STATIONARY ELECTRIC FIELD PARAMETERS CALCULATION IN QUASI-HOMOGENEOUS MEDIUM OF THE LOW TEMPERATURE MULTI-ELECTRODE COMPOSITE ELECTRIC HEATER COMPLEX SYSTEMS

V.V. Evstigneev, T.M. Khalina

The Altai State Technical University by I.I. Polzunov

ABSTRACT

In article the data about stationary electric field parameters calculation as applied to low temperature multi-electrode composite electric heater complex systems.

Keywords: electric field, low temperature, multi-electrode, composite, electric heater,

I. INTRODUCTION

From the point of view of electric power efficient utilization the most efficient way to heat biological and technical subjects is the application of the local surface-spread heating which can be implemented by means of the composite electric heaters [1].

The low temperature multi-electrode composite electric heaters (LTMCEH) present a complex system transforming electric power into that of the heat one and providing predetermined temperature on the heater surface in accordance with the LTMCEH electro-thermal physical parameters. The mentioned above demands the exact calculation of the given parameters with the reference to the system of electrodes located in the conductive composite material.

The task rigorous solution can be obtained only as the result of calculation of the stationary electric field created by the electrode system in the quasi-homogeneous medium. The heater calculation model is taken subject to the following boundary conditions: LTMCEH can be considered to possess lumped parameters as the batch processes under the frequency of f=50 Hz are regarded as quasi-stationary; the electrode surfaces along the full length can be regarded as equipotential subject to the medium low specific conductivity; the resistive material perimeter boundary can be considered as impenetrable for the electric field lines; as the electrode and resistive layer length is much greater than resistive layer cross-section dimension and the thickness is infinitesimal, the field between the electric heater electrode can be regarded as in-plane.

II. RESULTS AND DISCUSSION

The task solution has been implemented by means of the of electric field intensity direct determination [2] in combination with the conformal method. This method is based on the auxiliary function application γ (x, y) expressing the angle value formed by the in-plane electrical field vector at any point of the area at issue with one of the axes of the Cartesian coordinate system. The γ (x, y) function is harmonic and satisfying Laplace's two-dimensional equation as well as to the first type boundary conditions stated subject to the electric and equipotential lines orthogonality of the field in the parts where one of the condition is desired as follows:

U_s = const or $\left(\frac{\partial U}{\partial n}\right)_s = 0$.

The electric field of the LTMCEH axis-simmetric systems can be described by means of Laplace's equation in the meridian plane system equal in all the meridian planes in the cylindrical coordinates (R, φ, z). In case that $l/R > 1$ in the cylindrical coordinate system, the axis of which is congruent to the cylinder axis, the electric field proves to be in-plane and equal for any z.

The nonlinear transcendental equations have been derived as the result of insertion of the in-plane calculation model and conformal image of the original plane of the complex variable Z on to the plane of the new complex variable ζ subject to the necessary congruence of the original and image points of the planes.

The system solution has been implemented numerically by means of Newton's discrete method. Jacobi matrix of the function partial derivatives of the system is approximated by means of the first differences, in addition the minimum step of function argument is picked out under the criterion of the corresponding difference significance cancellation. The definite integrals appearing in equation system functions are calculated at each iterating by means of Newton-Cotes quadrature formula of the eighth order. Simultaneously with calculation of the subsequent values of the non-dimensional parameters a_i solution at each iterating the assessment of error of their determination is implemented.

