SCATTERING OF CHARGE CARRIERS AND LIGHT ABSORPTION IN LOW-DIMENSIONAL SYSTEMS

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In this work, we investigate the intraband optical transitions in semiconductor superlattices under the influence of a magnetic field, the absorption of electromagnetic radiation by free charge carriers via phonon scattering in quantum-dot-based superlattices, and the theory of free-carrier absorption in cylindrical quantum wires for the case when the radiation field is polarized along the wire axis and carriers scatter by acoustic phonons. The absorption of electromagnetic radiation by electrons in parabolic confinement potential quantum wires, including scattering by phonons, has also been studied.

Keywords: low-dimensional systems, quantum wire, confinement, absorption of light by free carriers, quantum constriction.

Among the achievements of modern science and technology, artificially created semiconductor structures occupy a special place. With the emergence of quantum effects, these semiconductor structures acquire unique physical properties. One of the most intensively studied among them is the size quantization effect, which arises when the characteristic dimensions of a system become comparable to the de Broglie wavelength of particles, thereby imposing restrictions on their motion.

There exist various types of low-dimensional systems: quantum wells, superlattices, quantum wires, quantum dots, quantum rings, quantum disks, etc. Advances in modern technology—such as molecular beam epitaxy under computer control—make it possible to fabricate semiconductor systems with any desired confinement, including those with parabolic potentials. In systems with a parabolic potential, the effect of dimensional quantization occurs even in quantum wells with sufficiently large widths (greater than 1000 Å), and at temperatures around T ~100 K, the quantization of the energy spectrum significantly affects the physical properties of the system. The quadratic dependence of the potential allows many characteristics of the system to be expressed analytically, which makes the analysis of the corresponding physical phenomena convenient.

When electric and magnetic fields influence optical radiation in semiconductor structures, new effects arise that lead to the development of more advanced optoelectronic devices. The study of how magnetic fields and the geometrical shape of nanostructures affect their optical and kinetic properties is one of the topical directions of nanoelectronics.

One of the main methods for studying the spectral characteristics of the electron gas in various low-dimensional structures is the analysis of intraband electronic transitions under the influence of electromagnetic radiation. The study of resonant absorption is more advantageous than kinetic measurements since it does not require electrical contacts that may affect the physical properties of the system. In recent decades, spin phenomena in semiconductors and nanostructures have been extensively investigated: the features of spin–orbit interaction, the dynamics of electron and hole spins, and the processes of photon angular momentum transfer in electronic systems. These effects have been found to result from the joint study of charge and spin degrees of freedom — a field known as spintronics.

Moreover, in low-dimensional structures, the relaxation time is longer compared to bulk materials. The main aspect of studying the intraband absorption of high-frequency electromagnetic radiation lies in the fact that, for discrete energy spectra, the absorption curve exhibits resonance peaks at certain frequencies where the radiation frequency equals the energy separation between electron levels. The resonance frequency determined in this way provides valuable information about lateral confinement and energy spectrum parameters in nanostructures.

The intraband optical transitions in semiconductor superlattices under the influence of a magnetic field have been studied [1]. When the magnetic field is directed perpendicular to the surface of the superlattice, Landau quantization occurs, and the energy spectrum becomes discrete. At the same time, the minibands formed due to the motion of electrons and holes along the z-direction remain continuous.

Under the influence of a longitudinal magnetic field $H = H_z$, the energy spectrum and the wave function of an electron in a superlattice with potential wells $U(z)$and period d_{SL}along the z-axis can be expressed as follows:

$$E_n(k_z) = (n + 1/2)\hbar\omega_c + \frac{\Delta}{2}(1 - \cos k_z d_{SL}) \quad (1)$$

$$\Psi_{nk_xk_z} = \frac{1}{L_y}\exp(ik_x x)\Phi_{nk_x}(y - y_0)\xi(k_z) \quad (2)$$

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Here, $\Phi_{nk_x}(y-y_0)$ is the harmonic oscillator function, L_x, L_y, L_z are the dimensions of the superlattice sample, Δ is the miniband width, ω_c is the cyclotron frequency, and $\xi_k(z)$ is the Bloch function along the z-direction.

