac{x}{L_p}\right) - \frac{\frac{G_o}{D_p}}{\alpha^2 - \frac{1}{L_p^2}} exp(-\alpha x)$$

Under short circuit $\Delta p(0) = 0$, and we can obtain B:

$$B = \frac{\frac{G_o}{D_p}}{\alpha^2 - \frac{1}{L_p^2}}$$

and

$$\Delta p(x) = \frac{\frac{G_o}{D_p}}{\alpha^2 - \frac{1}{L_p^2}} \left(exp\left(-\frac{x}{L_p}\right) - exp(-\alpha x) \right)$$

b) The short circuit current will be

$$J_{sc} = -qD_p \frac{d\Delta p}{dx}\bigg|_{x=0} = -qD_p \frac{\frac{G_o}{D_p}}{\alpha^2 - \frac{1}{L_p^2}} \left(-\frac{1}{L_p} + \alpha\right) = qG_0 \frac{1}{\alpha + \frac{1}{L_p}}$$

In order to obtain this result you should remember $(a+b)(a-b) = a^2 - b^2$, an useful result that we learned at school.

c) Let's calculate first the photogenerated current

$$J_{ph} = \int_0^\infty q\, G_0 e^{-\alpha x} dx = q\frac{G_0}{\alpha}$$

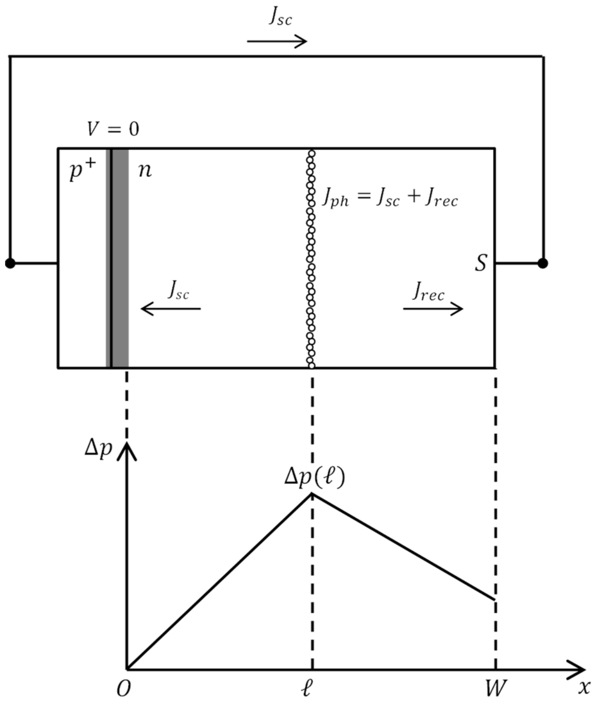
remember that the zone is thick enough to reach complete absorption.
Finally, the internal quantum efficiency is:

$$IQE = \frac{J_{sc}}{J_{ph}} = \frac{\alpha}{\alpha + \frac{1}{L_p}}$$

21. One of the reasons limiting the efficiency of solar cells is the waste of photons with energy less than the semiconductor energy gap. Nevertheless in the last years researchers are looking for strategies to overcome this limitation. Among these strategies the local introduction of nanoparticles (see figure) may result in an extra generation of carriers due to two photon absorption. The net result would be a J_{ph} due to those low energy photons.

Imagine a solar cell with a p⁺n structure whose n region has been modified including an extremely narrow region with nanoparticles at a distance ℓ of the junction. At the back contact of the solar cell we have a surface recombination velocity S. The solar cell is illuminated with a radiation whose energy is clearly lower than the bandgap energy and may be partially absorbed at the nanoparticles region. The generated carriers may diffuse towards the junction and be collected contributing to the photogenerated current or alternatively to diffuse towards the back contact where they will recombine.

Considering the n region short and the device under short circuit, the hole excess at $x = \ell$ is given by equation (i), where J_{ph} is the current density photogenerated at the nanoparticles region and D_p is the hole diffusion coefficient in the n region.



$$\Delta p(\ell) = \frac{J_{ph}/q}{\dfrac{D_p}{\ell} + \dfrac{D_p}{W-\ell}\dfrac{S}{S + \dfrac{D_p}{W-\ell}}} \qquad (i)$$

a) Find an expression for the collection efficiency, defined as the fraction of the photogenerated current collected under short circuit, $\chi = J_{sc}/J_{ph}$. Discuss the limiting situations for χ comparing S with $D_p/(W-\ell)$.

b) Can you obtain the expression (i) for $\Delta p(\ell)$?

a) Another impressive problem when you read it for the first time, but at the end of the day it reduces to calculate:

$$J_{sc} = -qD_p \frac{d\Delta p}{dx}\bigg|_{x=0}$$

In fact we do not need to worry about the signs, χ is defined positive or if you prefer taking as positive for J_{sc} given in the figure we can forget about the usual minus sign appearing in the hole diffusion current.

$$J_{sc} = qD_p \frac{d\Delta p}{dx}\bigg|_{x=0} = qD_p \frac{\Delta p(\ell)}{\ell}$$

the rest reduces again to work with the expressions.

$$J_{sc} = qD_p \frac{\Delta p(\ell)}{\ell} = q\frac{D_p}{\ell}\frac{J_{ph}/q}{\dfrac{D_p}{\ell} + \dfrac{D_p}{W-\ell}\dfrac{S}{S + \dfrac{D_p}{W-\ell}}}$$

$$\chi = \frac{J_{sc}}{J_{ph}} = \frac{1}{1 + \frac{\ell}{W-\ell}\frac{S}{S + \frac{D_p}{W-\ell}}}$$

The limiting cases are:

$$S \gg \frac{D_p}{W-\ell} \Rightarrow \chi \approx 1 - \frac{\ell}{W}$$

$$S \ll \frac{D_p}{W-\ell} \Rightarrow \chi \approx \frac{1}{1 + \frac{S}{D_p/\ell}}$$

b) The generated carriers at the nanoparticles layer partially diffuse towards the junction contributing to J_{sc} and the rest diffuse towards the backcontact where they recombine and are lost. We can write:

$$J_{ph} = J_{sc} + J_{rec}$$

$$J_{sc} = qD_p \frac{\Delta p(\ell)}{\ell}$$

$$J_{rec} = qD_p \frac{\Delta p(\ell) - \Delta p(W)}{W-\ell}$$

$$J_{rec} = qS\Delta p(W)$$

Now, a bit of algebra:

$$\Delta p(W) = \frac{J_{rec}}{qS}$$

$$J_{rec} = qD_p \frac{\Delta p(\ell) - \frac{J_{rec}}{qS}}{W-\ell} \Rightarrow J_{rec}\left(1 + \frac{\frac{D_p}{S}}{W-l}\right) = q\frac{D_p}{W-l}\Delta p(l)$$

$$J_{rec} = q\frac{D_p}{W-\ell}\Delta p(\ell)\frac{S}{S + \frac{D_p}{W-\ell}}$$

Coming back to $J_{ph} = J_{sc} + J_{rec}$ and substituting the expression for J_{rec}:

$$J_{ph} = qD_p \frac{\Delta p(\ell)}{\ell} + q\frac{D_p}{W-\ell}\Delta p(\ell)\frac{S}{S + \frac{D_p}{W-\ell}}$$

And finally:

$$\Delta p(\ell) = \frac{J_{ph}/q}{\frac{D_p}{\ell} + \frac{D_p}{W-\ell}\frac{S}{S + \frac{D_p}{W-\ell}}}$$

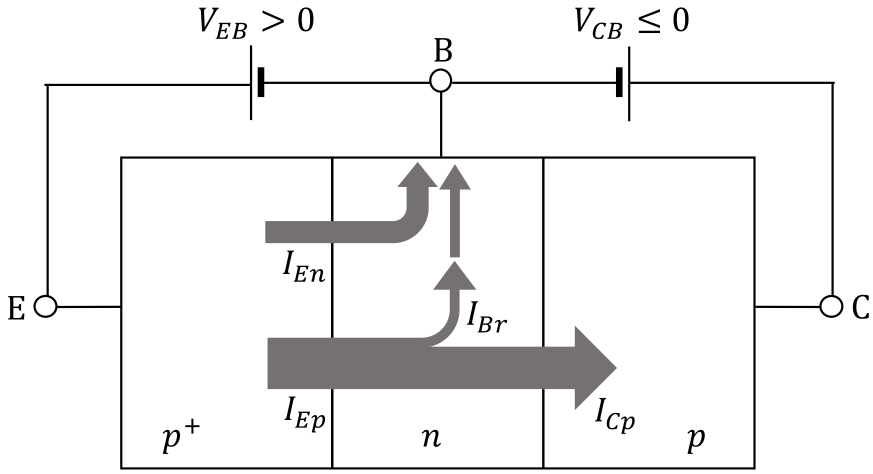
Chapter 3.

The Bipolar Junction Transistor

1. ***In a bipolar transistor define the base transport factor α_T. Show that it can be calculated with good approximation using:***

$$\boldsymbol{\alpha_T \approx 1 - \frac{W^2}{2L^2}}$$

where W is the neutral base width and L is the base minority carrier diffusion length. Justify all approximations. You may use either a NPN or PNP structure.



Before enter into the detail of the problem let's try to explain shortly the basic operation of a bipolar transistor: In a bipolar transistor the directly biased base emitter junction injects carriers from the emitter into the base, and also from the base into the emitter but let's forget them until later. Provided the base collector junction is reverse biased or short-circuited, most carriers injected into the base reach the collector (those which do not recombine in the base). The current through the base terminal will provide the carriers that will be injected by the base into the emitter (remember that the junction is directly biased) in addition to the flow of carriers needed to maintain the recombination in the base (under permanent regime we have carrier distributions constant with time, if carriers recombine a flow of incoming carriers is needed to maintain the distributions constant).

In what follows we restrict to small signal transistors forgetting power transistors. If the transistor is properly designed, the carriers injected by the emitter into the base are many more than the ones injected by the base into the emitter, if additionally we have low recombination in the base, the total base current will be small (much smaller than the others). At the end of the day a small change in the base emitter bias will produce a big change in the flow of carriers injected into the base and consequently in the collected flow of carriers at the collector. As a summary we can say that a good small signal transistor needs a small base current and for achieve this goal on one hand a low recombination into the base is needed, and on the other hand the injected current by the emitter into the base has to be much greater than the current injected by the base into the emitter.
We take a PNP transistor. If we consider a short base the minority carrier distribution is a straight line with boundary conditions:

$$\Delta p(0) = p_o \left(exp\left(\frac{V_{EB}}{V_T}\right) - 1 \right);\ \Delta p(W_B) = 0 \text{ for } V_{CB} = 0$$

Then, we can write the excess minority carrier distribution:

$$\Delta p(x) = \Delta p(0)\left(1 - \frac{x}{W_B}\right)$$

The hole current injected by the p-type emitter into the base will write:

$$I_{EP} = qAD_p \frac{\Delta p(0)}{W_B}$$

The hole current reaching the base-collector junction will be $I_{CP} = I_{EP} - I_{Br}$ where I_{Br} is the recombination within the base.

$$I_{Br} = \int_0^{W_B} qA \frac{\Delta p(x)}{\tau_p} dx = qA \frac{\Delta p(0)}{\tau_p} \int_0^{W_B} \left(1 - \frac{x}{W_B}\right) dx = qA \frac{\Delta p_n(0)}{\tau_p} \frac{W_B}{2}$$

The transport factor α_T is defined as the fraction of holes entering in the base that reaches the collector i.e:

$$\alpha_T = \frac{I_{CP}}{I_{EP}} = \frac{I_{EP} - I_{Br}}{I_{EP}} = 1 - \frac{I_{Br}}{I_{EP}}$$

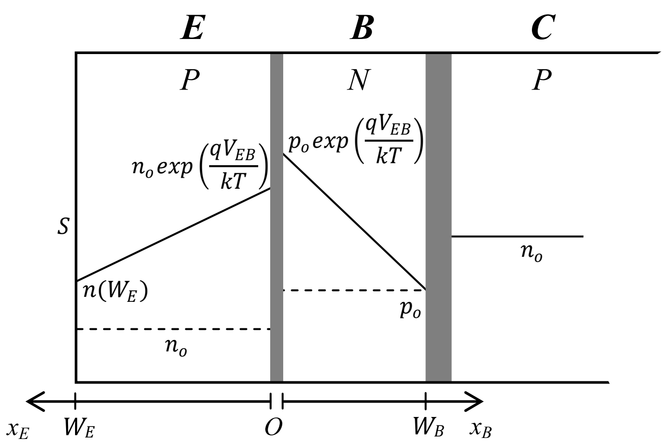
$$\alpha_T = 1 - \frac{qA \frac{\Delta p_n(0)}{\tau_p} \frac{W_B}{2}}{qAD_p \frac{\Delta p_n(0)}{W_B}} = 1 - \frac{{W_B}^2}{2D_p \tau_p} = 1 - \frac{{W_B}^2}{2L_p^2}$$

Remember that $L_p^2 = D_p \tau_p$.

If the explanation at the beginning of the problem has been useful you will agree with me that α_T needs to be near 1 in a good transistor.

May be the reader has already noticed some flaws in the reasoning. First we have started saying that the base is short. In fact the short zone approach is equivalent to neglect the recombination in the diffusion equation and if we neglect the recombination obviously all carriers entering in the base reach the collector and the transport factor is 1. In fact what we have done is slightly more subtle, we have assumed a short base in order to easily find an approximative minority carrier distribution in the base. Once a straight line carrier distribution is assumed, we calculate the recombination as proportional to the integral of the distribution (not taking derivatives at both extremes and subtracting as both derivatives would be equal and the difference zero) and we subtract this recombination from the incoming current. In practice is a very good approximation. For an exact solution we should have solved the diffusion equation in the base including the recombination term and then calculate the current reaching the collector as proportional to the derivative of the distribution at the collector base boundary.