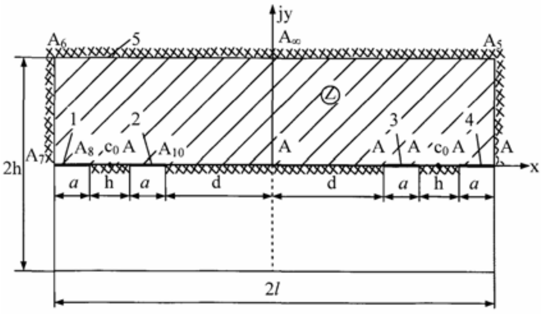
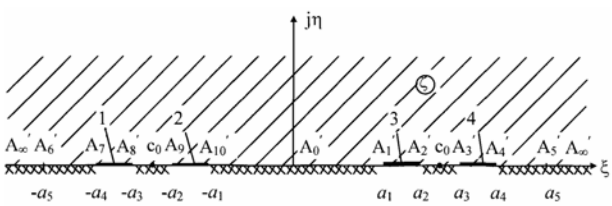
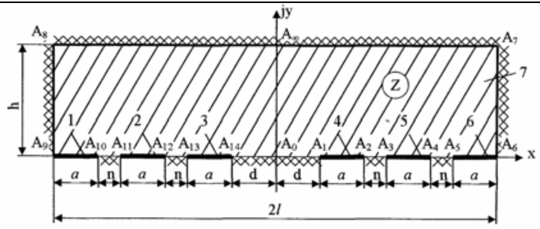
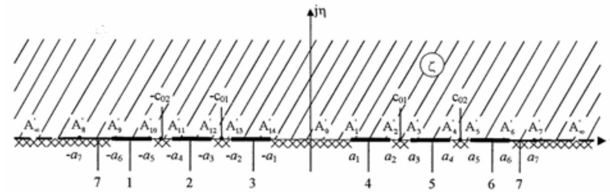
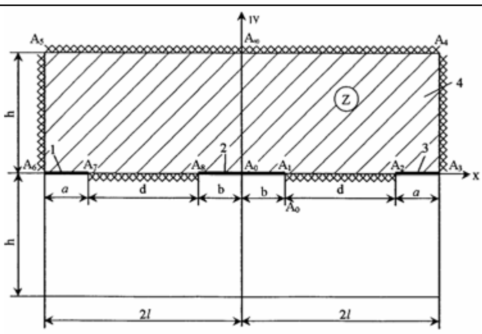
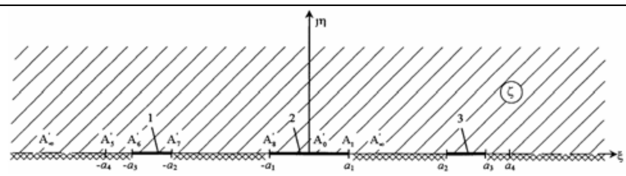
The given method has been applied in the electric parameters calculation of the most frequent LTMCEH systems :

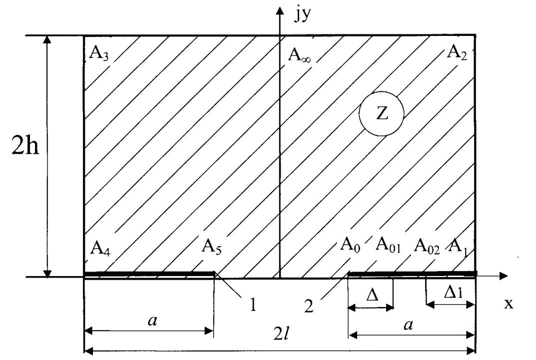
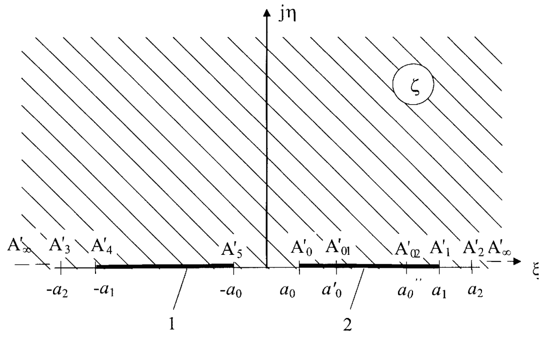
- conductivity between two pairs of the coplanar electrodes located in the rectangular cross section conductor (No. 1);
- conductivity between three-electrode systems (No. 2);
- conductivity of the low temperature composite electric heater (LTCEH) systems (No. 3);
- partial conductivities between the coplanar electrodes (No. 4);
- system conductivity subject to the electrode cross-section dimension and their bias with respect to each other (No. 5);
- LTCEH surface potential distribution (No. 6);
- conductivity of the two-electrode axis-symmetric system (No. 7);
- conductivity of the three-electrode axis-symmetric system (No. 8);
- conductivity of the multi-electrode axis-symmetric system (No. 9).