The energy spectrum given by (1) differs from that of both quasi-two-dimensional systems, where the magnetic field is perpendicular to the surface and leads to complete quantization, and quasi-one-dimensional systems, where quantization occurs only along one direction. When the polarization vector of the electromagnetic field is directed within the superlattice plane, the square of the matrix element for electron–photon interaction can be written as:

$$\left|\left\langle nk_x k_z \left| H_R \right| n'k'_x k'_z \right\rangle\right|^2 = \left(\frac{2\pi\hbar n_0}{\in(\omega)\omega}\right)\left(e\Delta d_{SL}\sin\left(k_z d_{SL}\right)/2\hbar\right)^2 \delta_{nn'}\delta_{k_x k'_x}\delta_{k_z k'_z} \tag{3}$$

From expression (3), it follows that direct intraband light absorption does not occur. Intraband absorption takes place through the participation of a third particle, such as phonons, in addition to electrons and photons. Here, $\vec{\varepsilon}$ is the polarization vector of the radiation field, $\in$ is the dielectric constant, while ω and c denote the frequency and velocity of light, respectively. When calculating the matrix elements of the operator H_R, the high-frequency field is considered homogeneous.

According to quantum-mechanical transition probabilities, charge carriers can either absorb photons while being scattered by phonons, or be scattered by phonons before absorbing a photon. We consider both polar and nonpolar phonon scattering processes. In this case, the absorption coefficient for light by free charge carriers can be expressed as:

$$\alpha = \frac{\in^{1/2}}{n_0 c}\sum_i W_i f_i \tag{4}$$

where n_0 is the photon number density in the radiation field, f_i is the distribution function of free carriers, and W_i is the transition probability defined as:

$$W_i = \frac{2\pi}{\hbar}\sum_{fq}\left[\left|\left\langle f\left|M_+\right|i\right\rangle\right|^2 \delta\left(E_f - E_i - \hbar\omega - \hbar\omega_q\right) + \left|\left\langle f\left|M_-\right|i\right\rangle\right|^2 \delta\left(E_f - E_i - \hbar\omega + \hbar\omega_q\right)\right] \tag{5}$$

here, E_i and E_f are the initial and final electron energies, $\hbar\omega_q$ is the phonon energy, and $\left\langle f\left|M_+\right|i\right\rangle$ the indices i, α, f correspond to the initial, intermediate, and final electronic states. V_S is the electron–phonon interaction operator. When charge carriers scatter by either polar or nonpolar phonons, the absorption coefficient includes $\frac{1}{\sqrt{1-\frac{4}{\Delta^2}\Theta_{1\pm}^2}}$ an oscillatory term dependent on both the magnetic field intensity $\Theta_{1\pm}(k_z d) = (n'-n)\hbar\omega_c \pm \hbar\omega_0 + \hbar\Omega + \frac{\Delta}{2}\cos k_z d$ and the incident light frequency, leading to the resonance condition: $N\omega_c = \Omega + \omega_0$ Each time this condition is satisfied $1-4\Theta_{1\pm}^2/\Delta^2 = 0$, the absorption coefficient exhibits sharp resonance peaks. Taking into account that the value of $1-4\Theta_{1\pm}^2/\Delta^2$ must be real and positive, the corresponding energy interval for possible absorption values can also be determined.

