

UNSTABLE THERMOMAGNETIC WAVES IN ANISOTROPIC MEDIA OF  
ELECTRONIC TYPE OF CHARGE CARRIERS

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In anisotropic media with an electronic type of charge carriers, an increasing thermomagnetic wave is excited under certain conditions. Analytical formulas are found for the frequency and for the increment of this wave. The analytical formulas for the tensor of electrical conductivity of the medium are indicated in the form of a table. A formula is found for the ratios of the temperature gradient.

**Keywords:** tensor, electrical conductivity, frequency, increment, hydrodynamic motion, temperature gradient, speed of sound waves, thermomagnetic wave.

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I. INTRODUCTION

In works [1-6] it is proved that in the presence of a temperature gradient  $\nabla T$  in a nonequilibrium plasma, an alternating magnetic field arises. In the presence of a temperature gradient, charge carriers oscillate in the plasma. Plasma with oscillatory motion of charge carriers differs significantly from ordinary plasma. Thermomagnetic waves arise in such a plasma without an external magnetic field. The frequency of this wave is directly proportional to the temperature gradient. If such a plasma is in an external magnetic field  $H$ , then the wave vector of the thermomagnetic wave is directed perpendicular to the magnetic field and is in the plane  $(H, \nabla T)$ . The velocity of the hydrodynamic motions of charge carriers  $\vec{V}$  and the magnetic field  $\vec{H}$  are directed perpendicularly  $\vec{\nabla} T$ . In such a plasma, an ordinary Alphen wave is split into two hydrothermomagnetic waves and the spectrum of magnetosonic waves will change. The speed of propagation of thermomagnetic waves and the speed of sound waves are approximately the same.

In a plasma with  $\nabla T = const$  or over a distance  $L = \frac{T}{\nabla T}$ , the change in the temperature gradient is very small. Then a weak magnetic field  $\Omega\tau \ll 1$ ,  $\Omega = \frac{eH}{mc}$  is excited in the plasma Larmor frequency,

$$\vec{j} = \sigma \vec{E}^* + \sigma' [\vec{E}^*, \vec{H}] - \alpha \vec{\nabla} T - \alpha' [\vec{\nabla} T \vec{H}] \tag{1}$$

Here  $E^* = \vec{E} + \frac{[\vec{V}\vec{H}]}{c} + \frac{T}{e n_0} \vec{\nabla} n$ ,  $e > 0$

$\frac{[\vec{V}\vec{H}]}{c}$  is the electric field created during hydrodynamic motions,  $T$  is the temperature of the medium,  $\vec{\nabla} n$  is the gradient of the electron concentration,  $n_0$  is the equilibrium value of the electron concentration,  $E$  is the external electric field. Substituting current density and in (1) we obtain the following vector equations for

$\tau$  is the time of collision of electrons. In ordinary solids, the flow of charge carriers with the speed of hydrodynamic movements  $\vec{V}$  is a plasma. A solid-state plasma under the influence of a temperature gradient is in a nonequilibrium state and the distribution of charge carriers is inhomogeneous. In a medium with such inhomogeneity, an alternating magnetic field arises and a thermomagnetic wave is excited. In isotropic media, the theory of thermomagnetic waves was constructed in [7-8]. In anisotropic media, the theoretical study of thermomagnetic waves was presented in [9] only in a limited limiting case for the conductivity of the medium.

In this theoretical work, we present the results of a theoretical study of thermomagnetic waves in anisotropic conducting media, taking into account all possible values of the electrical conductivity tensor  $\sigma_{ik}$  of the medium in the presence of a constant temperature gradient  $\nabla T = const$ .

II. Basic equations of the problem

In the presence of an electric field  $E$ , a temperature gradient  $\nabla T$  and the speed of hydrodynamic movements, the current density  $\vec{j}(\vec{r}, t)$  has the form:

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determining the electric field  $\vec{j} = \frac{c}{4\pi} \text{rot}\vec{H}$  and  $E^*$  to (1) we obtain the following vector equations for determining the electric field

$$\vec{E} = \vec{a} + [\vec{b}\vec{E}] \quad (2)$$

From (2) we get

$$\vec{E} = \vec{a} + [\vec{b}\vec{E}] + [\vec{b}[\vec{b}\vec{E}]] \quad (3)$$

$$\vec{E} = \zeta\vec{j} + \zeta'[\vec{j}\vec{H}] + \zeta''(\vec{j}\vec{H})\vec{H} + A'[\vec{\nabla}T\vec{H}] + A''(\vec{\nabla}T\vec{H}) + A\vec{\nabla}T \quad (4)$$