2. ***A PNP bipolar transistor is biased in the so called active zone with the base-emitter junction directly biased and the base-collector short-circuited. The base neutral region is short compared with the minority carrier diffusion length, consequently the transport factor (α_T) is close to 1. In the emitter, also short, we have a surface recombination velocity S at the front face. The boundary condition for the excess minority carrier concentration can be written as:***



$$\boldsymbol{qD_n \frac{d\Delta n}{dx_E}\Big|_{x_E=W_E} = -q\, S\, \Delta n(W_E)} \qquad \text{(signs according to the figure axis)}$$

a) ***Define the injection efficiency (γ_E) and find its expression for the proposed structure. Discuss the effect of S in γ_E. Find its value for the limiting cases $S \gg D_n/W_E$ and $S \ll D_n/W_E$.***

b) ***Define the current gain (β) and find its expression in this case. Discuss the effect of S in β and find the limiting results for $S \gg D_n/W_E$ and $S \ll D_n/W_E$.***

(Expressions would be function of the electron diffusion coefficient in the emitter D_n and holes in the base D_p, of the impurity concentration in the emitter N_E and base N_B, of the emitter and base neutral zone widths W_E and W_B and the surface recombination velocity at the front face S)

a) If in the previous problem we focused into α_T as a merit factor indicating the relative importance of the recombination in the base, now we are going to focus on another merit factor, the injection efficiency. The injection efficiency quantifies in what extent the current injected by the emitter into the base is larger than the current injected by the base into the emitter. By definition the injection efficiency γ_E is the ratio between the current injected by the emitter into the base (in a PNP transistor this will be hole current) and the total current through the emitter base junction. We write the emitter current $I_E = I_{EN} + I_{EP}$ Where I_{EN} is the electron current injected by the base into the emitter and I_{EP} is the hole current injected by the emitter into the base (in a PNP transistor). Then:

$$\gamma_E = \frac{I_{EP}}{I_{EN} + I_{EP}}$$

I_{EP} can be easily calculated. The excess minority carrier distribution is a straight line (see the figure). Then, and its derivative is constant all along the base:

$$I_{EP} = -q\, D_p \frac{d\Delta p(x)}{dx} = qD_p \frac{\Delta p(0)}{W_B}$$

$$I_{EP} = q \frac{D_p n_i^2}{W_B N_B}\left(exp\left(\frac{V_{EB}}{V_T}\right) - 1\right)$$

We neglect the area. All currents are proportional to the device area. Or if you prefer assume the area equal to 1 cm[2].

Now we calculate I_{EN}. In the emitter $\Delta n(x) = Ax + B$ with boundary values:

$$\Delta n(0) = B = n_0 \left(exp\left(\frac{V_{EB}}{V_T}\right) - 1 \right) = \frac{n_i^2}{N_E}\left(exp\left(\frac{V_{EB}}{V_T}\right) - 1 \right)$$

and

$$qD_n \frac{d\Delta n}{dx}\bigg|_{x_E = W_E} = -q\, S\, \Delta n(W_E)$$

with

$$\Delta n(W_E) = AW_E + \frac{n_i^2}{N_E}\left(exp\left(\frac{V_{EB}}{V_T}\right) - 1 \right)$$

We solve looking for A:

$$D_n A = -S\left(AW_E + \frac{n_i^2}{N_E}\left(exp\left(\frac{V_{EB}}{V_T}\right) - 1 \right) \right)$$

$$A = -\frac{S}{D_n + SW_E}\frac{n_i^2}{N_E}\left(exp\left(\frac{V_{EB}}{V_T}\right) - 1 \right)$$

We can now obtain:

$$I_{EN} = \; qD_n \frac{d\Delta n}{dx}\bigg|_{x_E=0} = -qD_n A = q\frac{D_n n_i^2}{W_E N_E}\frac{S}{\frac{D_n}{W_E} + S}\left(exp\left(\frac{V_{EB}}{V_T}\right) - 1 \right)$$

and finally:

$$\gamma_E = \frac{I_{EP}}{I_{EN} + I_{EP}} = \frac{q\dfrac{D_p n_i^2}{W_B N_B}}{q\dfrac{D_p n_i^2}{W_B N_B} + q\dfrac{D_n n_i^2}{W_E N_E}\dfrac{S}{\frac{D_n}{W_E} + S}} = \frac{1}{1 + \dfrac{D_n W_B N_B}{D_p W_E N_E}\dfrac{S}{\frac{D_n}{W_E} + S}}$$

For large values of S we find the classical result:

$$S \gg \frac{D_n}{W_E} \Longrightarrow \gamma_E \approx \frac{1}{1 + \dfrac{D_n W_B N_B}{D_p W_E N_E}}$$

whereas for low values of the surface recombination velocity S, as it may be the case in polysilicon emitter transistors, the current injected into the emitter decreases and as a consequence γ_E improves approaching 1.

$$S \ll \frac{D_n}{W_E} \Longrightarrow \gamma_E \approx \frac{1}{1 + \dfrac{W_B N_B S}{D_p N_E}}$$

b) The gain current β is defined as the ratio between collector and base current. The collector current is the hole current traversing the base and reaching the base collector boundary. As the base is short, the distribution a straight line and its derivative constant, equal to I_{EP} , on its hand the base current is only I_{EN} because we are neglecting the recombination in the base as it is short.

$$\beta = \frac{I_{EP}}{I_{EN}} = \frac{q\,\dfrac{D_p n_i^2}{W_B N_B}}{q\,\dfrac{D_n n_i^2}{W_E N_E}\,\dfrac{S}{\dfrac{D_n}{W_E}+S}} = \frac{D_p W_E N_E}{D_n W_B N_B\,\dfrac{S}{\dfrac{D_n}{W_E}+S}}$$

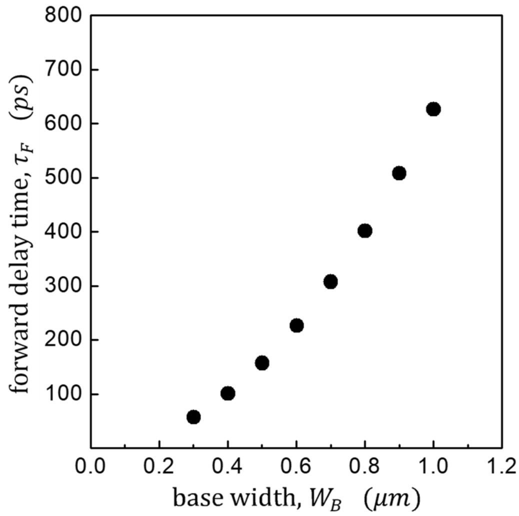
For high values of S (ohmic contact) we get the usual result:

$$S \gg \frac{D_n}{W_E} \Longrightarrow \beta = \frac{D_p W_E N_E}{D_n W_B N_B}$$

whereas for low values of S:

$$S \ll \frac{D_n}{W_E} \Longrightarrow \beta = \frac{D_p N_E}{S\, W_B N_B}$$

3. Consider a bipolar transistor biased in the active region. Changes in the total charge in the emitter-base junction are related with a delay time τ_F. This parameter τ_F limits the frequency performance of the transistor. It has been observed that for the usual values of W_B the delay time increases parabolically. Considering a short emitter and performing the approximations that you consider appropriate.



a) Find the expression for τ_F and verify that the values correspond with those shown in the figure. Calculate the cut-off frequency for a base width $W_B = 0.5\ \mu m$.

Now we evaluate for the extreme cases.

In first place, consider the case where the base width is much larger than the minority carrier diffusion length in the base.

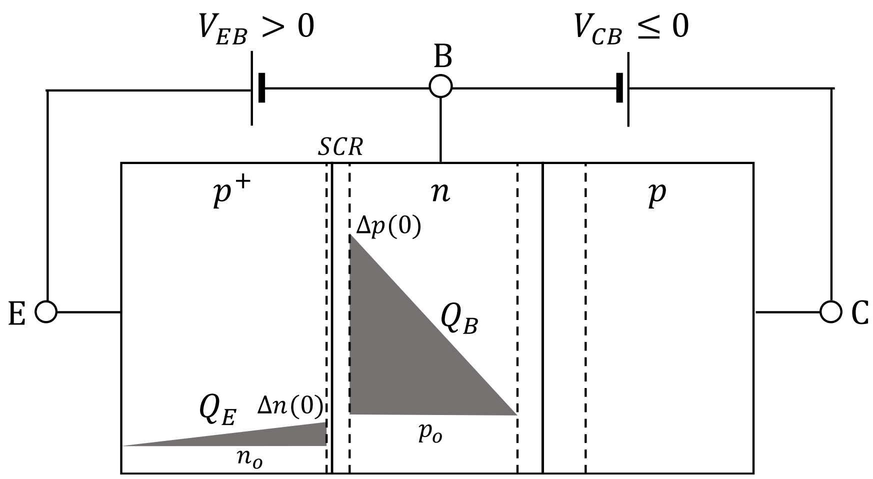
b) To what value will τ_F tend to saturate?

Consider now the other extreme case where the base width is very short.

c) What other term, usually negligible, limits the reduction of τ_F? Calculate τ_F for a base width of $W_B = 0.1\ \mu m$.

Data for the base: $D_{minB} = 8\ cm^2/s$, $\tau_{minB} = 1\ \mu s$, $N_B = 10^{17}\ cm^{-3}$

Data for the emitter: $D_{minE} = 6\ cm^2/s$, $N_E = 10^{18}\ cm^{-3}$, $W_E = 0.5\ \mu m$



a) The so called forward delay time τ_F is the ratio between the charge accumulated by the excess minority carrier in the transistor Q_F (part in the emitter and part in the base $Q_F = Q_E + Q_B$) and the collector current I_C.

$$\tau_F = \frac{Q_E + Q_B}{I_C}$$

The minority carrier diffusion length in the base will be $L_B = \sqrt{D_B \tau_B}$ = 28 μm.

As both Q_E and Q_B depend on $1/_{N_E}$ and $1/_{N_B}$ respectively and $N_E \gg N_B$, we can expect that $Q_E \ll Q_B$ and then $\tau_F \approx {Q_B}/_{I_C}$.

In the range of base widths in the figure, the base can be considered short, the excess charge Q_B is easily calculated integrating the excess minority carrier distribution (a straight line) assuming that the base collector junction is either short-circuited or reverse biased. Then:

$$\tau_F \approx \frac{q\frac{1}{2}\frac{n_i^2}{N_B}\left(exp\left(\frac{V_{EB}}{V_T}\right)-1\right)W_B}{q\frac{D_B}{W_B}\frac{n_i^2}{N_B}\left(exp\left(\frac{V_{EB}}{V_T}\right)-1\right)} = \frac{W_B^2}{2D_B} \quad \text{(base transit time)}$$

For $W_B = 1\ \mu m$, $\frac{W_B^2}{2D_B} = 625\ pS$

For $W_B = 0.3\ \mu m$, $\frac{W_B^2}{2D_B} = 56\ pS$

The corresponding values for the cut-off frequency will be $f_T = \frac{1}{2\pi\tau_F}$ leading to 0.25 and 2.8 GHz respectively.

b) If the base is much longer than the minority carrier diffusion length (a quite unrealistic situation indeed):

$$\tau_F \approx \frac{Q_B}{I_C} = \frac{q\Delta p(0)L_B}{q\frac{D_B}{L_B}\Delta p(0)} = \frac{L_B^2}{D_B} = \tau_B = 1\mu s$$

with increasing base width, the base transit time and consequently the forward transit time will saturate to the minority carrier lifetime, in this case $1\ \mu s$.

c) When the base becomes shorter, the accumulated excess charge Q_B also decreases. If the base is short enough we should consider Q_E which we neglected previously. Assuming a short emitter we get:

$$\tau_F = \frac{Q_E + Q_B}{I_C} \approx \frac{q\frac{1}{2}\Delta n(0)W_E + q\frac{1}{2}\Delta p(0)W_b}{q\frac{D_B}{W_B}\Delta p(0)} = \frac{1}{2}\frac{N_B W_B W_E}{N_E D_B} + \frac{W_B^2}{2D_B}$$

The first term decreases linearly with W_B, while the second decreases quadratically. For small enough values of W_B the first term will be dominant.