The calculation models and systems in the image plane are presented in Table 1.

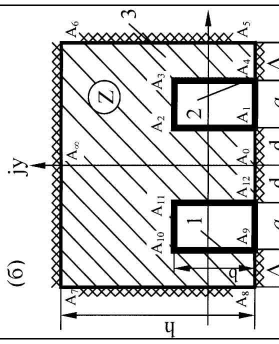
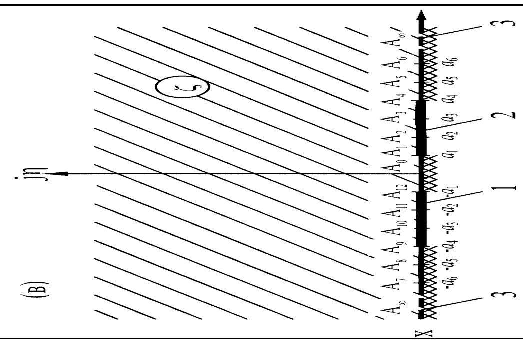
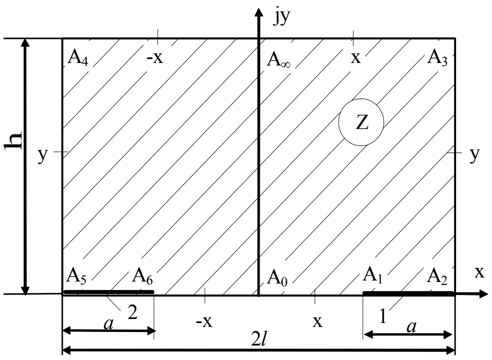
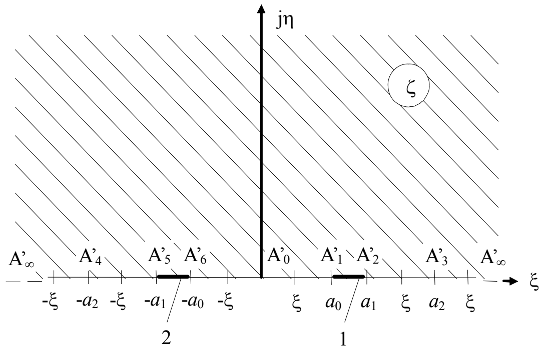
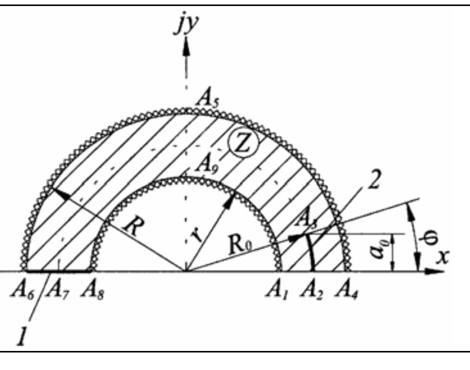
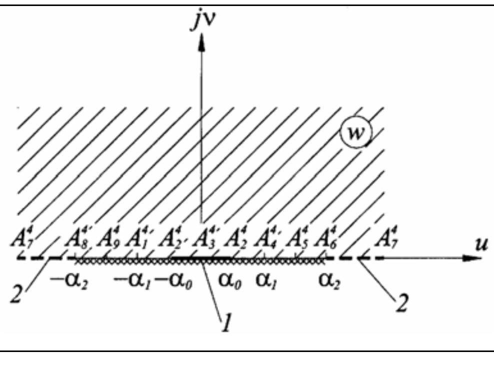
The imaging parameters and exact and approximate design formulas are presented in Table 2.