$A' = \frac{\sigma'\sigma - \alpha\sigma'}{\sigma}$ ,  $A$  is the thermoelectric power differential,  $A'$  is the Nernst – Ettingshausen coefficient. In anisotropic media, electric field (4) is the tensor

$$E_i = \zeta_{im}j_m + \zeta'_{im}[jH]_m + \zeta''_{im}(jH)H_m + A_{im}\frac{\partial T}{\partial x_m} + A'_{im}[\nabla TH]_m + A''_{im}(\nabla TH)H_m \quad (5)$$

In the absence of an external magnetic field,  $\vec{H}_0 = 0$  we obtain the following system of equations for our problem

$$\begin{cases} E'_i = \zeta_{im}j'_m + A'_{im}[\vec{\nabla}TH]_m \\ \text{rot}\vec{E}' = -\frac{1}{c}\frac{\partial\vec{H}'}{\partial t} \\ \text{rot}\vec{H}' = \frac{4\pi}{c}j' + \frac{1}{c}\frac{\partial E'}{\partial t} \end{cases} \quad (6)$$

Assuming that all variable quantities change in the form of a monochromatic wave, i.e.

$$(E', H') \sim e^{i(\vec{k}\vec{r} - \omega t)} \quad (7)$$

$\vec{k}$  is wave vector,  $\omega$  is wave frequencies from (6) we can easily obtain:

$$E'_i = \left( A\zeta_{ie}k_e k_m + B\zeta_{im} + \frac{cA'_{ie}}{\omega}k_e \frac{\partial T}{\partial x_m} \right) E'_m \quad (8)$$

Here

$$A = i\frac{c^2}{4\pi\omega}, \quad B = i\frac{\omega^2 - c^2k^2}{4\pi\omega} \quad (9)$$

Taking into account that  $E'_i = \delta_{im}E'_m$  from (8) we write  $\delta_{im} = \Phi_{im}$ ,  $\delta_{im} = \begin{cases} 1, i = m \\ 0, i \neq m \end{cases}$

$$\Phi_{im} = A\zeta_{ie}k_e k_m + B\zeta_{im} + \frac{cA'_{ie}}{\omega}k_e \frac{\partial T}{\partial x_m} \quad (10)$$

To obtain the dispersion equation, we must disclose the following determinant

$$|\Phi_{im} - \delta_{im}| = 0 \quad (11)$$

or

$$\begin{aligned} & (\Phi_{11} - 1)(\Phi_{22} - 1)(\Phi_{33} - 1) + \Phi_{21}\Phi_{13}\Phi_{32} + \Phi_{12}\Phi_{23}\Phi_{31} - \Phi_{13}\Phi_{31}(\Phi_{22} - 1) - \\ & - \Phi_{32}\Phi_{23}(\Phi_{11} - 1) - \Phi_{21}\Phi_{12}(\Phi_{33} - 1) = 0 \end{aligned} \quad (12)$$

$$\begin{aligned}
 \Phi_{11} &= \frac{i\omega}{4\pi} \zeta_{11}, \Phi_{12} = \frac{i\omega\zeta_{12}}{4\pi} - \frac{ic^2k^2}{4\pi\omega} \zeta_{12} + \frac{\omega_{11} - \omega_{12}}{\omega}, \omega_{11} = cA'_{11}k\nabla_2T, \omega_{12} = cA'_{12}k\nabla_1T \\
 \Phi_{13} &= \frac{i\omega}{4\pi} \zeta_{13} - i\frac{c^2k^2}{4\pi\omega} \zeta_{13} - \frac{\omega_{13}}{\omega}, \omega_{11} = cA'_{13}k\nabla_1T, \Phi_{21} = \frac{i\omega}{4\pi} \zeta_{21}, \\
 \Phi_{22} &= \frac{i\omega}{4\pi} \zeta_{22} - i\frac{c^2k^2}{4\pi\omega} \zeta_{22} + \frac{\omega_{21} - \omega_{22}}{\omega}, \omega_{21} = cA'_{21}k\nabla_2T, \omega_{22} = cA'_{22}k\nabla_2T \\
 \Phi_{23} &= \frac{i\omega}{4\pi} \zeta_{23} - i\frac{c^2k^2}{4\pi\omega} \zeta_{23} - \frac{\omega_{23}}{\omega}, \omega_{23} = cA'_{23}k\nabla_1T, \\
 \Phi_{31} &= \frac{i\omega}{4\pi} \zeta_{31}, \Phi_{32} = \frac{i\omega}{4\pi} \zeta_{32} - i\frac{c^2k^2}{4\pi\omega} \zeta_{32} + \frac{\omega_{31} - \omega_{32}}{\omega}, \\
 \omega_{31} &= cA'_{31}k\nabla_2T, \omega_{32} = cA'_{32}k\nabla_1T \\
 \Phi_{33} &= \frac{i\omega}{4\pi} \zeta_{33} - i\frac{c^2k^2}{4\pi\omega} \zeta_{33} - \frac{\omega_{33}}{\omega}, \omega_{33} = cA'_{33}k\nabla_1T
 \end{aligned} \tag{13}$$