4. ***The figure shows the minority carrier excess in the quasi-neutral emitter and base of a NPN bipolar transistor biased in the active zone. The device area is $A = 10^{-4}\ cm^2$.***

   a) ***V_{BC} =0. Calculate V_{BE} applied to the base-emitter junction.***

   b) ***Calculate both the electron and hole components of the emitter current. Give also the value for the injection efficiency γ_E***

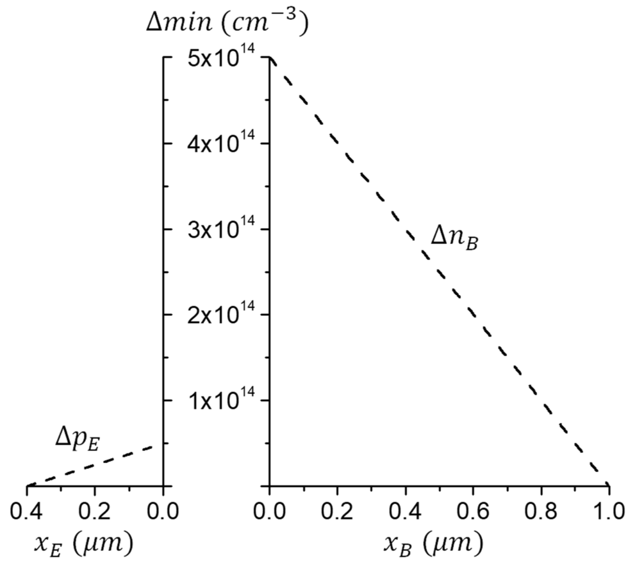
   c) ***Calculate the recombination current in the base and the corresponding transport factor α_T.***

   d) ***Calculate the currents at the terminals: I_E, I_B e I_C. What's the value of the current gain β of the transistor.***

   e) ***Calculate the small signal circuit parameters r_π y g_m.***

   f) ***Evaluate the charge stored in the device, the capacity C_π and the delay time τ_F.***

   g) ***Draw the small signal equivalent circuit and give an estimation of the cut-off frequency for which $|\beta| = 1$ (with the output short-circuited).***



Emitter data: $N_D = 10^{17}\ cm^{-3}$, $D_p = 1\ cm^2/s$
Base data: $N_A = 10^{16}\ cm^{-3}$, $D_n = 30\ cm^2/s$, $\tau_n = 0.2\ \mu s$
Other data: $n_i = 10^{10}\ cm^{-3}$, $q = 1.6 \times 10^{-19}\ C$, $k_B = 8.62 \times 10^{-5}\ eV/K$, $T = 300\ K$

a) The boundary condition at the emitter side of the emitter-base junction is, we take $\Delta p_E(0)$, from the figure:

$$\Delta p_E(0) = \frac{n_i^2}{N_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right) \Longrightarrow V_{BE} \approx \frac{kT}{q}\ln\left(\Delta p_E(0)\frac{N_E}{n_i^2}\right) = 0.637\ V$$

we can do the same starting from the boundary condition at the base side of the emitter-base junction.

b) We are in a NPN transistor I_{EN} is the electron current injected by the emitter into the base.
As the base is short and the excess carrier distribution is a straight line (see figure) we have:

$$I_{EN} = qD_N\frac{\Delta n_B(0)}{W_B}A = 2.4\ mA \qquad (A \text{ is the device area})$$

with

$$\Delta n_B(0) = \frac{n_i^2}{N_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)$$

Analogously:

$$I_{EP} = qD_P \frac{\Delta p_E(0)}{W_E} A = 0.02\ mA$$

with

$$\Delta p_E(0) = \frac{n_i^2}{N_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)$$

The injection efficiency will be:

$$\gamma_E = \frac{I_{EN}}{I_E} = \frac{I_{EN}}{I_{EN} + I_{EP}} = \frac{2.4}{2.42} = 0.9917$$

Note the difference with the expression used in the problem 2 as here we have a NPN transistor while in the former case it was PNP.

c) The recombination current:

$$I_{Br} = qA \int_0^{W_B} \frac{\Delta n(x)}{\tau_n} dx = qA \frac{1}{2} \frac{\Delta n(0) W_B}{\tau_n} = 2\ \mu A$$

The current at the collector boundary will equal the electron current entering from the emitter minus the recombination within the base.

$$I_{CN} = I_{EN} - I_{Br} = 2.398\ mA$$

and the transport factor

$$\alpha_T = \frac{I_{CN}}{I_{EN}} = \frac{2.398}{2.4} = 0.9992$$

d) Let's calculate the currents at the terminals:

$$I_E = I_{EN} + I_{EP} = 2.4\ mA + 0.002\ mA = 2.42\ mA$$

$$I_B = I_{EP} + I_{Br} = 0.02\ mA + 2\ \mu A = 22\ \mu A$$

with the base collector junction short-circuited

$$I_C = I_{CN} = 2.398\ mA$$

and

$$\beta = \frac{2.398\ mA}{0.022\ mA} = 109$$

e) In amplification applications, the transistor is biased in the active region (base emitter junction directly and collector base junction reversely biased). For doing so we need to use constant values for both polarizations, to this continuous values we superpose the small signal which has to be amplified. This signal value is much smaller than the polarization value. The first step when solving an amplifier is to solve it taking into account only the constant sources. Doing so we obtain the continuous values for the different variables (base, emitter, collector currents and

junction voltages). Knowing these values we calculate the components of the small signal equivalent circuit which relates between them the signal components of the different variables. From now on the rest of the problem refers to the small signal equivalent circuit.
A comment about notation, take for instance the base current, we write i_B the total value of the variable, I_B the polarization or continuous value and $i_b$the signal component. That means:

$$i_B = I_B + i_b$$

r_π is the dynamic resistance of the base emitter diode:

$$r_\pi = \frac{V_T}{I_B} = \frac{25\ mV}{22\ \mu A} = 1.1K\Omega$$

and $g_m$is the transconductance that relates the change in the output current (collector) with the variation in the input voltage (V_{EB})

$$g_m = \frac{I_C}{V_T} = \frac{2.398\ mA}{25\ mV} = 96\ mA/V$$

f) The total charge stored by excess minority carriers will write:

$$Q_F = qA\frac{\Delta p_E(0)W_E}{2} + qA\frac{\Delta n_B(0)W_B}{2} = 0.016\ pC + 0.4\ pC = 0.416\ pC$$

The transit time

$$\tau_F = \frac{Q_F}{I_{EN}} = \frac{0.416\ pC}{2.4\ \ mA} = 0.17\ ns$$

Finally,

$$\frac{i_c}{i_b} = \frac{r_\pi g_m v_{be}}{v_{be} + jC_\pi r_\pi \omega v_{be}}$$

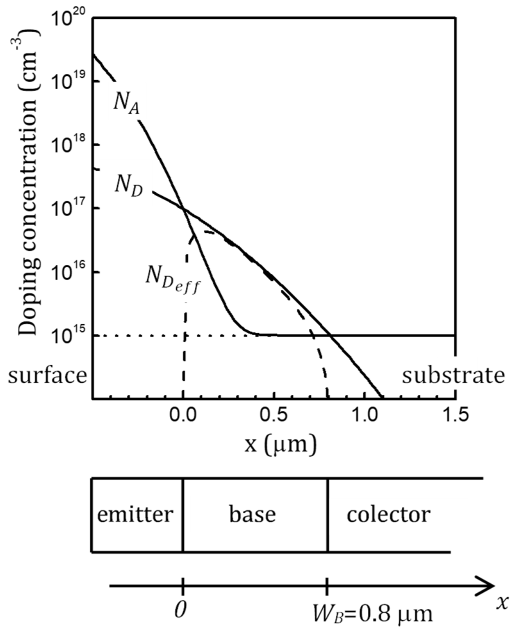
This quotient will be 1 when $\omega = \frac{g_m}{C_\pi}$

$$C_\pi = \frac{dQ_F}{dV_{BE}} = \tau_F \frac{I_C}{V_T}$$

and consequently

$$\omega = 2\pi f_T = \frac{1}{\tau_F} \Longrightarrow f_T = \frac{1}{2\pi\tau_F} = 936\ MHz$$

5. ***When fabricating a homojunction bipolar transistor we perform two diffusions with different doping on the same substrate. These diffusions produce the base and emitter regions, as it is shown in the figure for a PNP transistor. It can be seen that in general the doping concentration in the base in actual devices is clearly non uniform. Consider the n-type effective doping in the base can be approximated by an exponential profile:***



$$N_{D_{eff}} = N_D - N_A \approx N_o exp\left(-\frac{x}{\lambda}\right)$$

where $N_o = 10^{17}\ cm^{-3}$ and $\lambda = 0.17\ \mu m$.

a) Calculate the electric field in the base of the transistor.

b) Taking into account the magnitude and sign of the previously calculated electric field, discuss its effect regarding the transit time of the carriers injected by the emitter in the base when the transistor is biased in the active zone.

c) Following the same reasoning, discuss the effect of a non-uniform doping on the cut-off frequency. In particular consider the effect of λ comparing its value with the base width W_B.

Data: $k_B = 1.38 \times 10^{-23}\ J/K,\ \ T = 300\ K,$
$q = 1.6 \times 10^{-19}\ C$

a) The non-uniform doping within the base will generate a built-in electric field in order to stop the diffusion of electrons from the left towards the right, that is an electric field from left towards the right.

$$J_n = qn\mu_n E + qD_n\frac{dn}{dx} = 0$$

The concentration of electrons will be:

$$n \approx N_{D_{eff}} = N_o exp\left(-\frac{x}{\lambda}\right)$$

$$\frac{dn}{dx} = N_o\left(-\frac{1}{\lambda}\right)exp\left(-\frac{x}{\lambda}\right) = -\frac{n}{\lambda}$$

$$E = \frac{-qD_n\frac{dn}{dx}}{q\mu_n n} = \frac{D_n}{\mu_n}\frac{n/\lambda}{n} = \frac{kT}{q}\frac{1}{\lambda} \approx 1.5\ kV/cm$$

b) If $E > 0$ in the base it will accelerate the holes towards the right. In practice when biased in active zone, the emitter will inject holes into the base, the built-in electric field will accelerate the transit towards the collector.

c) If the total transit time τ_F is dominated by the base transit time, as it is usually the case, if the field is strong enough it will be somehow reduced in the presence of an internal electric field.

The transit time in the base due to diffusion (without electric field) is:

$$\tau_{tr} = \frac{W_b^2}{2D_p}$$

while the transit time in the base due to the drift will write:

$$\tau_{tr} = \frac{W_b}{\mu_p E}$$

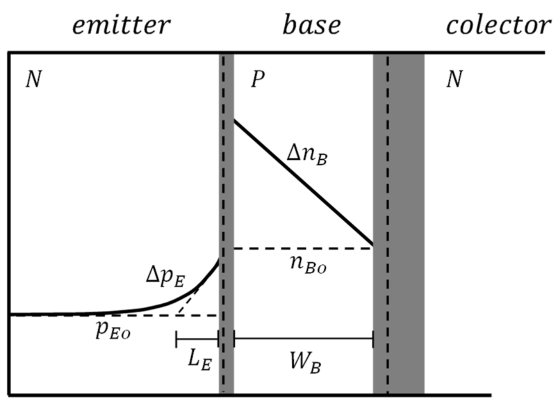
The reduction will be only noticeable if this late base transit time due to drift is smaller than the transit time due to diffusion i.e.

$$\mu_p E > \frac{2D_p}{W_b}$$

$$\mu_p \frac{kT}{q} \frac{1}{\lambda} > \frac{2\frac{kT\mu_p}{q}}{W_b} \rightarrow \lambda < \frac{W_b}{2}$$

In this case the base transit time will be reduced and then the cut-off frequency increased.

6. *Consider a NPN bipolar transistor whose base, short, defines a transport factor $\alpha_T \approx 1$. On its side the emitter can be considered long compared with the corresponding minority carrier diffusion length. The transistor is biased in the active zone and you can take $V_{BC} = 0\ V$*



a) *Find expression for the base and collector currents for a given value for V_{BE}>0.*

b) *Find expressions for the injection efficiency γ_E and for the current gain β. Give their numerical value.*

c) *Find expressions for the stored charge in the emitter Q_E and in the base Q_B for a given value of V_{BE}.*

d) *Find the expression for the delay time τ_F which will determine the frequency features of the transistor. Write τ_F in terms of both the hole lifetime in the emitter τ_E and the electron transit time in the base τ_{trB}. Calculate the τ_F value indicating the emitter and base contributions to this delay time.*

Emitter data: $N_E = 10^{19}\ cm^{-3}$, $D_E = 4\ cm^2/s$, $\tau_E = 100\ ps$
Base data: $N_B = 10^{17}\ cm^{-3}$, $D_B = 20\ cm^2/s$, $W_B = 1\ \mu m$

a) The total base current (with $V_{BC} = 0$) is the addition of the hole current injected by the base into the emitter and the hole current injected into the base to maintain the recombination. As the base is short and with little recombination (transport factor ≈ 1) we may suppose that the dominant term is the former one.
The emitter is long and then the minority carrier excess distribution will be exponential and will write (taking the x axis positive entering in the emitter and with the origin at the border of the space charge region at the emitter side).

$$\Delta p(x) = \Delta p(0) exp\left(-\frac{x}{L_E}\right)$$

The hole current injected in the emitter is:

$$I_{EP} = -qAD_E \frac{d\Delta p}{dx}\bigg|_{x=0} = qA\frac{D_E}{L_E}\frac{n_i^2}{N_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)$$

Then

$$I_B \approx I_{EP} = qA\frac{D_E}{L_E}\frac{n_i^2}{N_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)$$

being A the device area.
The collector current if we neglect the recombination in the base and remembering that $V_{BC} = 0$ will write as

$$I_C \approx qA\frac{D_B}{W_B}\frac{n_i^2}{N_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)$$

b) The injection efficiency γ_E is defined as the ratio between the current injected by the emitter into the base I_{EN} and the total emitter current $I_E = I_{EP} + I_{EN}$. On the other hand because the base is short and there is very little recombination in it $I_{EN} \approx I_C$. Then remembering that the diffusion length in the emitter can be calculated from the data as $L_E = \sqrt{D_E \tau_E} = 0.2\ \mu m$

$$\gamma_E = \frac{I_{EN}}{I_{EP} + I_{EN}} \approx \frac{\frac{D_B}{W_B}\frac{1}{N_B}}{\frac{D_B}{W_B}\frac{1}{N_B} + \frac{D_E}{L_E}\frac{1}{N_E}} = \frac{1}{1 + \frac{D_E W_B N_B}{D_B L_E N_E}} = 0.9901$$

The current gain β is defined as the ratio between the collector current I_C and the base current I_B

$$\beta = \frac{I_C}{I_B} \approx \frac{\frac{D_B}{W_B}\frac{1}{N_B}}{\frac{D_E}{L_E}\frac{1}{N_E}} = \frac{D_B}{D_E}\frac{L_E}{W_B}\frac{N_E}{N_B} = 100$$

c) The charge stored by the excess minority carrier in the emitter Q_E and the base Q_B are respectively :