Table 1

No.	Calculation Model	System in the Image Plane
1	2	3
1		
2		
3		

4		

Continue of table 1

1	2	3
5		
6		
7		

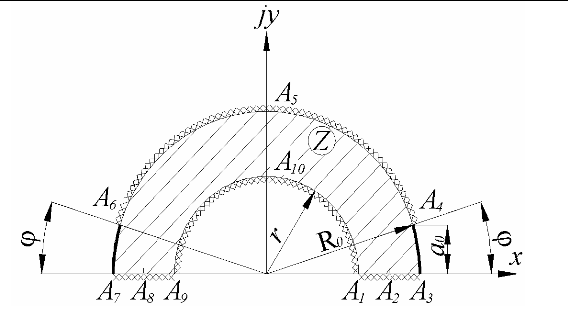
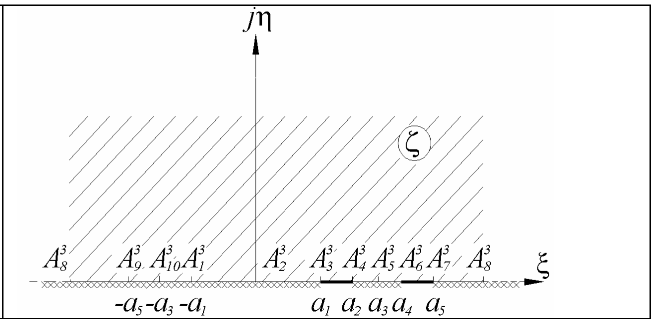
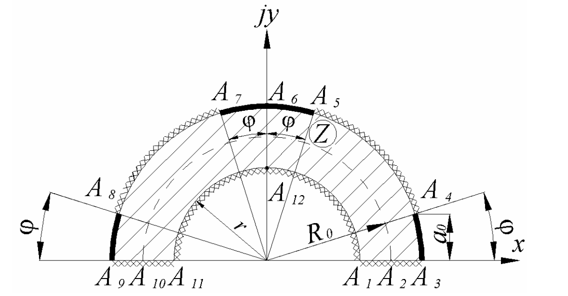
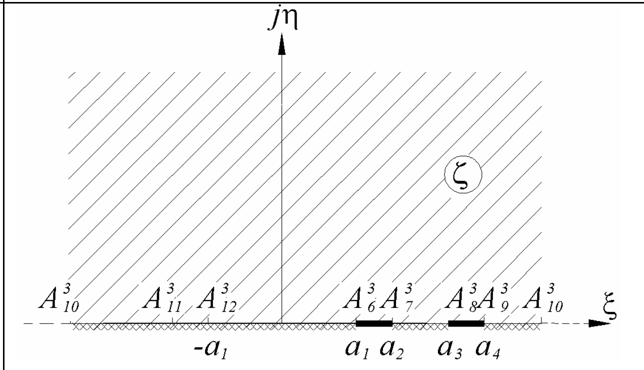
8		
9		

