DYNAMICS OF THE CURRENT STAGE OF EVOLUTION OF THE UNIVERSE IN THE GENERALIZED BASIC MODEL OF CHAPLYGIN GAS

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To describe the observed dynamics of the universe, we have to assume that dark energy represents the dominant component of the universe. One of the mechanisms explaining the occurrence of cosmological acceleration may be the vacuum energy (or, equivalently, the cosmological constant or the Λ-term) with $P = -\rho$. Another form of dark energy can be represented by a scalar field φ with a canonical kinetic term and a very small mass. The purpose of this article is to study the current stage of accelerated expansion of the universe, in which dark energy is a changing Chaplygin gas. This well-studied model of dark energy with a nonlinear equation of state has often been discussed in modern cosmology, it includes the energy-momentum tensor $T_{\mu\nu}$, density ρ, and pressure p, of an ideal liquid.

Keywords: scale factor, dark energy, accelerated expansion of the Universe, Chaplygin gas, Friedmann equation, polytropic equation

1. INTRODUCTION

Modern cosmology studies the large-scale structure and features of the evolution of the Universe. The last decades have been marked by a significantly new stage in cosmological research, which is caused by the improvement of observational methods and techniques, which made it possible to discover new properties and regularities inherent in the evolution of the Universe. If at the beginning of the last century the main result of cosmological observations was the discovery of an accelerated expansion of the Universe, then already today a number of observational manifestations are known that make it possible to study the dynamics of the early Universe and build theoretical models describing its further evolution.

We write out the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric for a homogeneous and isotropic space, which is determined by two quantities, a(t) and K, which are found from the Einstein equations.

$$d\tau^2 = dt^2 - a^2(t)\left[\frac{dx^2}{1-Kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right].$$

The basic equations of the general theory of relativity include the Hilbert-Einstein equations, which are presented in the following form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Einstein's equations are non-linear hyperbolic equations of the second order with respect to the components of the metric tensor $g_{\mu\nu}$, the right side of which depends on the components of the energy-momentum tensor of matter distributed in the space-time under study.

The tensor expression on the left side of the Einstein equations is usually called the Hilbert-Einstein tensor.

In a homogeneous isotropic model of the Universe, the (00)-component of the Einstein equations has the form:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{K}{a^2}.$$

This equation is called the Friedmann equation; it relates the rate of expansion of the Universe (the Hubble parameter $H = \dot{a}/a$ with the matter energy density ρ and spatial curvature.

Since we assume that ordinary matter does not interact with Chaplygin gas, we must write down separate non-explosive equations for the energy densities ρ and p in the Friedman—Lemaitre—Robertson-Walker geometry:

$$\dot{\rho} + 3H(p+\rho) = 0. \quad (1)$$

The generalized Chaplygin gas — is an ideal liquid with a polytropic equation of state:

$$p = \frac{A}{\rho^\alpha}. \quad (2)$$

2. STATE EQUATION $\rho = \frac{A}{\rho^\alpha}$

We will look for a solution to the Chaplygin gas problem for a flat universe (observational data indicate that for modern measurement accuracy $K = 0$). Substitute the barotropic equation (2) into equation (1):

$$a(\rho) = constexp\left(-\frac{1}{3}\int\frac{d\rho}{-\frac{A}{\rho^\alpha}+\rho}\right),$$

$$a(\rho) = constexp\left(-\frac{1}{3}\int\frac{d\rho^{\alpha+1}}{(\alpha+1)[\rho^{\alpha+1}-A]}\right),$$

from where follows

$$a(\rho) = \frac{1}{(\rho^{\alpha+1}-A)^{\frac{1}{3(\alpha+1)}}}. \quad (5)$$

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From equation (5) we obtain the dependence $\rho(a)$:

$$\rho = \left(A + \frac{B}{a^{3(\alpha+1)}}\right)^{\frac{1}{1+\alpha}}. \qquad (6)$$