$Q_E = qA\int_0^{W_E} \Delta p(x)dx$ and $Q_B = qA\int_0^{W_E} \Delta n(x)dx$

$$Q_E = qA\int_0^{W_E} \Delta p(0) e^{-\frac{x}{L_E}} dx = qA\frac{n_i^2}{N_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)L_E$$

$$W_E \gg L_E \Longrightarrow exp\left(-\frac{W_E}{L_E}\right) \to 0$$

On the other hand

$$Q_B = qA\int_0^{W_B} \Delta n(0)\left(1 - \frac{x}{W_B}\right)dx = qA\Delta n(0)\frac{W_B}{2} = qA\frac{n_i^2}{N_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)\frac{W_B}{2}$$

d) The forward delay time

$$\tau_F = \frac{Q_E + Q_B}{I_C} = \frac{Q_E}{I_C} + \frac{Q_B}{I_C} = \frac{\frac{L_E}{N_E}}{\frac{D_B}{W_B}\frac{1}{N_B}} + \frac{\frac{1}{N_B}\frac{W_B}{2}}{\frac{D_B}{W_B}\frac{1}{N_B}} = \frac{L_E^2}{\frac{D_B}{W_B}\frac{N_E}{N_B}L_E} + \frac{W_B^2}{2D_B}$$

$$\tau_F = \frac{\tau_E}{\frac{D_B}{D_E}\frac{N_E}{N_B}\frac{L_E}{W_B}} + \tau_{trB} = \frac{\tau_E}{\beta} + \tau_{trB} = \frac{100\ ps}{100} + 250\ ps$$

7. Consider a doped region with impurity concentration N, width W and minority carrier diffusion coefficient D. We define a characteristic parameter for this region called Gummel number:

$$\boldsymbol{G = \frac{NW}{D}}$$

Consider now a bipolar transistor with emitter and base shorts. We know the Gummel numbers for the emitter and base $\boldsymbol{G_E = 1.6 \times 10^{13}\ cm^{-4}s^{-1}}$ and base $\boldsymbol{G_B = 8 \times 10^{10}\ cm^{-4}s^{-1}}$.

a) Find an expression for the current gain $\boldsymbol{\beta}$ expressed in terms only of the Gummel numbers. Calculate the numerical value for $\boldsymbol{\beta}$.

b) Analogously find an expression for the injection efficiency and give its numerical value.

a) If we recover the expression for the current gain obtained in the last exercise we have:

$$\beta = \frac{I_C}{I_B} \approx \frac{\frac{D_B}{W_B}\frac{1}{N_B}n_i^2\left(exp\left(\frac{V_{BE}}{V_T}\right)-1\right)}{\frac{D_E}{W_E}\frac{1}{N_E}n_i^2\left(exp\left(\frac{V_{BE}}{V_T}\right)-1\right)} = \frac{D_B}{D_E}\frac{W_E}{W_B}\frac{N_E}{N_B}$$

In this expression we have the same intrinsic concentration for both the emitter and the base. If we re-organize the former expression we get:

$$\beta = \frac{D_B}{D_E}\frac{W_E}{W_B}\frac{N_E}{N_B} = \frac{\frac{W_E N_E}{D_E}}{\frac{W_B N_B}{D_B}} = \frac{G_E}{G_B} = \frac{1.6 \times 10^{13}}{8 \times 10^{10}} = 200$$

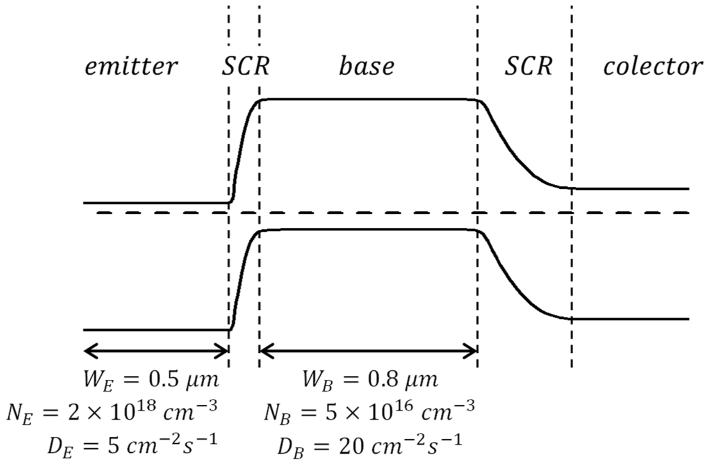
b) The expression for the injection efficiency is

$$\gamma_E = \frac{I_{EN}}{I_{EP} + I_{EN}} \approx \frac{\frac{D_B}{W_B}\frac{1}{N_B}}{\frac{D_B}{W_B}\frac{1}{N_B} + \frac{D_E}{W_E}\frac{1}{N_E}} = \frac{1}{1 + \frac{D_E W_B N_B}{D_B W_E N_E}} = \frac{1}{1 + \frac{G_B}{G_E}} = \frac{1}{1 + \frac{8 \times 10^{10}}{1.6 \times 10^{13}}} = 0.995$$

8. ***Consider an NPN Silicon bipolar transistor (BJT) as shown in the figure. The base can be considered short (transport factor $\alpha_T \approx 1$).The emitter is also short.***
   a) ***Calculate the current gain β.***

   Consider now a heterojunction bipolar transistor (HBT) with an emitter of $Al_{0.1}Ga_{0.9}As$ ($E_g = 1.5\ eV$) grown on a GaAs base $(E_g = 1.4\ eV)$:

   b) ***Draw qualitatively the band diagram of the base-emitter junction under equilibrium. Give an estimation about the increase in β due to the heterojunction.***



In GaAs based ternary alloys for simplicity suppose that electron affinity, diffusion coefficients, and effective densities of states do not vary much compared with GaAs.

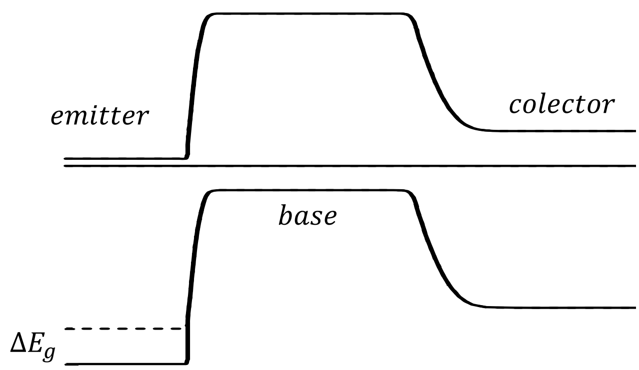
Data: $q = 1.6 \times 10^{-19}\ C,\ k_B = 1.38 \times 10^{-23} J/K,\ T = 300\ K$

a) This exercise explores the effect of ideal heterojunctions on bipolar transistor performance. HBT are important devices nowadays when high power and frequency are required.
First we are asked about the current gain in a homojunction bipolar transistor with emitter and base electrically short.
If the base is short the recombination within the base is generally negligible and in particular smaller than the current injected by the base into the emitter and on the other hand the collector current (with V_{BC} =0) is nearly equal to the current injected by the emitter in the base.

$$\beta = \frac{I_C}{I_B} \approx \frac{\frac{D_B}{W_B}\frac{1}{N_B}n_{iB}^2\left(exp\left(\frac{V_{BE}}{V_T}\right)-1\right)}{\frac{D_E}{W_E}\frac{1}{N_E}n_{iE}^2\left(exp\left(\frac{V_{BE}}{V_T}\right)-1\right)} = \frac{D_B}{D_E}\frac{W_E}{W_B}\frac{N_E}{N_B} = 100$$

where n_{iB} and n_{iE} are the intrinsic concentration in the base and the emitter respectively. As the semiconductor is Silicon in both cases $n_{iB} = n_{iE}$.

b) Now we analyze the case of a transistor where the emitter has a larger bandgap than the base.



Under certain conditions the main electrical effect of the heterojunction comes from the asymmetry of barriers for electrons and holes. This asymmetry results in a strong asymmetry between hole and electron currents.
If we recover the expression for the current gain

$$\beta = \frac{I_C}{I_B} \approx \frac{D_B}{D_E}\frac{W_E}{W_B}\frac{N_E}{N_B}\frac{n_{iB}^2}{n_{iE}^2}$$

Now due to the heterojunction we have different materials at emitter and base de intrinsic concentration is no longer equal. In fact if we consider constant effective density of states in the GaAs alloys we have:

$$n_{iE}^2 = n_{iB}^2 exp\left(-\frac{\Delta E_G}{kT}\right) \text{ , with } \Delta E_g = E_{gE} - E_{gB} = 0.1\ eV$$

The expression for the current gain is

$$\beta_{HBT} = \frac{I_C}{I_B} \approx \frac{D_B}{D_E}\frac{W_E}{W_B}\frac{N_E}{N_B}\ exp\left(\frac{\Delta E_G}{kT}\right) = \beta_{BJT}\ exp\left(\frac{\Delta E_G}{kT}\right) = 100 \cdot 54 = 5400$$

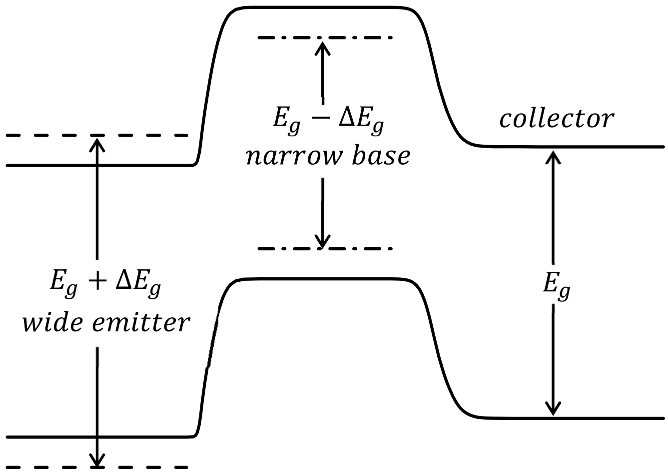
9. Consider a npn bipolar transistor, base and collector are fabricated using the same semiconductor with an energy gap E_g. We will refer to it as reference BJT. The base can be considered short and consequently the recombination can be neglected. The emitter also can be considered short. The base emitter junction is directly biased with voltage V_{BE} while the base collector junction is short circuited $V_{BC} = 0\ V$.

For the reference transistor:

a) Find and expression for the current gain β as a function of the given data.

b) Find expressions for the small signal parameters r_π and g_m.

c) Find and expression for the delay time τ_F, separate the base and emitter contributions ($\tau_F = \tau_E + \tau_B$).



n_i: intrinsic concentration for E_g
N_E , N_B: emitter and base doping concentrations
D_E , D_B: emitter and base diffusivities
W_E , W_B: emitter and base widths
q: electron charge, kT: thermal energy
A: device area

Consider now two heterojunction transistors. One of them, called wide emitter (WE), the emitter has an energy gap $E_g + \Delta E_g$ larger than the base and collector. The other, called narrow base (NB), the base has an energy gap $E_g - \Delta E_g$narrower than collector and emitter

d) Complete the table below showing how the different parameters change referred to the reference BJT.

$BJT\ reference$	$HBT\ wide\ emitter$	$HBT\ narrow\ base$
$\boldsymbol{\beta}$	$\boldsymbol{\beta_{we} = \beta \cdot exp\left(\frac{\Delta E_g}{kT}\right)}$	$\boldsymbol{\beta_{nb}} =$
$\boldsymbol{r_\pi}$	$\boldsymbol{r_{\pi,we}} =$	$\boldsymbol{r_{\pi,nb}} =$
$\boldsymbol{g_m}$	$\boldsymbol{g_{m,we}} =$	$\boldsymbol{g_{m,nb}} =$
$\boldsymbol{\tau_E}$	$\boldsymbol{\tau_{E,we}} =$	$\boldsymbol{\tau_{E,nb}} =$
$\boldsymbol{\tau_B}$	$\boldsymbol{\tau_{B,we}} =$	$\boldsymbol{\tau_{B,nb}} =$

a) The problem reviews the different features of HBT's (Heterojunction Bipolar Transistors). The focus is on one hand on wide gap emitter HBT's, for instance $Ga_{1-x}Al_xAs/GaAs/GaAs$, and on the other hand narrow gap base HBT's, the best known $Si/Si_{1-x}Ge_x/Si$. As we have already seen in problem 8 if the band diagram is smooth enough, without spikes and notches, the analysis of HBT's is rather simple, in fact it reduces to consider the proper intrinsic concentration in each case.

Let's start writing the expressions for I_C, I_B, Q_E and Q_B. Both emitter and base are shorts.

$$I_C = qAD_B \frac{n_{iB}^2}{N_B W_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right) \;\; ; \;\; I_B = qAD_E \frac{n_{iE}^2}{N_E W_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)$$

where n_{iB} and n_{iE} are the intrinsic concentration at base and emitter respectively.

$$Q_E = qA \frac{n_{iE}^2}{N_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)\frac{W_E}{2} \;\; ; \;\; Q_B = qA \frac{n_{iB}^2}{N_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)\frac{W_B}{2}$$

With that we can write

$$\beta = \frac{I_C}{I_B} \approx \frac{qAD_B \frac{n_{iB}^2}{N_B W_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)}{qAD_E \frac{n_{iE}^2}{N_E W_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)} = \frac{D_B}{D_E}\frac{W_E}{W_B}\frac{N_E}{N_B}\frac{n_{iB}^2}{n_{iE}^2}$$

For the reference transistor $n_{iB} = n_{iE}$ and $\beta = \frac{D_B}{D_E}\frac{W_E}{W_B}\frac{N_E}{N_B}$

b) The dynamic resistance of the base emitter junction is

$$r_\pi = \frac{V_T}{I_B} = \frac{V_T}{qAD_E \frac{n_{iE}^2}{N_E W_E}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)}$$

And the transconductance

$$g_m = \frac{I_C}{V_T} = \frac{qAD_B \frac{n_{iB}^2}{N_B W_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)}{V_T}$$

c) The forward transit time

$$\tau_F = \frac{Q_F}{I_C} = \frac{Q_E}{I_C} + \frac{Q_B}{I_C} = \tau_E + \tau_B$$

$$\tau_B = \frac{Q_B}{I_C} = \frac{qA \frac{n_{iB}^2}{N_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)\frac{W_B}{2}}{qAD_B \frac{n_{iB}^2}{N_B W_B}\left(exp\left(\frac{V_{BE}}{V_T}\right) - 1\right)} = \frac{W_B^2}{2D_B} = \tau_{trB}$$

and

$$\tau_E = \frac{Q_E}{I_C} = \frac{qA\frac{n_{iE}^2}{N_E}\left(exp\left(\frac{V_{BE}}{V_T}\right)-1\right)\frac{W_E}{2}}{qAD_B\frac{n_{iB}^2}{N_B W_B}\left(exp\left(\frac{V_{BE}}{V_T}\right)-1\right)} = \frac{\frac{W_E^2}{2D_E}}{\frac{D_B}{D_E}\frac{N_E}{N_B}\frac{W_E}{W_B}}\frac{n_{iE}^2}{n_{iB}^2} = \frac{\tau_{trE}}{\beta}$$

$$\tau_F = \frac{W_B^2}{2D_B} + \frac{\frac{W_E^2}{2D_E}}{\beta}\frac{n_{iE}^2}{n_{iB}^2}$$

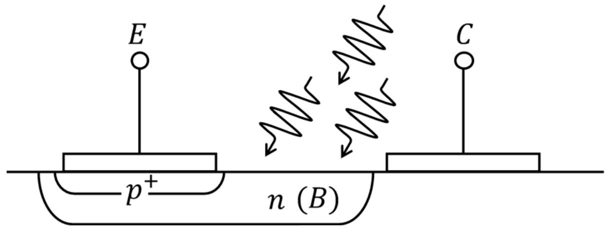
If the material is homogenous $n_{iB} = n_{iE}$ and $\tau_F = \frac{W_B^2}{2D_B} + \frac{W_E^2}{\beta 2D_E}$

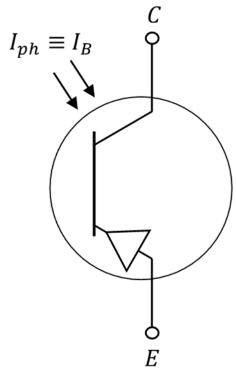
d) The expressions giving the different parameters are the same as previously, with the only difference that now both in wide gap emitter and narrow base we have $\frac{n_{iB}^2}{n_{iE}^2} = exp\left(\frac{\Delta E_g}{kT}\right)$

Remember that ΔE_g is defined positive in both cases.