Table 2

No.	Imaging Parameters	Exact and Approximate Designed Formulas
1	2	3
1	$\zeta = a_4 \mathrm{sn}(K_0 \frac{z}{l}, k_0)$; $\frac{a_1}{a_4} = \mathrm{sn}(K_0 \frac{d}{l}, k_0)$; $\frac{a_2}{a_4} = \mathrm{sn}(K_0 \frac{d+a}{l}, k_0)$; $\frac{a_3}{a_4} = \mathrm{sn}(K_0 \frac{d+a+n}{l}, k_0)$; $\frac{h}{l} = \frac{K'_0}{K_0}$; $k_0 = \frac{a_4}{a_5}$; $k'_0 = \sqrt{1-k_0^2}$, where K_0 и K'_0 - complex elliptic integrals of the fist kind with moduli k_0 и k'_0	$\frac{G_l}{\gamma} = \frac{I_l}{\Delta U} = \frac{I_7 - I_5 + \frac{I_1}{I_2}(I_6 - I_8)}{\frac{I_1 I_4}{I_2} - I_3}$ The integrals $I_1 \div I_8$ are hyperelliptic and must be found numerically in according to [3].
2	$\frac{a_1}{a_6} = \mathrm{sn}\left(K_0 \frac{d}{l}, k_0\right)$; $\frac{a_2}{a_6} = \mathrm{sn}\left(K_0 \frac{d+a}{l}, k_0\right)$; $\frac{a_3}{a_6} = \mathrm{sn}\left(K_0 \frac{d+a+n}{l}, k_0\right)$; $\frac{a_4}{a_6} = \mathrm{sn}\left(K_0 \frac{d+2a+n}{l}, k_0\right)$ $\frac{a_5}{a_6} = \mathrm{sn}\left(K_0 \frac{d+2a+2n}{l}, k_0\right)$; $\frac{a_7}{a_6} = \frac{1}{k_0}$	$\frac{G_l}{\gamma} = \frac{D_1 - D_2 + D_3}{I_7 - \left(c_{01}^2 + c_{02}^2\right) I_8 + c_{01}^2 c_{02}^2 I_9}$, where D_1 - $I_{10} - I_{13} + I_{16}$ D_2 - $\left(c_{01}^2 + c_{02}^2\right)\left(I_{11} - I_{14} + I_{17}\right)$ D_3 - $c_{01}^2 c_{02}^2 \left(I_{12} - I_{15} + I_{18}\right)$ The integrals $I_1 \div I_{18}$ are hyperelliptic and must be found numerically in according to [3].

3	$\zeta = a_3 \mathrm{sn}\left(K_{01}\frac{Z}{2l}, k_{01}\right)$, $k_{01} = a_3/a_4$; $\frac{a_1}{a_3} = \mathrm{sn}\left(K_{01}\frac{b}{2l}, k_{01}\right)$; $\frac{a_2}{a_3} = \mathrm{sn}\left(K_{01}\frac{2l-a}{2l}, k_{01}\right)$. $k_{01} = 4\sqrt{q_1}\left[\frac{1+q_1^{1\cdot 2}+q_1^{2\cdot 3}+q_1^{3\cdot 4}+\cdots+q_1^{n(n+1)}}{1+2q_1+2q_1^4+2q_1^9+\cdots+2q_1^{n^2}}\right]^2$, where $q_1 = e^{-\frac{\pi h}{2l}}$.	$\frac{G_3}{\gamma} = 4\frac{K'(k)}{K(k)}$, $k = \sqrt{\frac{1-a_1^2/a_2^2}{1-a_1^2/a_3^2}}$, where $K'(k)$ и $K(k)$ - complex elliptic integrals with moduli k and $k' = \sqrt{1-k^2}$ [4].
4	$\zeta = a_1 \mathrm{sn}(K_0 \frac{z}{l}, k_0)$; $\frac{h}{l} = \frac{K_0'}{K_0}$; $k_0 = \frac{a_1}{a_2}$ $\frac{a_0}{a_1} = \mathrm{sn}\left(K_0\frac{l-a}{l}, k_0\right)$; $\frac{a_0'}{a_1} = sn\left(K_0\frac{l-a+\Delta}{l}, k_0\right)$; $\frac{a_0''}{a_1} = sn\left(K_0\frac{l-\Delta_1}{l}, k_0\right)$	$\frac{G_{12}}{G_{13}} = \frac{F(\varphi, k')}{F(\varphi_1, k')}$ $\varphi \approx \sqrt{\frac{1-a_0^2/a_0'^2}{1-a_0^2/a_1^2}}$ $\varphi_1 \approx \sqrt{\frac{1-a_0''^2/a_1^2}{1-a_0^2/a_1^2}}$ $\frac{\widetilde{G}_{12}}{\widetilde{G}_{13}} \approx \frac{\varphi}{\varphi_1} \approx \sqrt{\frac{1-a_0^2/a_0'^2}{1-a_0''^2/a_1^2}}$
5	$z = A\int_0^\zeta \sqrt{\frac{(\zeta^2-a_2^2)(\zeta^2-a_3^2)}{(\zeta^2-a_1^2)(\zeta^2-a_4^2)(\zeta^2-a_5^2)(\zeta^2-a_6^2)}}d\zeta$ $(l-a-\Delta)/l = I_1/I_7$; $b/l = I_2/I_7$; $a/l = I_3/I_7$; $h/l = I_6/I_7$; $I_2 = I_4$; $I_1 + I_3 + I_5 = I_7$; The integrals $I_1 \div I_7$ are hyperelliptic and must be found numerically in according to [5].	$G_{эl} = \gamma\frac{K(k')}{K(k)}$; $k = a_1/a_4$; $k' = \sqrt{1-k^2}$ $k = \frac{a_1}{a_4} = \frac{\mathrm{sn}\left(K_0\frac{d}{l}, k_0\right)}{\mathrm{sn}\left(K_0\frac{d+a}{l}, k_0\right)}$, for b = 0.

