THERMOELECTRIC AMPLIFICATION OF PHONONS IN NANOSTRUCTURES.

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ABSTRACT

The damping coefficient of a acoustical phonons interacting with electrons confined in quantum wires modeled with a parabolic potential subjected to an external temperature gradient in presence and absence magnetic field is calculated. In quantum wire the amplification coefficient is found to be enhanced over its bulk value. Results show that in presence magnetic field phonon amplification coefficient to be enhanced over its value in absence magnetic field.

Keywords: damping coefficient, quantum, temperature gradient, potential subjected

I. INTRODUCTION

The possibility of fabrication of quantum structures of submicron dimensions has, the investigation of thermoelectric effects in nanostructures is attracting considerable interest [1-8]. The problem of acoustic wave amplification by an external temperature gradient in semiconductors has already been considered [9-15]. To describe the interaction of phonons with electrons in the presence of external temperature gradient, a quantum mechanical approach was employed to derive an expression for the damping (amplification) coefficient for the acoustic wave from which the conditions for the phonon instability was obtained.

In this paper we present a theory for the phonon instability in parabolic quantum well wires under a temperature gradient and a quantizing magnetic field.

The kinetic equation for the acoustic-phonon population is as follows[16],

$$\frac{dN_q(t)}{dt} = \gamma_q N_q - \left(\frac{N_q(t) - N_q^0}{\tau_q}\right) \qquad (1)$$

Where, τ_q is the acoustic-phonon relaxation time assumed to be described by the Landay-Rumer loss, N_q^0 is the equilibrium phonon distribution function, and γ_q is the phonon growth rate due to the collision with the electrons in the temperature gradient which is given by [16-17]

$$\gamma_q = \frac{2\pi}{\hbar}\sum_{knn'}\left|\langle kn|M|k'n'\rangle\right|^2 \left(f_{k-q,n'} - f_{k,n}\right)\times$$
$$\times\delta\left(E_{k,n} - E_{k-q,n'} - \hbar\omega_q\right)(2)$$

In Eq.(2) $f_{k\ ,n}^{0}$ is the electron distribution function and ω_q is the frequency of acoustic phonons, k represents the wave vector of the electron along the *x* direction. To evaluate Eq.(2), we consider the cases of short-and open –circuited samples. For short-circuited samples, the electron distribution function in the presence of the temperature gradient is, within the relaxation time approximation, given by [9,16]

$$f_{k,n} = f_{k,n}^0\left\{1 - \frac{\hbar\tau}{m^*}k\left[\frac{\nabla T}{T}\left(E_{n,k}/K_B T\right)\right]\right\}(3)$$

Where $\tau$the relaxation time of electrons is, ∇T is the temperature gradient, and k_B is the Boltzmann constant. Here we choose the equilibrium electron distribution $f_{k_z,n}^0$ such that $2\sum f_{k_z,n}^0 = N_0$, where N_0 is the total number of electrons. For a nondegenerate electron gas it can be shown to be

$$f_{k,n_z}^0 = \frac{\sqrt{2\pi}\hbar n_1}{\gamma\left(m^* K_B T\right)^{1/2}}\exp\left(-\frac{\hbar\omega(n+1/2)}{K_B T}\right)\times$$
$$\times\exp\left(-\frac{\hbar^2 k^2}{2m^* K_B T}\right),$$

(4)

where $\delta_n = \sum_n \exp\left[-\frac{\hbar\omega(n+1/2)}{K_B T}\right]$, n_1 is the one-dimensional concentration of electrons (electrons per cm^{-1}).

For open –circuited samples, the electron distribution function changes into [9,18]

$$f_{k,n}=f^0_{k,n}\left\{1-\frac{\hbar\tau}{m^*}k\left[\frac{\nabla T}{T}\left(E_{n,_zk}/K_BT\right)-\frac{e\nabla\varphi}{K_BT}\right]\right\} \quad (5)$$

where e is the charge of the electron, and φ is the electrostatic potential. Applying the zero-current condition $j=-\frac{e\hbar}{m^*}\sum f_{k,n}k=0,$ to eliminate $\nabla\varphi$ of Eq.(5).