$BJT\ reference$	$HBT\ wide\ emitter$	$HBT\ narrow\ base$
β	$\beta_{we} = \beta \cdot exp\left(\frac{\Delta E_g}{kT}\right)$	$\beta_{nb} = \beta \cdot exp\left(\frac{\Delta E_g}{kT}\right)$
r_π	$r_{\pi,we} = r_\pi exp\left(\frac{\Delta E_g}{kT}\right)$	$r_{\pi,nb} = r_\pi$
g_m	$g_{m,we} = g_m$	$g_{m,nb} = g_m exp\left(\frac{\Delta E_g}{kT}\right)$
τ_E	$\tau_{E,we} = \tau_E exp\left(-\frac{\Delta E_g}{kT}\right)$	$\tau_{E,nb} = \tau_E exp\left(-\frac{\Delta E_g}{kT}\right)$
τ_B	$\tau_{B,we} = \tau_B$	$\tau_{B,nb} = \tau_B$

10. *A phototransistor is a light detector device able to internally amplify the photogenerated current. Its physical structure is similar to a bipolar junction transistor (BJT) but without a base contact. Consider a pnp phototransistor where a long enough wavelength (resulting in a uniform generation profile) illuminates the base-collector junction. Assume the base short and that the photogenerated current I_{ph} is due exclusively to optical absorption in the collector region. The collector region can be assumed long (minority carrier diffusion length L_C). Electrons generated into the collector reaching the base act from all points of view as an effective I_B. Recombination within the base can be neglected because it is short. Emitter can be also assumed short. We apply between the terminals a voltage $V_{EC} = 5\ V$. Using the rest of data included in the figure:*





Data:

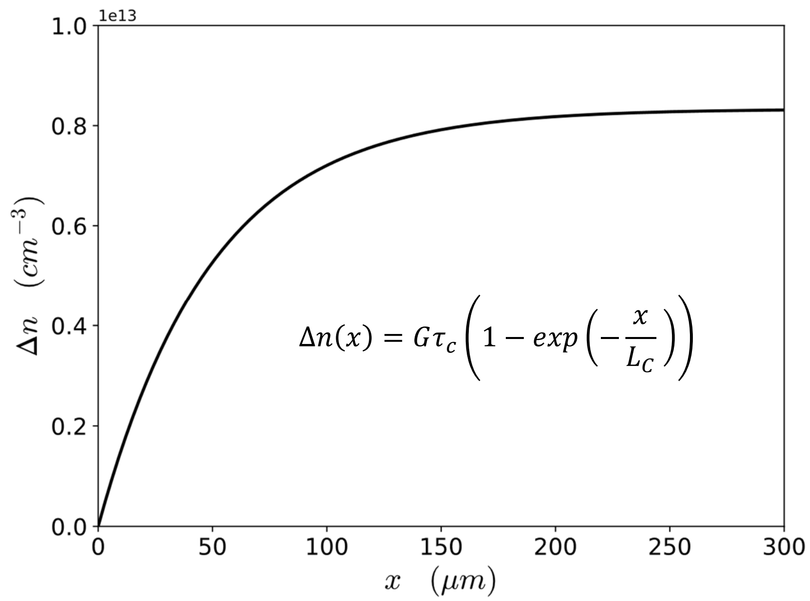
$N_E = 3 \times 10^{18}\ cm^{-3}$
$D_E = 6\ cm^2 s^{-1}$
$W_E = 1\ \mu m$
$N_B = 6 \times 10^{16}\ cm^{-3}$
$D_B = 12\ cm^2 s^{-1}$
$W_B = 1\ \mu m$
$L_C = 50\ \mu m$
$A = 0.01\ cm^2$
$G = 10^{19}\ cm^{-3} s^{-1}$
$q = 1.6 \times 10^{-19}\ C$
$n_i = 10^{10}\ cm^{-3}$
$V_T = 25\ mV$

a) Calculate the current I_{ph} generated in the collector region, which is injected in the base.

b) Calculate the voltage V_{EB} biasing the emitter-base junction. Calculate also the voltage drop at the collector base junction?

c) Calculate the collector current I_C and compare with the previously calculated I_{ph}. As a light detector, what is the current gain of this phototransistor?

a) For details about the calculation of the photocurrent see problem 19 chapter 2. The excess carrier distribution in the collector will be:



$$I_{ph} = qAD_C \left.\frac{d\Delta n}{dx}\right|_{x=0} = qAD_C \frac{G\tau_c}{L_C} = qAGL_C = 80\ \mu A$$

b) The current injected into the base will play the role of the base current. As the base is short and consequently we assume no recombination in it, this current will be injected into the emitter.

$$I_{ph} = I_B \approx I_{EN} = qA\frac{D_E}{W_E}\frac{n_i^2}{N_E}\left(exp\left(\frac{V_{EB}}{V_T}\right) - 1\right)$$

and

$$V_{EB} = V_T \ln\left(1 + I_{ph}\frac{N_E W_E}{D_E n_i^2}\frac{1}{qA}\right) = 598\ mV \approx 0.6\ V$$

$$V_{EC} = V_{EB} - V_{CB} \rightarrow V_{CB} = V_{EB} - V_{EC} = 0.6V - 5V = -4.4V$$

c) The collector current will be

$$I_C \approx I_{EP} = qA\frac{D_B}{W_B}\frac{n_i^2}{N_B}\left(exp\left(\frac{V_{EB}}{V_T}\right) - 1\right) = 7.8\ mA$$

The light generated current (at least the collected part of it) is $I_{ph} = 80\mu A$ but the collector is β times larger with (remember than the recombination current in the base is assumed to be zero)

$$\beta = \frac{D_B}{D_E}\frac{W_E}{W_B}\frac{N_E}{N_B} = 10$$

Acting as photodetector we have a gain of β.

Chapter 4.

The Metal-Oxide-Semiconductor Field-Effect Transistor

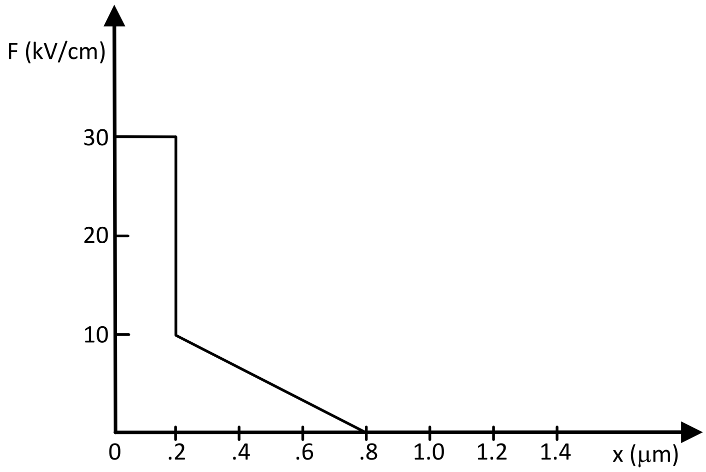
1. ***The figure shows the electric field distribution within a MOS structure for a given potential V_G applied to the gate. Assuming the flat band potential V_{FB} (potential for which the bands are flat) is zero, find:***

   a) ***The potential in the bulk of the semiconductor ψ_B***

   b) ***The potential at the surface ψ_S***

   c) ***The threshold voltage V_T***

   d) ***Find the value of the applied potential V_G***



Data:

$q = 1.6 \times 10^{-19}\ C$

$\varepsilon_{Si} = 10^{-12}\ F/cm$

$\varepsilon_{ox} = 3 \times 10^{-13}\ F/cm$

$n_i = 1.5 \times 10^{10}\ cm^{-3}$

$k_B = 1.38 \times 10^{-23}\ J/K$

$T = 300\ K$

N.B. We take arbitrarily the origin of potentials at the point where $E_{Fi} = E_F$ and will be positive if $E_F > E_{Fi}$ and negative otherwise.

a) At a given point x the potential is defined as:

$$\psi(x) = \frac{E_F - E_{Fi}(x)}{q}$$

Then, at equilibrium, the electron and hole concentrations can be written as:

$$n(x) = n_i exp\left(\frac{q\psi(x)}{kT}\right) \qquad p(x) = n_i exp\left(-\frac{q\psi(x)}{kT}\right)$$

Coming back to the problem. In the semiconductor the electric field will be:

$$E = \int_0^x \frac{\rho(x)}{\varepsilon_{Si}} dx$$

The charge within the space charge region in the semiconductor beneath the surface will be $\rho(x) = -qN_A$ and:

$$E(x) = -q\frac{N_A}{\varepsilon_{Si}}x + C$$

At the end of the space charge region the electric field will be zero.

$$E(W) = -q\frac{N_A}{\varepsilon_{Si}}W + C = 0 \Longrightarrow C = q\frac{N_A}{\varepsilon_{Si}}W$$

and

$$E(x) = q\frac{N_A}{\varepsilon_{Si}}(W - x)$$

Now, from the maximum value of the field:

$$E_{max} = q\frac{N_A}{\varepsilon_{Si}}W \Longrightarrow N_A = \frac{\varepsilon_{Si}}{q}\frac{E_{max}}{W} = 10^{15}\ cm^{-3}$$

In the bulk the hole concentration will be equal to N_A. Then, we write:

$$N_A = n_i e^{-\frac{q\psi_B}{kT}} \Longrightarrow \psi_B = -\frac{kT}{q}\ln\frac{N_A}{n_i} = -0.28\ V$$

b) The potential drop in the semiconductor is the difference between the potential at the surface ψ_S and the the potential at the bulk of the semiconductor ψ_B:

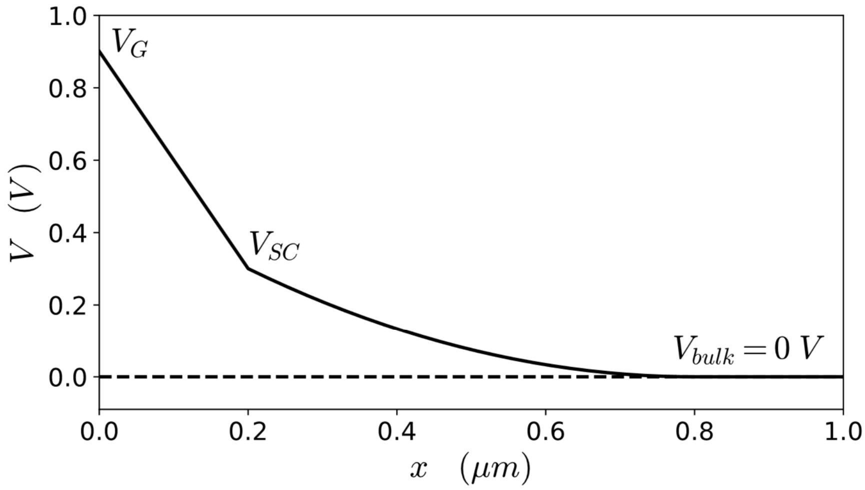
$$V_{SC} = \psi_S - \psi_B$$

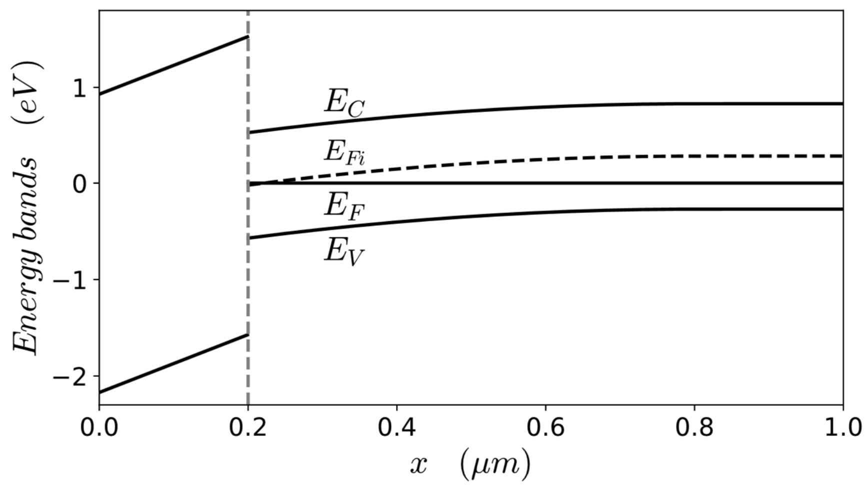
On the other hand the potential drop in the semiconductor V_{SC} is the integral of the electric field:

$$V_{SC} = \frac{E_{max}W}{2} = 0.3\ V$$

$$\psi_S = V_{SC} + \psi_B = 0.02\ V$$

The potential distribution taking as a reference the potential at the bulk:





c) The threshold voltage with zero flat band voltage $V_{FB} = 0$ writes:

$$V_T = -\frac{Q_B}{C_{ox}} + 2|\psi_B|$$

The charge Q_B is the charge in the space charge region of the semiconductor $-qN_AW$ and W the width of the space charge region:

$$W = \sqrt{\frac{2\varepsilon_{Si}}{q}\frac{2|\psi_B|}{N_A}} = \sqrt{2\varepsilon_{Si}qN_A2|\psi_B|}$$

$$V_T = \frac{\sqrt{2\varepsilon_{Si}qN_A2|\psi_B|}}{C_{ox}} + 2|\psi_B| = 1.45\ V$$

with C_{ox} the gate capacity by unit surface, $C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = 15\ nF/cm^2$

d) The applied potential at the gate will be the addition of the potential drop at the dielectric and the potential drop in the semiconductor

$$V_G = V_{OX} + V_{SC} = 30 \times 10^3 \frac{V}{cm} \cdot 0.2 \times 10^{-4} cm + \frac{1}{2} \cdot\ 10 \times 10^3 \frac{V}{cm} \cdot 0.6 \times 10^{-4} cm$$

$$V_G = V_{OX} + V_{SC} = 0.6\ V + 0.3\ V = 0.9\ V$$

As we see the applied voltage is lower than the threshold.