The absorption of electromagnetic radiation by free charge carriers through phonon scattering in quantum-dot superlattices has been investigated [2]. It is assumed that in such superlattices, the electron gas is confined by an anisotropic parabolic potential. Within the strong-coupling approximation, the normalized eigenfunction of an electron $\Psi_{n,l,k_z}(r)$ and its energy eigenvalues in the conduction band $E_{n,l}(k_z)$ are expressed, respectively, as follows [3]:

$$\Psi_{n,l,k_z}(r) = \frac{1}{\sqrt{L_z}}\Psi_n(x)\Psi_l(y)\xi_{k_z}(z)\ , \tag{6}$$

$$E_{n,l}(k_z) = (n+\frac{1}{2})\hbar\omega_x + (l+\frac{1}{2})\hbar\omega_y + \frac{\Delta}{2}(1-\cos k_z d) = \varepsilon_{n,l} + \varepsilon(k_z), \tag{7}$$

where m^* is the effective electron mass, ω_x and ω_y are the confinement frequencies in the *x*- and *y*-directions, respectively, $n = 0,1,2,\ldots$ and $l = 0,1,2,\ldots$ are the subband indices of the electron, and k_z is the wave vector component along the *z*-axis. Here, $\Psi_n(x)$ and $\Psi_l(y)$ are the eigenfunctions of a simple harmonic oscillator.

By using the wave function (6), the matrix element for electron–photon interaction can be obtained from Eq. (3) by replacing $\delta_{k_x k_x}$ the function symbols accordingly $\delta_{ll'}$, where V is the crystal volume, and the radiation field is polarized along the z-axis. The matrix element of electron–phonon interaction is expressed as:

$$\left|\langle k_z' n' l' | V_s | k_z n l \rangle\right| = C_j' \; J_{nn'}(x) J_{ll'}(y) I(q_z) \tag{8}$$

where V_S is the electron–phonon interaction operator and C_j is the function characterizing the interaction between electrons and phonons, given by [4]:

$$J_{nn'}(q_x) = \int_{-\infty}^{\infty} e^{iq_x x} dx \Psi_n(x) \Psi_{n'}(x) \qquad J_{ll'}(q_y) = \int_{-\infty}^{\infty} e^{iq_y y} dy \Psi_l(y) \Psi_{l'}(y)$$

$$I(q_z) = \int_0^d \xi_{k_z}(z) \xi_{k_k'}(z) e^{iq_z z} dz \qquad C_j'^2 = C_j^{\,2} F_j(q)$$

The exact expression of C_j and $F_j(q)$ depends on the type of phonons involved in the scattering process. For polar and nonpolar optical phonon scattering, the absorption coefficient takes the following forms:

$$\alpha_{pol} = \frac{4\pi e^4 \Delta d \omega_0 L_z}{c\Omega^3 \in^{1/2} \hbar^3} \left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right) \sum_{n'l'} \sum_{nl} \sum_{\pm} \int_{-\pi/d}^{\pi/d} dk_z f_{nlk_z} \left(N_0 + \frac{1}{2} \pm \frac{1}{2}\right) \frac{\sqrt{1 - \frac{4}{\Delta^2}\theta_\pm^2} - \sin k_z d}{\sqrt{1 - \frac{4}{\Delta^2}\theta_\pm^2}} \times$$

$$\times \int_0^\infty \int_0^\infty \frac{\left|I_{nn'}(q_x)\right|^2 \left|I_{ll'}(q_y)\right|^2}{\left(q_x^{\,2} + q_y^{\,2}\right) + a_\pm^{\,2}} dq_x dq_y \;, \quad a_\pm^2 = \frac{1}{d^2}\left(k_z d - \arccos\left(\frac{2}{\Delta}\Theta_\pm(k_z d)\right)\right)^2 \tag{9}$$

$$\alpha_{n.pol} = \frac{D^2 e^2 \Delta d}{\pi c \rho \omega_0 \Omega^3 L_x L_y \in^{1/2}} \sum_{n'l'} \sum_{nl} \sum_{\pm} \int_{-\pi/d}^{\pi/d} dk_z f_{nek_z} \left(N_0 + \frac{1}{2} \pm \frac{1}{2}\right) \frac{\sqrt{1 - \frac{4}{\Delta^2}\Theta_\pm^2} - \sin k_z d}{\sqrt{1 - \frac{4}{\Delta^2}\Theta_\pm^2}} \times$$

$$\times \frac{1}{n' - n} \cdot \frac{1}{l' - l} \;, \qquad \Theta_\pm(k_z d) = (n' - n)\hbar\omega_x + (l' - l)\hbar\omega_y \pm \hbar\omega_0 + \hbar\Omega + \frac{\Delta}{2}\cos k_z d \tag{10}$$

Considering that the absorption coefficient $1 - 4\Theta_i^2 / \Delta^2$ must be real and positive, the possible energy interval for absorption can be determined accordingly.