When obtaining expressions (13), we choose the following coordinate system

$$k = k_1, k_2 = k_3 = 0, \frac{\partial T}{\partial x_1} = \nabla_1T \neq 0, \frac{\partial T}{\partial x_2} = \nabla_2T \neq 0, \frac{\partial T}{\partial x_3} = 0.$$

To obtain the dispersion equation from (12) taking into account (13), we calculate each term that in (12).

$$\begin{aligned}
 1) \quad & (\Phi_{11} - 1)(\Phi_{22} - 1)(\Phi_{33} - 1) = \frac{\sqrt{2}\Omega_2}{4\pi} \left( \frac{\zeta_{11}}{4\pi} + \frac{i}{\omega} \right), \Omega_2 = \omega_{21} - \omega_{22}, \zeta_{22} = \zeta_{33} = \frac{1}{ck}, ck = \sqrt{2}\Omega_2 \\
 2) \quad & \Phi_{21}\Phi_{13}\Phi_{32} = -i\frac{\Omega_3\omega_{13}\zeta_{21}}{\pi\omega}, \Omega_3 = \omega_{31} - \omega_{32}, \zeta_{32} = \frac{1}{ck}, \zeta_{13} = \frac{\omega_{13}}{ck\Omega_3} \\
 3) \quad & \Phi_{12}\Phi_{31}\Phi_{23} = -\frac{i\omega\zeta_{31}}{8\pi} \left( \frac{c^2k^2\zeta_{12}\zeta_{23}}{16\pi^2} + \frac{\Omega_4\omega_{23}}{c^2k^2} \right), \zeta_{12}\omega_{23} = \Omega_4\zeta_{23}, \Omega_4 = \omega_{11} - \omega_{12} \\
 4) \quad & \Phi_{13}\Phi_{31}(\Phi_{22} - 1) = -\frac{ck\zeta_{31}}{8\pi^2}; \zeta_{13} = \frac{2}{ck}, \Omega_2\zeta_{13} = \zeta_{22}\omega_{13}, \Omega_2 = \omega_{21} - \omega_{22} \\
 5) \quad & \Phi_{32}\Phi_{23}(\Phi_{11} - 1) = -\frac{c^2k^2}{\omega^2} \left( \frac{i\omega}{4\pi} \zeta_{11} - 1 \right) \left( \frac{c^2k^2\zeta_{32}\zeta_{23}}{16\pi^2} + \frac{\Omega_3\omega_{23}}{c^2k^2} \right), \Omega_3 = \omega_{31} - \omega_{32}, \zeta_{23}\Omega_3 = \zeta_{32}\omega_{23} \\
 6) \quad & \Phi_{21}\Phi_{12}(\Phi_{33} - 1) = \frac{ick}{4\pi} \zeta_{21} \left( -\frac{c^2k^2\zeta_{12}}{16\pi^2\omega} + i\frac{ck\zeta_{12}}{4\pi} - \frac{\Omega_4\omega_{33}}{ck\omega} - \frac{\Omega_4}{\omega} \right), \zeta_{12} = \frac{\Omega_4}{ck\omega_{33}}
 \end{aligned}$$

Substituting all terms (1-6) in (12), we obtain the following dispersion equations for determining the vibration frequency inside the medium at

$$\frac{\nabla_2T}{\nabla_1T} = \frac{A'_{32}}{A'_{31}}, \zeta_{21} = \frac{\sqrt{2}\Omega_2}{4\Omega_3\omega_{13}}, \zeta_{11} = \frac{ck\Omega_4}{4\Omega_3\omega_{13}\omega_{33}}, ck = \sqrt{2}\omega_{33} \tag{14}$$

$$\omega^2 + \left( i\frac{4\omega_{33}^3}{\Omega_4\omega_{23}} - 4\omega_{33} \right) \omega - \frac{6\omega_{33}^4}{\Omega_4\omega_{23}} = 0 \tag{15}$$

Let us denote  $\varphi = \frac{\omega_{33}^2}{\Omega_4\omega_{23}}$  and then from (15) we get:

$$\omega_{1,2} = 2\omega_{33} - 2i\omega_{33}\varphi \pm 2\omega_{33} \left( 1 - 2i\varphi - \varphi^2 + \frac{3}{4}\varphi \right)^{1/2} \tag{16}$$