2. ***We fabricate a capacitance, one plate is a metal sheet and the other a n doped semiconductor, both "plates" are separated by a dielectric. Although the doped semiconductor presents a high conductivity, the electric field enters (a very short distance) into its volume. As a consequence a small fraction of the voltage applied to the capacitance will drop in the semiconductor.***

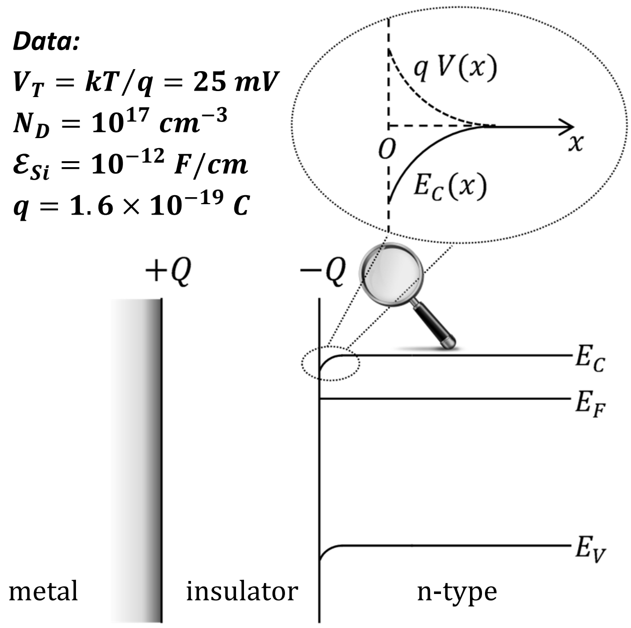
   a) ***In agreement with the figure, find the expression for the charge density within the semiconductor $\rho(x)$ as a function of the voltage $V(x)$ which bends the bands. Assuming $V \ll kT/q$ develop $\rho(x)$ to the first order in $V(x)$.***

   b) ***Show that the voltage within the semiconductor quickly drops following an exponential shape:***

$$V(x) = V(0)\ exp\left(-\frac{x}{L_D}\right),\ \textit{where}\ L_D = \sqrt{V_T \frac{\varepsilon_{Si}}{qN_D}}$$

   L_D is called Debye length. Calculate its value with the given data

Data:
$V_T = kT/q = 25\ mV$
$N_D = 10^{17}\ cm^{-3}$
$\varepsilon_{Si} = 10^{-12}\ F/cm$
$q = 1.6 \times 10^{-19}\ C$



a) Charge $\rho(x)$ will be at the semiconductor side:

$$\rho(x) = q(N_D - n(x)) = q\left(N_D - N_C\ exp\left(-\frac{E_C(x) - E_F}{kT}\right)\right)$$

From the figure we can write:

$$E_C(x) = E_C(\infty) - qV(x)$$

Then replacing in the former expression:

$$\rho(x) = q\left(N_D - N_C\ exp\left(-\frac{E_C(\infty)-E_F}{kT}\right) exp\left(\frac{qV(x)}{kT}\right)\right) = qN_D\left(1 - exp\left(\frac{qV(x)}{kT}\right)\right)$$

where we have used that $N_D = N_C\ exp\left(-\frac{E_C(\infty)-E_F}{kT}\right)$.

If we develop the exponential to the first order following the suggestion we have:

$$\rho(x) = qN_D\left(1 - \left(1 + \frac{qV(x)}{kT}\right)\right) = -\frac{q^2N_D}{kT}V(x)$$

b) From the Gauss's law:

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_{Si}} = -\frac{q^2N_D}{kT\varepsilon_{Si}}V(x)$$

Then, as $E = -\frac{dV}{dx}$ we write:

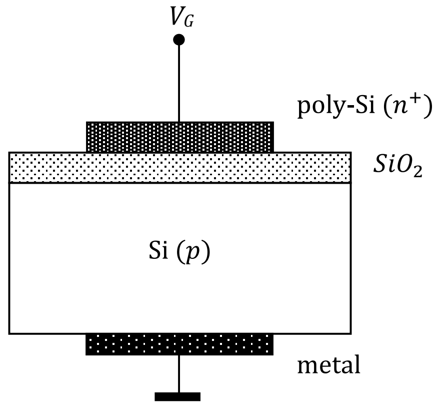
$$\frac{d^2V}{dx^2} = \frac{q^2N_D}{kT\varepsilon_{Si}}V(x) \Rightarrow \frac{d^2V}{dx^2} - \frac{1}{L_D^2}V(x) = 0$$

The solution of the differential equation will be $V(x) = V(0)\ exp\left(-\frac{x}{L_D}\right)$ where:

$$L_D = \sqrt{\frac{kT\varepsilon_{Si}}{q^2 N_D}} = \sqrt{V_T \frac{\mathcal{E}_{Si}}{qN_D}}$$

is called the Debye length. For $N_D = 10^{17} cm^{-3}$ we obtain a value of $L_D \approx 12\ nm$.

3. Consider the MOS structure shown in the figure. The charge within the oxide is $Q_{ox} = 10^{-8}\ C/cm^2$, the flat band voltage is $V_{FB} = -1\ V$.



a) Calculate the impurity concentration in the substrate.

b) Calculate the threshold voltage V_T for the MOS structure.

If we use this structure for the fabrication of a N channel MOSFET transistor and we bias in the saturation zone with $V_{GS} = 4\ V$ and $V_{DS} = 10\ V$:

c) Define the transconductance and calculate its value.

Data : $q = 1.6 \times 10^{-19}\ C$, $k_B = 1.38 \times 10^{-23} J/K$, $T = 300\ K$,

$E_g = 1.1\ eV$, $n_i = 1.5 \times 10^{10}\ cm^{-3}$, $\varepsilon_o = 8.85 \times 10^{-14} F/cm$,

$\varepsilon_{Si} = 11.9$, $\varepsilon_{ox} = 3.9$, $t_{ox} = 350$ Å, $\mu_n = 1300\ cm^2/Vs$, $W = L = 1\ \mu m$.

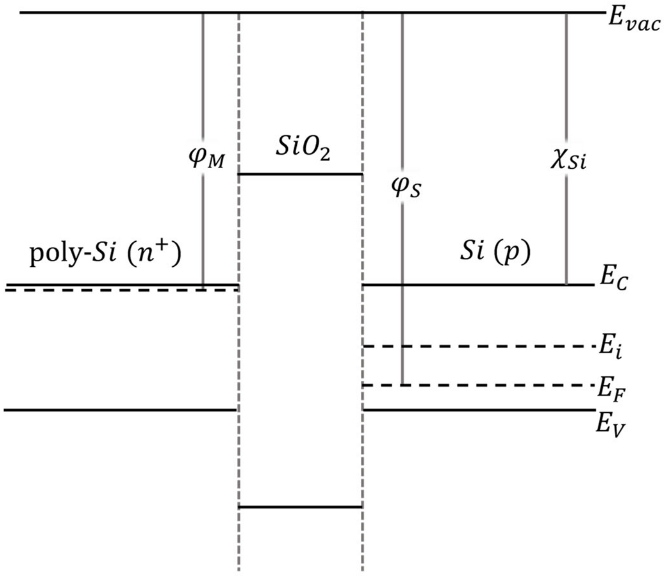
a) The impurity concentration in the substrate is directly related to the bulk potential ψ_B:

$$p(x) = n_i exp\left(-\frac{q\psi_B}{kT}\right) = N_A$$

We will find first ψ_B. The flat band voltage is:

$$V_{FB} = \varphi_{MS} - \frac{Q_{ox}}{C_{ox}} \quad , \text{ with } \varphi_{MS} = \varphi_M - \varphi_S$$

φ_M and φ_S are respectively the work functions of the gate electrode and the substrate. Remember than the work function is the energy difference between the Fermi level and the vacuum level (we can say that the work function is the energy that should be given to an electron hypothetically located at the Fermi level to extract it from the material).



The gate electrode is polysilicon, heavily n-doped, consequently the Fermi level will be nearly at the conduction band ($E_F \approx E_C$) and the work function of the gate electrode is roughly equal to the electron affinity in it, $\varphi_M \approx \chi_{Si}$.

On the other hand, the work function in the semiconductor φ_S is the distance between the Fermi level and the vacuum level at the semiconductor side.

$$\varphi_S = \chi_{Si} + \frac{E_G}{2} + q|\psi_B|$$

$$\varphi_{MS} = \varphi_M - \varphi_S = \chi_{Si} - \left(\chi_{Si} + \frac{E_G}{2} + q|\psi_B|\right)$$

$$V_{FB} = \varphi_{MS} - \frac{Q_{ox}}{C_{ox}} = \chi_{Si} - \left(\chi_{Si} + \frac{E_G}{2} + q|\psi_B|\right) - \frac{Q_{ox}}{C_{ox}}$$

$$\frac{Q_{ox}}{C_{ox}} = \frac{Q_{ox}}{\varepsilon_{ox}/t_{ox}} \text{ , with } t_{ox} \text{ the oxide thickness} \Rightarrow \frac{Q_{ox}}{C_{ox}} = 0.1\ V$$

$$\psi_B = -0.35\ V$$

The substrate is P type, the Fermi level is below the intrinsic Fermi level and the potential at the bulk negative.

$$N_A = n_i exp\left(-\frac{q\psi_B}{kT}\right) = 1.1 \times 10^{16} cm^{-3}$$

b) The threshold voltage is $V_T = V_{FB} + V_{TO}$

$$V_{TO} = \frac{\sqrt{2\varepsilon_{Si} q N_A 2|\psi_B|}}{C_{ox}} + 2|\psi_B| = 1.21\ V$$

$$V_T = V_{FB} + V_{TO} = -1\ V + 1.21\ V = 0.21\ V$$

c) The transconductance is, for a given bias point, the incremental relationship between the drain current and the gate voltage:

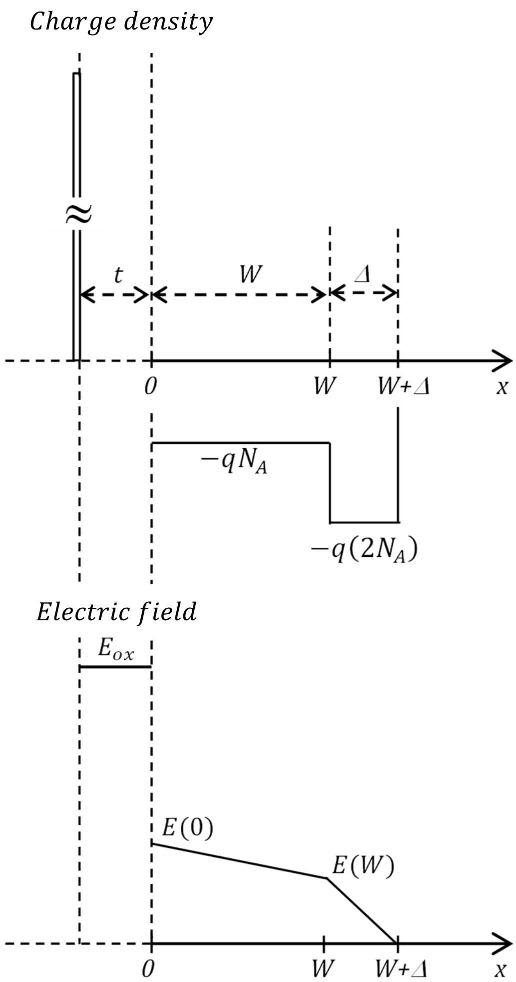
$$g_m = \left.\frac{di_D}{dv_{GS}}\right|_Q$$

being Q the polarization point. For the given values for V_{GS} and V_{DS} the transistor will be in the saturation zone. Then:

$$i_D = \frac{1}{2}\ \mu_n C_{ox} \frac{W}{L} (v_{GS} - V_T)^2$$

$$g_m = \left.\frac{di_D}{dv_{GS}}\right|_Q = \frac{1}{2}\ \mu_n C_{ox} \frac{W}{L} 2\left(V_{GSQ} - V_T\right) = 0.48\ mA/V$$

4. A given MOS structure has a gate electrode, an oxide without fixed charge and thickness $t = 200\ nm$ and a substrate of p type Silicon. A first zone in the semiconductor substrate with thickness $W = 0.4\ \mu m$ has a doping concentration $N_A = 10^{15}\ cm^{-3}$, and $2 \times 10^{15}\ cm^{-3}$ in the rest of the substrate. When we apply $V_G = 1\ V$ at the gate a space charge region is created in the semiconductor with a total thickness of 0.56 μm.



a) Calculate the values of the electric field at $x = 0$ and $x = W$, $E(0)$ and $E(W)$.

b) Calculate the electric field E_{ox} inside the dielectric.

c) Calculate the total potential drop in the semiconductor and the oxide. Verify that the addition of both gives the gate voltage $V_G = 1\ V$.

d) Calculate the voltage drop in the semiconductor needed if the surface has to reach the inversion threshold. Explain the reason why for $V_G = 1\ V$ the surface has not been inverted yet.