Continue of table 2

1	2	3

6	$\zeta = a_1 sn\left(K_0 \frac{x+jy}{l}, k_0\right)$ где $k_0 = a_1 / a_2$; $h/l = K_0' / K_0$	$\Delta U_{A_6 A_0} = \frac{F(\varphi,k)}{2K(k)} U_1$; $\varphi = \arcsin \frac{\sqrt{k^2 - \mathrm{sn}^2\left(K_0 \frac{x}{l}, k_0\right)}}{k \mathrm{cn}\left(K_0 \frac{x}{l}, k_0\right)}$; $\Delta U_{A_5 A_4} = \frac{U_1}{2} \frac{F(\varphi_1,k)}{K(k)}$; $\Delta U_{A_4 A_\infty} = \frac{U_1}{2} \frac{F(\varphi_2,k)}{K(k)}$; $\Delta U_{A_\infty A_3} = U_1\left(1 - \frac{F(\varphi_2,k)}{2K(k)}\right)$; $\Delta U_{A_3 A_2} = U_1\left(1 - \frac{F(\varphi_1,k)}{2K(k)}\right)$, $\varphi_1 = arcsin \frac{K_0' sn\left(K_0' \frac{y}{h}, k_0'\right)}{\sqrt{1 - k^2 dn^2\left(K_0' \frac{y}{h}, k_0'\right)}}$ $\varphi_2 = arcsin \frac{dn\left(K_0 \frac{x}{l}, k_0\right)}{\sqrt{1 - k^2 k_0^2 sn^2\left(K_0 \frac{x}{l}, k_0\right)}}$
7	$Z_1 = \ln Z$; $Z_2 = Z_1 - (d+l)$; $\zeta = a_1 sn\left(K_0 \frac{Z_2}{l}, k_0\right)$; $\omega = \sqrt{\zeta^2 + b^2}$; $\frac{\alpha_0}{\alpha_1} = sn\left(K_0 \frac{a}{l}, k_0'\right)$; $\frac{\alpha_1}{\alpha_2} = \frac{k_0}{dn\left(K_0 \frac{a}{l}, k_0\right)}$, where $k_0' = \sqrt{1 - k_0^2}$; $dn\left(K_0 \frac{a}{l}, k_0'\right)$ – delta amplitudinis; $\frac{a}{l} = \frac{2a_0}{\sqrt{Rr} \ln \frac{R}{r}}$ - constructive parameters of the electric heater.	$G_l = \gamma \frac{K(k)}{K(k')}$; $k = \frac{a_0}{a_1} \cdot \frac{a_1}{a_2} = k_0 \frac{sn(u,k_0')}{dn(u,k_0')}$, where $u = K_0 \frac{2a_0}{R\sqrt{\frac{r}{R}} \ln \frac{R}{r}}$ $\widetilde{G}_l \approx \gamma \frac{R-r}{2\pi r\left(1 + \frac{R-r}{\pi^2 r} \ln \frac{R-r}{\pi a_0}\right)}$
8	$k_0 = \frac{a_1}{a_5}$; $\frac{a_1}{a_2} = \frac{a_4}{a_5} = dn(K_0', \frac{h_1}{h}, k_0')$; $\frac{a_1}{a_4} = \frac{a_2}{a_5} = \frac{k_0'}{dn(K_0', \frac{h_1}{h}, k_0')}$, ; $\frac{h_1}{h} = \frac{a_0}{\pi R_0}$ $k_0 = 4l^{\frac{-\pi^2}{\ln R_0 / r}}$.	$G_l = 4\gamma \frac{K(k)}{K(k')}$ $k = \sqrt{\frac{(1 - a_4/a_5)(1 - a_1/a_2)}{(1 - a_2/a_5)(a_4/a_2 - a_1/a_2)}} = \sqrt{k_0} \frac{1 - dnu}{dnu - k_0}$ where $u = K_0 \frac{2a_0}{R_0 \ln \frac{R_0}{r}}$ $G_l \approx \frac{4(R_0 - r)}{\pi r\left(1 - \frac{4(R_0 - r)}{\pi^2 r} \ln sh \frac{\pi a_0 r}{2R_0(R_0 - r)}\right)}$