Let us write (16) in the following form

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$$\begin{aligned}\omega_{1,2} &= 2\omega_{33} - i2\omega_{33}\varphi \pm 2\omega_{33}(\alpha + i\beta)^{1/2} = 2\omega_{33} - i2\omega_{33}\varphi \pm 2\omega_{33}(x + iy) = \\ &= 2\omega_{33} - i2\omega_{33}\varphi \pm 2\omega_{33} \frac{1}{\sqrt{2}} \left[ \left( \sqrt{\alpha^2 + \beta^2} - \alpha \right)^{1/2} + i \left( \sqrt{\alpha^2 + \beta^2} + \alpha \right)^{1/2} \right]\end{aligned}\quad (17)$$

$$\alpha = 1 - \varphi^2 + \frac{3}{4}\varphi; \beta = -2\varphi$$

From (17) we write

$$\omega_1 = \omega_{10} + i\gamma_1, \omega_2 = \omega_{20} + i\gamma_2 \quad (18)$$

It is clear that a wave with frequency  $\omega_{20}$  decays, and a wave with frequency  $\omega_{10}$  can grow if  $\gamma_1 > 0$ ,

$$\sqrt{2} \left( \sqrt{\alpha^2 + \beta^2} + \alpha \right)^{1/2} > 2\varphi \quad (19)$$

Substituting the values  $\alpha$  and  $\beta$  in (19), we obtain for the growth of the wave with the frequency

$$\omega_{10} = 2\omega_{33} \left[ 1 + \frac{1}{\sqrt{2}} \left( \sqrt{\alpha^2 + \beta^2} - \alpha \right)^{1/2} \right] \quad (20)$$

the following inequalities

$$\varphi^2 - \frac{3}{4}\varphi - 1 < 0 \quad (21)$$

We are interested in the fulfillment of inequality (20) at a certain value  $\varphi$ . Therefore, we restrict ourselves to the values

$$\varphi = \pm 1 \quad (22)$$

It is easy to see that at  $\varphi = 1$  for inequality (20) becomes  $-\frac{3}{4} < 0$  and the instability condition (21) is satisfied, then

$$\varphi = \frac{\omega_{33}^2}{\omega_{11}\omega_{23}} = \frac{(\Lambda'_{33})^2 (\nabla_1 T)^2}{\Lambda'_{11} \nabla_2 T \cdot \Lambda'_{23} \nabla_1 T} = \frac{(\Lambda'_{33})^2}{\Lambda'_{11} \Lambda'_{23}} \cdot \frac{\nabla_1 T}{\nabla_2 T} = 1 \quad (23)$$

Considering the relations we used  $\frac{\nabla_1 T}{\nabla_2 T} = \frac{\Lambda'_{31}}{\Lambda'_{32}}$

we get

$$\frac{(\Lambda'_{33})^2 \Lambda'_{31}}{\Lambda'_{11} (\Lambda'_{23}) \Lambda'_{32}} = 1$$

Thus, an increasing thermomagnetic wave with frequency (20) is excited in anisotropic conducting media of the electronic type of charge carriers. The values of the tensor of electrical conductivity in different directions are indicated in the following table.

Table 1.

Conductivity values by direction

$\sigma_{11} = \sigma_{22} \frac{8\Lambda'_{33}}{\sqrt{2}\Lambda'_{11}} \cdot \Lambda'_{31} \nabla_1 T$	$\sigma_{12} = \frac{\Lambda'_{31}}{\Lambda'_{11}} \sigma_{22}$	$\sigma_{13} = \frac{ck}{2}$
$\sigma_{21} = \sigma_{22} \frac{8\Lambda'_{31} \nabla_1 T}{\sqrt{2}}$	$\sigma_{22} = ck$	$\sigma_{23} = \sigma_{22} \frac{\Lambda'_{31} \nabla_2 T}{\Lambda'_{23} \nabla_1 T}$
$\sigma_{31} = \sigma_{22} \frac{16\Lambda'_{31}}{\sqrt{2}\Lambda'_{11}} \cdot \Lambda'_{33} \nabla_1 T$	$\sigma_{32} = \sigma_{22} = ck$	$\sigma_{33} = \sigma_{22} = ck$

Table 1 shows that all components of the electrical conductivity tensor are expressed in terms of the component  $\sigma_{22}$ . This means that with measurement

$\sigma_{22}$  it is possible to determine all other components  $\sigma_{ik}$ . However, excited in the above medium wave is

purely thermomagnetic. This wave grows at  $\frac{\omega_{33}^2}{\Omega_1 \omega_{23}} = I$ , i.e. at

$$\frac{\nabla_1 T}{\nabla_2 T} = \frac{A'_{11} A'_{23}}{(A'_{33})^3} \quad (24)$$

### III. DISCUSSION OF THE RESULTS

All tensor values  $\sigma_{ik}$  are determined by coefficient values  $A'_{ik}$ . Experimental measurement of the coefficients  $A'_{ik}$  using table 1. determine the electrical conductivity  $\sigma_{ik}$ . Thus, the excited growing thermomagnetic wave with frequency (20) makes it possible to determine all components of electrical conductivity  $\sigma_{ik}$ .

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