II.THERMOELECTRIC AMPLIFICATION OF PHONONS IN ABSENCE OF MAGNETIC FIELD

We consider electrons confined in a wire of dimensions L_x, L_y, L_z such that $L_x, L_z << L_x=L$. The confinement of electrons in the z direction is modeled with a triangular well. This gives rise to electric subbands. We will consider electron densities such that only the lowest subband with energy E_z^0 is occupied in the z direction. The corresponding eigenfunctions is denoted by $\Psi_0(z)$. For confinement in the lateral direction y for a parabolic well with frequency ω the one-electron eigenfunctions and eigenvalues are given by

$$\Psi_{n,k}(r)=\sqrt{1/L}H_n(y)e^{ikx}\Psi_0(z) \quad (6)$$

$$E_{n,k}=\hbar\omega\left(n+\frac{1}{2}\right)+\frac{\hbar^2k^2}{2m^*}+E_z^0$$

where $H_n(y)$ is a Hermite polynomial, L is the length of the wire, m^* is the conduction band mass and $\Psi_0(z)=b_0^{3/2}ze^{-b_0z/2}/2$, $\langle L_z\rangle=3/b_0$.

To evaluate γ we need to know the matrix element connecting the initial electron state (n,k) to the final state (k',n') due to an interaction with a phonon. Using the wave function (6), it can easily be seen that

$$\left|\langle k_z,n|M|k_{z',n'}\rangle\right|^2=\frac{E_d^2\hbar q}{2\rho\upsilon_s\Omega_0}\left(1+\frac{q_z^2}{b_0}\right)^{-3}\times \quad (7$$
$$\times\frac{u^{n'-n-1/2}}{l_\omega n!n'!}e^{-u}\left[L_n^{n'-n}(u)\right]^2$$

Here $L_n^m(x)$ are the associated Laguerre polynomials, $u=l_\omega^2q_y^2/2,\quad l_\omega^2=\hbar/m^*\omega$.

Inserting Eq.(7) into Eq.(2), assuming an energy independent relaxation time, and changing the summation over k_z into an integral one obtains, for $\hbar\omega_q<K_BT$:

$$\gamma=A\exp\left(-\frac{\hbar^2\left(q_z^2+b'^2\right)}{2m^*K_BT}\right)\exp\left(-\frac{\hbar\omega(n+n'+1)}{2K_BT}\right)\times$$
$$\times\left\{\begin{array}{l}1+\frac{\tau\nabla T}{m^*T}\left[\begin{array}{l}\frac{\hbar^2}{2m^*}\left(\frac{q_z^2}{4}+b'^2\right)+\frac{\hbar\omega_k}{2}+\\+\frac{\hbar\omega}{2}(n+n'+1)\end{array}\right]\times\\ \times\left(\frac{1}{\upsilon_s}+\frac{m^*\omega(n'-n)}{q_zK_BT}-\frac{m^*\upsilon_s}{K_BT}\right)\end{array}\right\} \quad (8)$$

where

$$A=\frac{\sqrt{\pi m^*}E_d^2n_1q}{\sqrt{2}(K_BT)^{3/2}\rho L_yL_z\delta_n}\left(1+\frac{q_z^2}{b_0}\right)^{-3}\frac{u^{n'-n-1/2}}{l_\omega n!n'!}\times,$$
$$\times e^{-u}\left[L_n^{n'-n}(u)\right]^2$$
$$b'=\frac{m^*\left[\omega(n-n')-\omega_k\right]}{\hbar q_z}$$

III. THERMOELECTRIC AMPLIFICATION PHONONS IN MAGNETIC FIELD

In the presence of a perpendicular magnetic field HII OZ, the one-electron eigen-functions and eigen-values are

$$\Psi_{Nk}=\Phi_n(y-y_0)e^{ikx}\psi_0(z)/\sqrt{L} \quad (9)$$

$$E_{Nk}=\left(N+\frac{1}{2}\right)\hbar\tilde{\omega}+\frac{\hbar^2k^2}{2\tilde{m}^*}+E_z$$

respectively, N is the Landay-level index, k is the wave vector in the x direction, $\tilde{\omega}=\sqrt{\omega_c^2+\omega^2}$, $\tilde{m}=m^*\tilde{\omega}^2/\omega$, $y_0=\tilde{b}l_b^2k$, with $\tilde{b}=\omega_c/\tilde{\omega}$ and $\tilde{l}_b=\hbar c/m^*\tilde{\omega}$.