***Data:** $q = 1.6 \times 10^{-19}\ C$, $n_i = 10^{10}\ cm^{-3}$, $\varepsilon_{Si} = 10.62 \times 10^{-13}\ F/cm$, $\varepsilon_{ox} = 3.54 \times 10^{-13}\ F/cm$, $k_B = 8.62 \times 10^{-5}\ eV/K$, $T = 300\ K$*

a) From Gauss's law:

$$E = \int_0^x \frac{\rho(x)}{\varepsilon_{Si}} dx$$

If we integrate from $W + \Delta$ to W we obtain:

$$E(W) = \frac{2qN_A\Delta}{\varepsilon_{Si}} = 484\ V/cm$$

If we integrate up to the origin:

$$E(0) = q\frac{(2N_A\Delta + N_AW)}{\varepsilon_{Si}} = 10847\ V/cm$$

b) The electric displacement has to be continuous at the boundary (0) between the oxide and the semiconductor.

$$E_{ox}\varepsilon_{ox} = E_{Si}\varepsilon_{Si}$$

$$E_{ox} = \frac{E_{Si}\varepsilon_{Si}}{\varepsilon_{ox}} = 32541\frac{V}{cm}$$

c) The voltage drop in the semiconductor is the integral of the field.

$$V_{SC} = E(W)\frac{\Delta}{2} + E(W)W + \big(E(0) - E(W)\big)\frac{W}{2} = 0.3519\ V$$

$$V_{ox} = E_{ox}t = 0.6508\ V$$

$$V_G = V_{SC} + V_{ox} \approx 1\ V$$

d) Let's find first the potential at the bulk:

$$N_A = n_i e^{-\frac{q\psi_B}{kT}} \Longrightarrow \psi_B = -0.29\ V$$

$$2|\psi_B| = 0.58\ V$$

The potential drop in the semiconductor is:

$$V_{SC} = 0.3519\ V < \ 2|\psi_B|$$

Consequently, the surface is not inverted yet.

5. The figure shows the potential distribution in a n cannel MOS structure (p substrate). The dielectric is silicon oxide, without fixed charge and thickness $t = 0.1\ \mu m$. Assume null flat band voltage. With the information given in the figure:

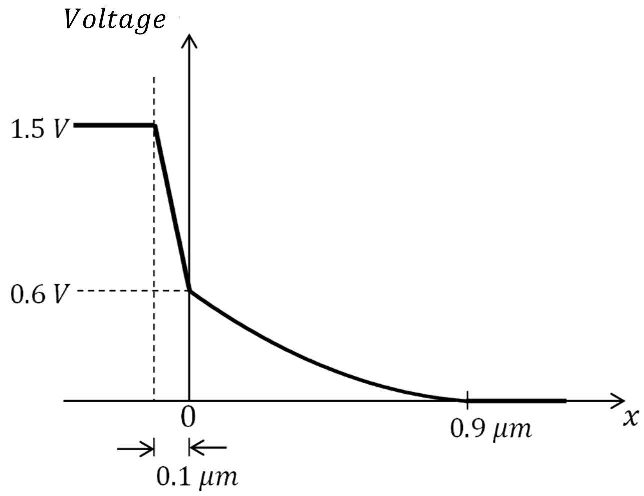
a) Calculate the substrate doping N_A and the bulk potential ψ_b

b) Calculate the threshold voltage V_T for the MOS structure

c) Justify that the semiconductor surface is inverted

d) Calculate the electric field in the oxide.

e) Calculate the total charge in the MOS capacitance. Which part is due to free carriers in the inverted channel?



Data:

$\boldsymbol{q = 1.6 \times 10^{-19}\ C,\ n_i = 10^{10}\ cm^{-3},}$

$\boldsymbol{k_B = 8.62 \times 10^{-5}\ eV/K,\ T = 300\ K,}$

$\boldsymbol{\varepsilon_{Si} = 10.62 \times 10^{-13}\ F/cm,\ \varepsilon_{ox} = 3.54 \times 10^{-13}\ F/cm}$

a) Now the figure shows the potential distribution i.e. the integral of the field. Let's start finding the electric field:

$$E = \int_0^x \frac{\rho(x)}{\varepsilon_{Si}} dx$$

The charge within the space charge region in the semiconductor beneath the surface will be $\rho(x) = -qN_A$ and:

$$E(x) = -q\frac{N_A}{\varepsilon_{Si}}x + C$$

At the end of the space charge region $x = W$, the electric field will be zero.

$$E(W) = -q\frac{N_A}{\varepsilon_{Si}}W + C = 0 \Longrightarrow C = q\frac{N_A}{\varepsilon_{Si}}W$$

$$E(x) = q\frac{N_A}{\varepsilon_{Si}}(W - x)$$

On the other hand:

$$E = -\frac{dV}{dx} \Longrightarrow dV = -E\ dx$$

$$\int_0^W dV = -\int_0^W Edx = -\int_0^W q\frac{N_A}{\varepsilon_{Si}}(W - x)dx$$

$$V(W) - V(0) = -\frac{qN_A}{\varepsilon_{Si}}\left[Wx - \frac{x^2}{2}\right]_{x=0}^{x=W} = -\frac{qN_A}{\varepsilon_{Si}}\frac{W^2}{2}$$

The potential drop within the semiconductor

$$V_{SC} = \frac{qN_A}{\varepsilon_{Si}}\frac{W^2}{2}$$

From the figure, $V_{SC} = 0.6\ V$ and $W = 0.9\ \mu m$. We can calculate the doping concentration:

$$N_A = \frac{2\varepsilon_{Si}}{qW^2}V_{SC} \approx 10^{15} cm^{-3}$$

$$N_A = p = n_i exp\left(-\frac{q\psi_B}{kT}\right) \Rightarrow \psi_B = -\frac{kT}{q}\ln\frac{N_A}{n_i} = -0.28\ V$$

b) The flat band voltage is zero then the threshold voltage will write

$$V_T = -\frac{Q_B}{C_{ox}} + 2|\psi_B|$$

The charge Q_B is the charge in the space charge region of the semiconductor $-qN_AW$ and W the width of the space charge region:

$$W = \sqrt{\frac{2\varepsilon_{Si}}{q}\frac{2|\psi_B|}{N_A}} = \sqrt{2\varepsilon_{Si}qN_A 2|\psi_B|} \Rightarrow Q_B = -14.3\ nC/cm^2$$

$$V_T = \frac{\sqrt{2\varepsilon_{Si}qN_a 2|\psi_B|}}{C_{ox}} + 2|\psi_B| \approx 1\ V$$

with C_{ox} the gate capacity by unit surface, $C_{ox} = \varepsilon_{ox}/t_{ox} = 35.4\ nF/cm^2$

c) From the figure the total gate voltage is 1.5 $V > V_T$. So, the surface is inverted.

d) The electric field in the oxide is $E_{ox} = \frac{V_{ox}}{t_{ox}} = 90\ kV/cm$

e) From Gauss's law, the electric field at the oxide is

$$E_{ox} = \frac{\text{total charge}}{\varepsilon_{ox}} = \frac{\text{charge in the channel} + \text{charge in the space-charge-region}}{\varepsilon_{ox}}$$

$$E_{ox} = \frac{Q_{inv} + Q_B}{\varepsilon_{ox}}$$

$$Q_{inv} = E_{ox}\varepsilon_{ox} - Q_B = 17.5\ nC/cm^2$$

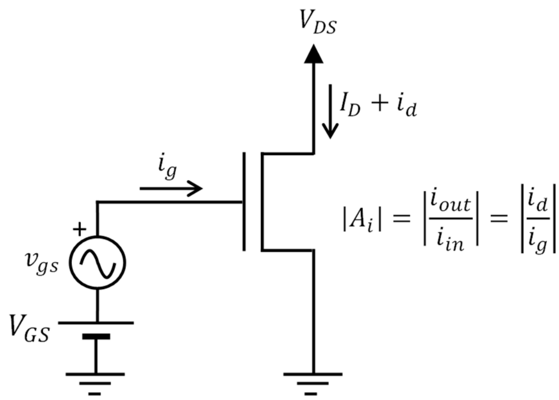
Alternatively, the excess over V_T i.e $V_G - V_T = \frac{Q_{inv}}{C_{ox}}$

$$Q_{inv} = C_{ox}(V_G - V_T) = 17.5\ nC/cm^2$$

6. ***Consider the circuit shown in the figure, using it we try to analyze the frequency features of a n channel MOSFET.***

   a) ***Considering the values (DC) of V_{GS} and V_{DS}, determine in what region is the transistor biased. Calculate the corresponding value for I_D.***

   b) ***Calculate the main parameters of the transistor small signal model in its operating point taking into consideration the region of operation where the transistor is biased.***



   c) ***Analyzing the small signal circuit, find the transistor cut-off frequency f_T. The cut-off frequency is defined as the frequency for which $|A_i| = 1$. Discuss the effect on f_T of the following dimensions: oxide thickness (t_{ox}), width (W) and channel length (L).***

Data: $\boldsymbol{V_{GS} = V_{DS} = 2.5\ V,\ V_T = 1\ V, W = 50\ \mu m,\ \ L = 1\ \mu m}$
$\boldsymbol{\varepsilon_{r_{ox}} = 3.9,\ \varepsilon_o = 8.85 \times 10^{-14}\ F/cm,\ \ t_{ox} = 80\ nm, \mu_n = 900\ cm^2/Vs}$

a) $V_{GS} > V_T$ and $V_{GS} = V_{DS}$. Finally, $V_{DS} > V_{GS} - V_T$ and the transistor is in saturation.

$$I_D = \frac{1}{2}K(V_{GS} - V_T)^2 \ ; \ K = \mu_n C_{ox}\frac{W}{L} = 1.9\ mA/V^2 \ ; \ I_D = 2.18\ mA$$

b) The capacitance $C_{gs} = \frac{2}{3}C_{ox}WL = 14\ fF$.

Remember the convention:

- Capital letter with capital letter subscript for the bias value M_M
- Small letter with capital letter subscript for the total value of the variable m_M
- Small letter with small letter subscript for the signal part m_m

$$g_m = \left.\frac{di_D}{dv_{GS}}\right|_Q = K\left(V_{GSQ} - V_T\right) = 2.91\ mA/V$$

c) Fort the current gain:

$$i_g = v_{gs}j\omega C_{gs} \implies |A_i| = \left|\frac{i_d}{i_g}\right| = \frac{g_m v_{gs}}{v_{gs}C_{gs}\omega} = \frac{g_m}{C_{gs}\omega}$$

If we look at what frequency the current gain drops to unit we get:

$$\frac{g_m}{C_{gs}\omega_T} = 1 \implies \omega_T = 2\pi f_T = \frac{g_m}{C_{gs}}$$

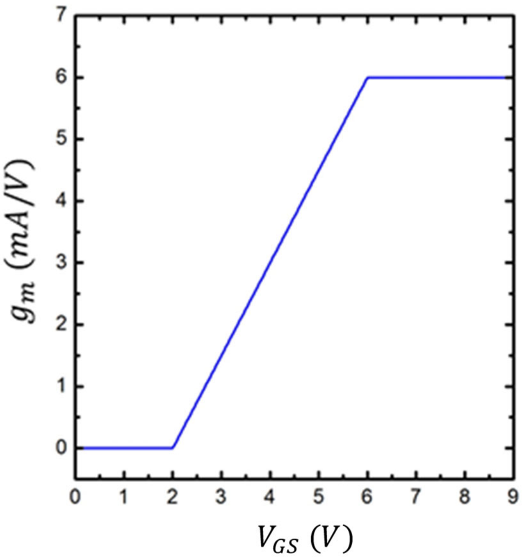
$$f_T = \frac{1}{2\pi}\frac{g_m}{C_{gs}} = \frac{1}{2\pi}\frac{K\left(V_{GSQ} - V_T\right)}{{}^2/_3\,C_{ox}WL} = \frac{1}{2\pi}\frac{\mu_n C_{ox}\frac{W}{L}\left(V_{GSQ} - V_T\right)}{{}^2/_3\,C_{ox}WL} = 32\ GHz$$

If we have a look to the final expression for f_T we see that it does not depend neither on the oxide thickness t_{ox} neither on the channel width. On the other hand, it depends inversely on the square of the channel length $f_T \propto {}^1/_{L^2}$.

7. *The figure shows the transconductance g_m of a n channel MOSFET as a function of V_{GS}, measured for a given value of V_{DS}.*

- ***a) Identify in the figure which regions correspond to cut-off, linear zone and saturation.***
- ***b) What is the transistor threshold voltage V_T?***
- ***c) What is the value of V_{DS} for which the measurements have been done?***
- ***d) Calculate the mobility of the electrons in the channel.***

Data: $\boldsymbol{W = 70\ \mu m,\ L = 10\ \mu m,\ t_{ox} = 20\ nm,}$
$\boldsymbol{\varepsilon_{r_{ox}} = 3.9,\ \varepsilon_o = 8.85 \times 10^{-14}\ F/cm}$



a) b) The transconductance in saturation has a linear dependence with V_{GS}:

$$g_m = \left.\frac{di_D}{dv_{GS}}\right|_Q = K\left(V_{GSQ} - V_T\right)$$

An n channel MOSFET is in saturation if $V_{DS} > V_{GS} - V_T$, i.e., $V_{GS} < V_{DS} + V_T$. If we look at the figure the linear dependence is from $V_{GS} > 2V$ until $V_{GS} = 6V$. That means that in this interval $2V \leq V_{GS} \leq 6V$ the MOSFET is in saturation. If $V_{GS} \geq 6V$ the MOSFET is in the ohmic zone and in cut-off for $V_{GS} \leq 2V$. Then, $V_T = 2\ V$.