From Eqs. (9) and (10), it follows that distinct peaks appear in the absorption spectrum at certain incident photon frequencies. For electron–phonon scattering, the resonant condition for the absorption coefficient is obtained as:

$$N\hbar\omega_x + P\hbar\omega_y \pm \hbar\omega_0 = \hbar\Omega \tag{11}$$

where $N = n' - n = 1,2,3, \ldots$ and $P = l' - l = 1,2,3, \ldots$.

Equation (11) shows that electrons can scatter from subbands characterized by indices *n(l)* to subbands $n'(l')$. Through the absorption or emission of LO phonons of energy $\hbar\omega_0$, simultaneously absorbing or emitting photons of energy $\hbar\Omega$.

Expression (11) serves as the basis for determining the spectral line shape of the absorption process, allowing the analysis of resonant effects in semiconductors.

It should be noted that the summation over subbands in Eq. (9) involves three types of transitions: (i) $n' \neq n, l' = l$, (ii) $n' = n, l' \neq l$ və (iii) $n' \neq n, l \neq l'$

1. Transitions only between quantized subbands along the *x*-direction.
2. Transitions only between quantized subbands along the *y*-direction.
3. Combined transitions involving both *x*- and *y*-directions.

From Eq. (10), it follows that for nonpolar phonon scattering, only the third type of transition (simultaneous change in both *x* and *y* subband indices) is possible.

The theory of light absorption by free charge carriers in cylindrical quantum wires has been developed for the case where the radiation field is polarized along the wire axis and carriers are scattered by acoustic phonons [5]. Electrons move freely along the axis of a cylindrical quantum wire of radius $R$and length *L*, while their motion across the wire cross-section is confined. Within the effective mass approximation, the electron wave function in the quantum wire can be expressed as:

$$\Psi_{nlK}(r)=\frac{\exp(iKz)\exp(il\vartheta)}{(\pi R^2L)^{1/2}}\cdot\frac{J_{nl}(k_{nl}\rho)}{J_{l+1}(k_{nl}R)}\quad, \tag{12}$$

where $l=0,1,2,\ldots$, $n=1,2,3,\ldots$, J_l is the Bessel function of the first kind, r - is (ρ,ϑ,z), ϑ is the azimuthal angle, and K is the wave vector component along the wire axis.

Using the wave function, the matrix elements of the electron–photon interaction Hamiltonian can be written as:

$$\langle n'l'K'|H_R|nlK\rangle=-\frac{e\hbar}{m^*}\left(\frac{2\pi\hbar n_0}{V\Omega\in}\right)^{1/2}(\varepsilon K)\delta_{KK'}\delta_{ll'}G_{nl,n'l'}(R) \tag{13}$$

The electron–phonon interaction matrix element depends on the scattering mechanism and, for acoustic phonons, can be expressed as:

$$\langle f|V_s|\alpha\rangle=C_{nl}(q_z)I_{n'l',n''l''}(q_{nl})\delta_{K',K''+q_z} \tag{14}$$

In our analysis, we consider the interaction between electrons and transversely confined acoustic phonons through the deformation potential.