Continue of table 2

1	2	3
9	$\zeta = a_1 sn(K_0 \frac{Z_3}{l}, k_0)$, где $k_0 = a_1/a_4$; $\frac{h_3}{l} = \frac{K_0'}{K_0}$; $\xi = \frac{a_1}{dn(K_0' \frac{y}{h_3}, k_0')}$; $\xi = a_2 = \frac{a_1}{dn(K_0^I \frac{h_4}{h_3}, k_0^I)}$; $\xi = a_3 = \frac{a_1}{dn(K_0' \frac{h_3 - h_5}{h_3}, k_0')}$; $k_0 = 4e^{\frac{-\pi h_3}{2l}} = 4e^{\frac{-\pi^2}{2\ln R_0/r}}$ $k = \sqrt{\frac{\left(1-\frac{a_3}{a_4}\right)\left(\frac{a_2}{a_4}-\frac{a_1}{a_4}\right)}{\left(1-\frac{a_2}{a_4}\right)\left(\frac{a_3}{a_4}-\frac{a_1}{a_4}\right)}} = \frac{1-dnu}{dn\,u - k_0}\sqrt{k_0}$; $u = K_0' \frac{a_0}{R_0 h_3} = K_0 \frac{2a_0}{R_0 \ln \frac{R_0}{r}} \approx \pi \frac{a_0}{R_0 \ln \frac{R_0}{r}}$	$G_l = 8\gamma \frac{K(k)}{K(k')}$ $\widetilde{G}_l \approx \frac{8\ln(R_0/r)}{\pi}\left[1-\frac{8\ln R_0/r)}{\pi^2}\ln sh\frac{u}{2}\right]^{-1}$ $\frac{8(R_0-r)}{\pi r}\left[1-\frac{8(R_0-r)}{\pi^2 r}\ln sh\frac{\pi a_0}{2R_0}\frac{r}{(R_0-r)}\right]^{-1}$

III. CONCLUSION

Thus, determination of the stationary electric field parameters is implemented by the desired structural dimensions by means of the imaging parameters and exact or approximate expression choice.

The expressions presented illustrate that notwithstanding the methodical identity they are individual for each system type.

For the purpose of designing LTCEH with the desired parameters the following expressions have been derived: the expressions for calculation of the partial conductivity between the electrodes, the expressions for calculation of the conductivity subject to the electrode cross-section dimension, their bias with respect to each other and LTCEH surface potential distribution.

The derived formulas illustrate that electric heater conductivity dependency on the electrode width possesses the logarithmic character, the increasing ratio of the conductive layer inner radius to that of the outer one leads to the reduction of the non-dimensions conductivities due to the constant values of the electrode width ratio to the difference of the radii mentioned above, and the increase of the latter ratio due to the constant ratio of the conductive layer radii contributes to increasing of the electric heater conductivity.

The calculation model of the LTCEH complex systems in combination with the derived complex of the exact and approximate expressions of the stationary electric field parameters present the theoretical ground of the engineering design procedure of the electric heaters structural parameters.

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