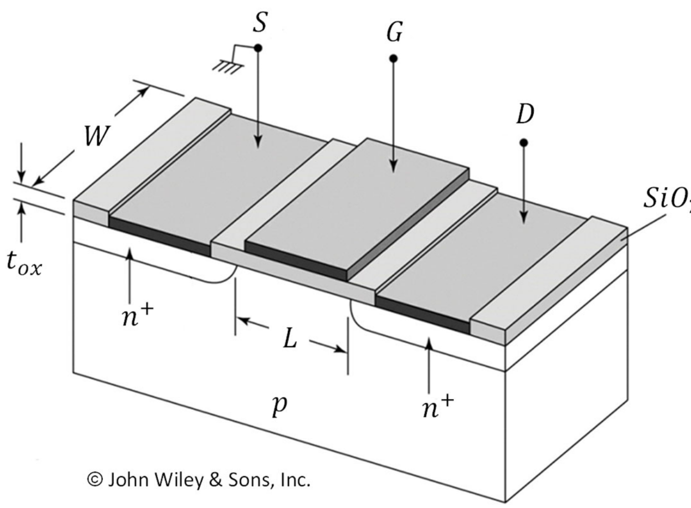
c) If $V_T\ = 2V$ the transition from saturation to ohmic zone happens at $V_{GS} = 6V$, then $V_{DS} = 4V$.

d) The expression in saturation for the transconductance is $g_m = \left.\frac{di_D}{dv_{GS}}\right|_Q = K\left(V_{GSQ} - V_T\right)$

If we take the value $g_m = 6\ mA/V$ for $V_{GS} = 6V$, we obtain $K = 1.5\ mA/V^2$. Then:

$$K = \mu_n C_{ox}\frac{W}{L} \Longrightarrow \mu_n = \frac{K}{C_{ox}}\frac{L}{W} = \frac{1.5 \times 10^{-3} \cdot 20 \times 10^{-7}}{3.9 \cdot 8.85 \times 10^{-14}}\left(\frac{10}{70}\right) = 1241\ \ cm^2/V \cdot s$$

8. ***The objective of the Dennard scaling is to achieve that the consumed power by a MOSFET scales with the area. Then, although in an integrated circuit the density of transistors increases, the consumed power by unit area will remain more or less constant. As an example consider an n channel MOSFET with Aluminum gate ($\phi_m = 4.2\ eV$), fabricated on a p type substrate and doping $N_A = 10^{16}\ cm^{-3}$. The gate dielectric is silicon dioxide without fixed charge and thickness t_{ox}.***



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a) ***Calculate the threshold voltage V_T for an oxide thickness $t_{ox} = 100\ nm$. Verify that if we reduce t_{ox} by a factor 2, the value of V_T will reduce also approximatively in a factor 2.***

Then if we reduce t_{ox} by a factor 2, we can operate the MOSFET with around half voltages V_{DS} and V_{GS}. Consider also reduce by a factor 2 the width W and the channel length L, (the aspect ratio W/L will remain constant).

b) ***How will the current I_D vary compared with the value for the initial dimensions of t_{ox}, W and L?***

c) ***Justify that the consumed power ($P \approx V_{DS} I_D$) is proportional to the device area $W \times L$.***

d) ***Finally evaluate how the transistor cut-off frequency f_T will vary when the dimensions (t_{ox}, W y L) are reduced by a factor of 2. f_T is the frequency for which the small signal current gain $|i_d/i_g| = 1$.***

Data: $q = 1.6 \times 10^{-19}\ C, k_B = 1.38 \times 10^{-23}\ J/K,\ T = 300\ K,\ n_i = 10^{10}\ cm^{-3}$, $\chi_{Si} = 4.05\ eV,\ \varepsilon_o = 8.85 \times 10^{-14}\ F/cm,\ \varepsilon_{r_{Si}} = 11.9,\ \varepsilon_{r_{Ox}} = 3.9$

a) Let's calculate first the threshold voltage V_T.

$$V_T = \frac{\emptyset_{MS}}{q} + 2|\psi_B| + \frac{\sqrt{2\varepsilon_{Si} q N_A 2|\psi_B|}}{C_{ox}}$$

with $\emptyset_{MS}$ the difference in work function between the metal ϕ_M, in this case Aluminum, and the semiconductor ϕ_S.

The work function in the semiconductor writes as:

$$\phi_S = \chi_{Si} + \frac{E_G}{2} + q|\psi_B|$$

where χ_{Si} is the electron affinity in the semiconductor and ψ_B is the distance between the intrinsic Fermi level in the bulk (far away from the surface) of the semiconductor and the actual position of the Fermi level:

$$\psi_B = \frac{(E_F - E_{Fi})}{q} = -\frac{kT}{q} ln \frac{N_A}{n_i} = -0.345\ V$$

$$\phi_S = \chi_{Si} + \frac{E_G}{2} + q|\psi_B| = 4.05\ eV + 0.55\ eV + 0.345\ eV \approx 4.95\ eV$$

Finally,

$$\emptyset_{MS} = \phi_M - \phi_S \approx -0.75\ eV$$

$$V_T = \frac{\emptyset_{MS}}{q} + 2|\psi_B| + \frac{\sqrt{2\varepsilon_{Si} q N_A 2|\psi_B|}}{C_{ox}} \approx -0.75V + 0.7V + 1.39\ V \approx 1.35\ V$$

for an oxide thickness of $t_{ox} = 100\ nm$.

If $t_{ox} = 50\ nm$ then:

$$V_T = \frac{\emptyset_{MS}}{q} + 2|\psi_B| + \frac{\sqrt{2\varepsilon_{Si} q N_A 2|\psi_B|}}{C_{ox}} \approx -0.75V + 0.7V + 0.7\ V \approx 0.65\ V$$

Roughly the threshold reduces to a half when the oxide thickness reduces by a factor 2.

b) If we have a look to the expression of the current in saturation.

$$I_D = \frac{1}{2}\,\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2$$

Now if we reduce by a factor of 2 the dimensions t_{ox}, W, L and also the voltages $V_{GS}$and V_T we get

$$I_D \sim \frac{1}{1/t_{ox}} \frac{W}{L} V^2 \sim t_{ox} \frac{W}{L} V^2$$

and when we scale

$$I_D \sim \frac{1}{2} \cdot \frac{1/2}{1/2} \cdot (1/2)^2 \sim \frac{1}{2}$$

c) The power $P_D = V_{DS} I_D$. Scaling dimensions (and consequently currents as we have just seen) and also the voltages by a factor of 2 the dissipated power will scale by a factor of 4.

Summarizing if we scale dimensions and voltages by a factor of 2, the area will reduce by a factor of 4 and so will do the dissipated power. The dissipated power by unit area will remain constant.

d) The cut-off frequency

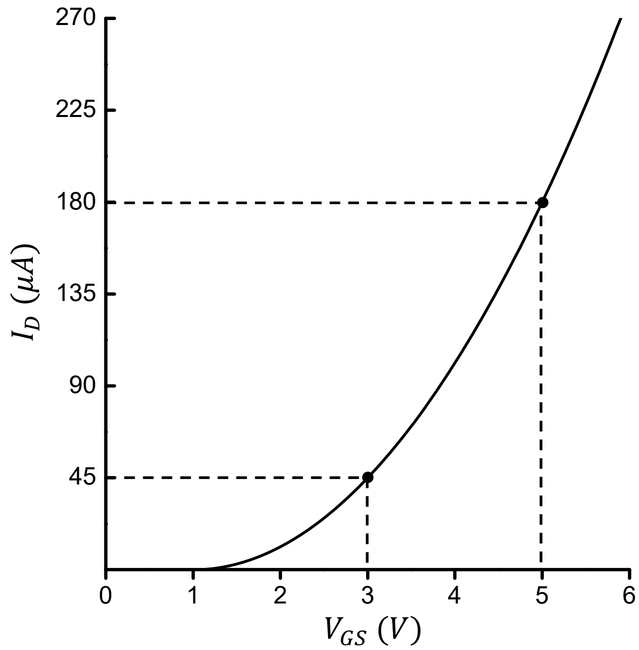
$$f_T = \frac{1}{2\pi} \frac{g_m}{C_{gs}} = \frac{1}{2\pi} \frac{K(V_{GSQ} - V_T)}{2/3\, C_{ox} W L} = \frac{1}{2\pi} \frac{\mu_n C_{ox} \frac{W}{L} (V_{GSQ} - V_T)}{2/3\, C_{ox} W L}$$

will scale if we reduce dimensions and voltages by a factor of two we get:

$$f_T \sim \frac{1/t_{ox} \frac{W}{L} V}{1/t_{ox}\, W L} \sim 2$$

The cut-off frequency will increase by a factor of 2.

9. The figure shows the transfer characteristic I_D-V_{GS} for a MOSFET measured in saturation (we have fixed $V_{DS} = V_{GS}$ during the measurement) We have also measured the gate capacitance (WLC_{ox}), and we obtain 25×10^{-17} F. The gate is polysilicon p^+, square with $1\,\mu m$ side. The substrate resistivity is $2.1\ \Omega \cdot cm$.



- ***a) Calculate the threshold voltage V_T.***
- ***b) Is a n or p channel transistor? Why?***
- ***c) Calculate the transconductance g_m at $V_{GS} = 5\ V$, under saturation conditions.***
- ***d) Find the electron mobility in the channel and the oxide thickness.***
- ***e) Calculate the substrate doping, knowing that the ratio $\frac{\mu_p}{\mu_n} = \frac{1}{3}$.***
- ***f) Calculate the flat-band voltage V_{FB} and the fixed charge in the oxide.***

$\boldsymbol{Data}$: $\varepsilon_{ox} = 3.45 \times 10^{-13}\ F/cm$, $\varepsilon_{Si} = 10.5 \times 10^{-13}\ F/cm$, $E_g = 1.1\ eV$, $n_i = 10^{10}\ cm^{-3}$, $kT/q = 0.025\ V$, $q = 1.6 \times 10^{-19}\ C$

a) In saturation

$$I_D = \frac{1}{2}\,\mathrm{K}\frac{W}{L}(V_{GS} - V_T)^2$$

Taking two points of the curve ($V_{GS} = 3V$ and $V_{GS} = 5V$), we get:

$$\frac{180}{45} = \frac{(5 - V_T)^2}{(3 - V_T)^2} \Longrightarrow V_T = 1V$$

we can obtain also $K = 22.5\ \mu A/V^2$

b) It's a N channel MOSFET because $V_{GS}, V_{DS} > 0$, $V_T > 0$, $I_D > 0$

c) The transconductance g_m is defined as:

$$g_m = \left.\frac{di_D}{dv_{GS}}\right|_Q = K\left(V_{GSQ} - V_T\right)$$

Calculating for $V_{GSQ} = 5V$ we obtain $g_m = 90\ \mu A/V$

d) The gate has $L = W = 1\mu m$ and the $C_{ox} = \frac{25 \cdot 10^{-17}\ F}{10^{-8} cm^2} = 25\ nF/cm^2$

$$t_{ox} = \frac{\varepsilon_r \varepsilon_{ox}}{C_{ox}} \approx 140\ nm$$

On the other hand,

$$K = \mu_n C_{ox} \frac{W}{L} \Longrightarrow \mu_n = \frac{K}{C_{ox}} \frac{L}{W} = 900 \ cm^2/V \cdot s$$

e) The resistivity $\rho = \frac{1}{\sigma} = \frac{1}{q\mu_p N_A}$. Remember that we are in a NMOS and the substrate is p type,

$$\mu_p = \frac{\mu_n}{3} = 300 \ cm^2/V \cdot s \ \Longrightarrow N_A = \frac{1}{q\mu_p \rho} \ \approx 10^{16} \ cm^{-3}$$

f) The threshold voltage V_T can be written as the addition of the value of the threshold voltage V_{T0} when the bands are flat and the flat-band voltage V_{FB} i.e the needed voltage in order to flatten the bands

$$V_T = V_{FB} + V_{T0}$$

On the one hand:

$$V_{TO} = \frac{\sqrt{2\varepsilon_{Si} q N_A 2|\psi_B|}}{C_{ox}} + 2|\psi_B| \ \text{ with } \ \psi_B = -\frac{KT}{q} ln \frac{N_A}{n_i} = -0.345 \, V$$

$$V_{TO} = 2.6 \, V$$

The flat-band voltage V_{FB} will be:

$$V_T - V_{TO} = -1.6 \, V$$

from V_{FB} we can calculate the fixed charge in the oxide:

$$V_{FB} = \frac{\phi_{MS}}{q} - \frac{Q_{ox}}{C_{ox}}$$

but we need to calculate first $\phi_{MS} = \phi_M - \phi_S$.

The work function at the gate electrode is the difference between the Fermi level at the gate electrode and the vacuum level. As the gate is fabricated in heavily p-type doped polysilicon, $\phi_M \approx \chi_{Si} + E_G$, while on the semiconductor side $\phi_S = \chi_{Si} + \ E_G/2 + q|\psi_B|$.

Finally,

$$\phi_{MS} = \frac{E_G}{2} + q|\psi_B| = 0.205 \, V$$

$$V_{FB} = \frac{\phi_{MS}}{q} - \frac{Q_{ox}}{C_{ox}} \Longrightarrow Q_{ox} = C_{ox} \left(\frac{\phi_{MS}}{q} - V_{FB} \right) = 45 \ nC/cm^2$$