In this case, the function $C_{n,l}(q_z)$ appearing in Eq. (13) is given by [6]:

$$\left|C^{LA}_{nl,q_z}\right|^2=\frac{D^2\hbar\sqrt{k_{nl}^2R^2+R^2q_z^2}}{2\pi RL\mu c_sJ^2_{n+1}(k_{nl}R)} \tag{15}$$

Using Eqs. (4), (5), and (12)–(14), it was found that the absorption coefficient of light by free charge carriers, when they scatter from acoustic phonons, decreases as the radius of the cylindrical quantum wire increases. Physically, this is due to the reduction of quantum confinement effects with increasing wire radius, leading to smoother energy spectra and weaker resonance conditions for phonon-assisted transitions.

The simultaneous absorption of electromagnetic radiation and phonon scattering by electrons in quantum wires with a parabolic confinement potential has been studied [7].

It is assumed that the electron gas in the quantum wire is confined by an anisotropic parabolic potential with frequencies ω_x and ω_z along the *x*- and *z*-directions, respectively. For asymmetric parabolic potential quantum wires, the eigenfunction and energy eigenvalue of an electron in the conduction band can be written as follows:

$$\Psi_{n,m,p_y}(r)=\frac{1}{\sqrt{2\pi\hbar}}\Psi_n(x)\Psi_m(z)\exp(ip_yy)\quad, \tag{16}$$

$$E_{n,m}(p_y)=(n+\frac{1}{2})\hbar\omega_x+(m+\frac{1}{2})\hbar\omega_z+\frac{p_y^2}{2m}, \tag{17}$$

where *n (= 0,1,2,...)* and *m (= 0,1,2,...)* are the subband indices of the quantized levels, and p_y is the momentum component along the wire axis (the *y*-direction). Here, $\Psi_n(x)$ and $\Psi_m(z)$ are the eigenfunctions of a simple harmonic oscillator. Summation is performed over all initial states “i” of the system. The transition probability W_i between states mnP_y and $m'n'P'_y$ is determined from Eq. (5). Using the wave function (16), the matrix element of the electron–photon interaction can be expressed as:

$$\langle n'l'K'|H_R|nlK\rangle=-\frac{e}{m^*}\left(\frac{2\pi\hbar n_0}{V\Omega\in}\right)^{1/2}(\varepsilon P_y)\delta_{P_yP_y}\delta_{mm'}\delta_{nn'}\quad, \tag{18}$$

where the radiation field is polarized along the y-axis.

The distribution function of electrons $f_0(E_{nmP_y})$ satisfies the following normalization condition:

$$\frac{L_Y}{2\pi\hbar}\sum_{m=-\infty}^{\infty}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}f_0(E_{mnP_y})dP_y=N \tag{19}$$

where N is the electron concentration per unit volume, and $L_y$is the length of the quantum wire along the y-axis. The electron–phonon interaction matrix element is given by:

$$\left\langle n''m''P_y'' \right| V_s \left| nmP_y \right\rangle = D_q \sqrt{N_q + \frac{1}{2} \pm \frac{1}{2}} \left\langle n''m''P_y'' \right| e^{\pm iqr} \left| nmP_y \right\rangle \quad (20)$$

For deformation (DO-phonons) and polar optical (PO-phonons), the coupling constants are expressed as [8]:

$$\left|D_q\right|^2 = \frac{2\pi\hbar^2 \alpha_l \omega_0}{m*} \begin{cases} \sqrt{2m^*\hbar\omega_0} \Big/ q^2 - PO \\ 4\hbar^2 \Big/ \sqrt{2m^*\hbar\omega_0} - DO \end{cases} \quad (21)$$

Using the displacement operator formalism $exp(\pm aP_x x/\hbar)\,\Phi(x) = \Phi(x \pm a)$, the electron–phonon interaction matrix elements are calculated.

The absorption coefficient α is evaluated using Eqs. (19), (20), which include both the electron–photon and electron–phonon interaction terms. It has been established that in asymmetric parabolic potential quantum wires, the position of the peaks in the intraband optical absorption coefficient depends on the characteristic confinement frequencies

$N\omega_x + P\omega_y \pm \omega_0 = \Omega$.

Thus, varying the parameters of the confining potential allows tunable optical absorption, making such structures promising for infrared and terahertz optoelectronic applications.

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