The expression where ρ is a function of the scale factor is substituted into the energy equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\left(A + \frac{B}{a^{3(\alpha+1)}}\right)^{\frac{1}{1+\alpha}},$$

$$\frac{da}{a\left(A + \frac{B}{a^{3(\alpha+1)}}\right)^{\frac{1}{2+2\alpha}}} = \sqrt{\frac{\kappa}{3}}dt,$$

$$\frac{(a^{-3(1+\alpha)}B + C))^{-1/(2+2\alpha)}da}{a} - \frac{\sqrt{\kappa}dt}{\sqrt{3}} = 0,$$

$$\frac{(A + a^{-3(\alpha+1)}B)^{-1/(2+2\alpha)}}{a} = \frac{\sqrt{\kappa}\frac{dt(a)}{da}}{\sqrt{3}},$$

$$\frac{dt(a)}{da} = \frac{\sqrt{3}(A + a^{-3-3\alpha}B)^{-1/(2+2\alpha)}}{\sqrt{\kappa}a}.$$

We will save the general solution for the scale factor in the form of quadratures

$$t(a) = \int \frac{\sqrt{3}(A + a^{-3-3\alpha}B)^{-1/(2+2\alpha)}}{\sqrt{\kappa}a}da.$$

Now let's consider the special cases when $\alpha = 1,2,3$. Since the late stage of the evolution of the Universe is considered, when a is large, the resulting expression (6) can be decomposed into a series at $a \to \infty$:

$$\rho = \left(A + \frac{B}{a^{3(\alpha+1)}}\right)^{\frac{1}{1+\alpha}} = A^{\frac{1}{1+\alpha}}\left[1 + \frac{B}{Aa^{3(\alpha+1)}}\right]^{\frac{1}{1+\alpha}},$$

$$\left(1 + \frac{B}{Aa^{3(\alpha+1)}}\right)^{\frac{1}{1+\alpha}} = 1 + \frac{B}{(1+\alpha)Aa^{3(\alpha+1)}} - \frac{\alpha B^2}{2(1+\alpha)^2A^2a^{6(\alpha+1)}} + O\left(\left(\frac{B}{Aa^{3(\alpha+1)}}\right)^n\right)$$

$$\rho(a) = \sqrt[1+\alpha]{A} + \frac{B\sqrt[1+\alpha]{A}}{(1+\alpha)Aa^{3(\alpha+1)}} - \frac{\sqrt[1+\alpha]{A}\alpha B^2}{2(1+\alpha)^2A^2a^{6(\alpha+1)}},$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{6}\frac{a^{-6(1+\alpha)}A^{-2+\frac{1}{1+\alpha}}(-B^2\alpha + 2a^{3+3\alpha}AB(1+\alpha) + 2a^{6+6\alpha}A^2(1+\alpha)^2)}{(1+\alpha)^2}.$$

So, let's find a solution for $\alpha = 1$:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{24}\left(-\frac{B^2}{\sqrt{A^3}a^{12}} + 4\frac{B}{\sqrt{A}a^6} + 8\sqrt{A}\right),$$

$$\frac{da}{\left[8\sqrt{A}a^2 + \frac{4B}{\sqrt{A}a^4} - \frac{B^2}{\sqrt{A^3}a^{10}}\right]^{\frac{1}{2}}} = \sqrt{\frac{\kappa}{24}}dt,$$

$$t(a) = \int \frac{2\sqrt{6}a^5}{\sqrt{\kappa}\sqrt{\frac{8A^2a^{12} + 4ABa^6 - B^2}{A^{\frac{3}{2}}}}}da,$$

after integration we get

$$t(a) = \frac{ln\left(\left|4\sqrt{16A^2a^{12} + 8ABa^6 - 2B^2} + 16Aa^6 + 4B\right|\right)}{\sqrt[4]{A}\sqrt{12\kappa}}$$

Given that a is large, we will leave only the leading terms

$$t(a) = \frac{ln|32Aa^6|}{\sqrt[4]{A}\sqrt{12\kappa}}$$

Denote the constants as $C = \frac{1}{\sqrt{12\kappa}\sqrt[4]{A}}$, then decomposing the resulting expression into a series up to the first three terms, we get:

$$t(a) = C\left[32Aa^6 - 1 - \frac{(32Aa^6 - 1)^2}{2} + \frac{(32Aa^6 - 1)^3}{3}\right]$$

The resulting expression describes the evolution of the scale factor as a function of time for a flat universe in the Chaplygin Gas model for a given parameter $\alpha = 1$.