Using the wave function (), the square matrix element can easily be seen that

$$\left|\langle kN|M|k'N'\rangle\right|^2=\frac{E_в^2\hbar q}{2\rho\upsilon_c\Omega_0}\left(1+\frac{q_z^2}{b_0^2}\right)^{-3}\frac{N!e^{-u}\upsilon^{N'-N}}{N'!}\times$$
$$\times\left[L_N^{N'-N}(\vartheta)\right]^2$$

Here $\vartheta=\tilde{l}_b^2\left(q_x^2+\tilde{b}q_y^2\right)$

Inserting Eq.() into Eq.(), assuming an energy independent relaxation time, and changing the summation over k into an integral one obtains, for $\hbar\omega_{й}<K_BT$

$$\gamma = A' \exp\left(-\frac{\hbar^2 (q_z^2 + b'^2)}{2m^* K_B T}\right) \exp\left(-\frac{\hbar\tilde{\omega}(N+N'+1)}{2K_B T}\right) \times$$

$$\left\{ 1 + \frac{\tau \nabla T}{m^* T} \left[\frac{\hbar^2}{2m^*} \left(\frac{q_z^2}{4} + b''^2 \right) + \frac{\hbar\omega_k}{2} + \frac{\hbar\tilde{\omega}(N+\tilde{N}'+1)}{2} \right] \times \times \left(\frac{1}{\upsilon_s} - \frac{\tilde{m}^* \tilde{\omega}(N-N')}{q_z K_B T} - \frac{\tilde{m}^* \upsilon_s}{K_b T} \right) \right\} \quad (11)$$

where

$$A' = \frac{\sqrt{\pi m^*} E_d^2 n_1 q}{\sqrt{2} (K_B T)^{3/2} L_y L_z \delta_N \rho} \left(1 + \frac{q_z^2}{b_0^2} \right)^{-3} \frac{N! e^{-u} \upsilon^{N'-N}}{N'!} \times \times \left[L_N^{N'-N}(\vartheta) \right]^2$$

$$, \; b'' = \frac{\tilde{m}^* [\tilde{\omega}(N-N') - \omega_k]}{h q_z}$$

IV. CONCLUSION

It follows from Eqs.(8) and (11) that if γ>0 the phonon population is amplified, whereas if γ<0 is damped. Of course, a net growth of the acoustic phonon population (amplification) can only be achieved provided the growth rate γ due to electron-phonon collisions is greater than the losses τ^{-1} due to other effects than phonon emission or absorption by electrons. It is clear from Eqs.(8)-(11) that, the condition for amplification is defined as

$$\left(\frac{1}{\upsilon_s} + \frac{m^* \omega(n'-n)}{q_z K_B T} - \frac{m^* \upsilon_s}{K_b T} \right) > 1 \qquad \text{or}$$

$$\frac{K_B T}{m^* \upsilon_s^2} + \frac{\omega(n'-n)}{q_z \upsilon_s} > 1$$

It is found that the phonon amplification coefficients follow the inverse area of the cross-section of the wire so that they can be enhanced over their correspondent bulk values [16] by decreasing transverse dimension of the quantum wire. For judicious parameters of the GaAs/$Ga_{1-x}Al_x$as parabolic quantum wire it has been obtained that $\gamma \geq \gamma_{volume}$ and $\gamma_H \geq \gamma \geq \tau_q^{-1}$

From comparison between (8) and (11) it is seen that the coefficient of the thermoelectric phonon amplification in the quantum wire in the presence of a magnetic field increases. This is associated with that in a magnetic field carriers in the quantum wire are more heavily localized, because of this phonon carrier scattering processes occur more actively. In [19], attention was drawn to the effective amplification of the electron scattering in the ultra-quantum limit with increase in the magnetic field. From (8) and (11) it is seen that the coefficient of the thermoelectric phonon amplification increases with decreasing wire cross section. Therefore a decrease in dimensionality of the quantum system, i.e. an increase in the band carrier localization, leads to a rise in the coefficient of the thermoelectric phonon amplification.

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