We will perform similar calculations for the second special case when $\alpha = 2$:

$$\left(\frac{\dot{a}}{a}\right)^2 = \kappa\left(-\frac{B^2}{27\sqrt[3]{A^5}a^{18}} + \frac{B}{9\sqrt[3]{A^2}a^9} + \frac{\sqrt[3]{A}}{3}\right),$$

$$\frac{da}{\left[\frac{a^2\sqrt[3]{A}}{3} + \frac{B}{9\sqrt[3]{A^2}a^7} - \frac{B^2}{27\sqrt[3]{A^5}a^{16}}\right]^{\frac{1}{2}}} = \sqrt{\kappa}dt,$$

$$t(a) = \int \frac{3^{\frac{3}{2}}A^{\frac{5}{6}}a^8}{\sqrt{\kappa}\sqrt{9A^2a^{18} + 3ABa^9 - B^2}},$$

from where we find

$$t(a) = \frac{ln\left(\left|2\sqrt{9A^2a^{18} + 3ABa^9 - B^2} + 6Aa^9 + B\right|\right)}{3^{\frac{3}{2}}\sqrt[6]{A}\sqrt{\kappa}}.$$

decomposing in a row, we get:

$$t(a) = C\left[12Aa^9 - 1 - \frac{(12Aa^9-1)^2}{2} + \frac{(12Aa^9-1)^3}{3}\right]$$

And finally, consider the last special case for αgiven 3:

$$\left(\frac{\dot{a}}{a}\right)^2 = \kappa\left(-\frac{B^2}{32\sqrt[4]{A^7}a^{24}} + \frac{B}{12\sqrt[4]{A^3}a^{12}} + \frac{\sqrt[4]{A}}{3}\right),$$

$$\frac{da}{\left[\frac{a^2\sqrt[4]{A}}{3} + \frac{B}{12\sqrt[4]{A^3}a^{10}} - \frac{B^2}{32\sqrt[4]{A^7}a^{22}}\right]^{\frac{1}{2}}} = \sqrt{\kappa}dt,$$

$$t(a) = \int \frac{4\sqrt{6}A^{\frac{7}{8}}a^{11}}{\sqrt{\kappa}\sqrt{32A^2a^{24} + 8ABa^{12} - 3B^2}},$$

from where we get

$$t(a) = \frac{\sqrt{6}ln\left(\left|2^{\frac{7}{2}}\sqrt{32A^2a^{24} + 8ABa^{12} - 3B^2} + 64Ca^{12} + 8c\right|\right)}{3\cdot 2^{\frac{5}{2}}\sqrt[8]{A}\sqrt{\kappa}}.$$

The final expression will take the form:

$$t(a) = C\left[128Aa^{12} - 1 - \frac{(128Aa^{12}-1)^2}{2} + \frac{(128Aa^{12}-1)^3}{3}\right]$$

the obtained three expressions for t(a) describe the growth of the scale factor with time in the modern stage of the evolution of the Universe, the stage of secondary inflation corresponding to accelerated expansion. The main result of cosmological research over the past few years is this: the existence of dark energy and the anti-gravity created by it is reliably and has now been definitively proven. The establishment of the mechanism causing cosmological acceleration is currently one of the central problems of modern cosmology and fundamental physics.

3. CONCLUSION

At the moment, the cosmological Lambda Cold Dark Matter (ΛCDM) has a number of disadvantages, which is why alternative models describing the accelerated expansion of the Universe at the modern cosmological stage are being considered. In this paper, we obtain a general solution of the Chaplygin gas problem for a flat universe in quadratures:

$$t(a) = \int \frac{\sqrt{3}(A + a^{-3-3\alpha}B)^{-1/(2+2\alpha)}}{\sqrt{\kappa}a}da.$$

and three numerical solutions for three special cases:

$$t(a) = C\left[32Aa^6 - 1 - \frac{(32Aa^6-1)^2}{2} + \frac{(32Aa^6-1)^3}{3}\right],$$

$$t(a) = C\left[12Aa^9 - 1 - \frac{(12Aa^9-1)^2}{2} + \frac{(12Aa^9-1)^3}{3}\right],$$

$$t(a) = C\left[128Aa^{12} - 1 - \frac{(128Aa^{12}-1)^2}{2} + \frac{(128Aa^{12}-1)^3}{3}\right].$$

This model should be expanded and this will be a task for future research.

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