MAGNON SPECTRUM ANALYSIS IN ANTIFERROMAGNETIC NANOTUBES VIA THE GREEN'S FUNCTION METHOD

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This work analyzes spin-wave dispersion in hexagonal antiferromagnetic nanotubes using a Green's-function framework. With a Hamiltonian limited to nearest-neighbor exchange and a crystal-field term, dispersion relations are derived via symmetry and Fourier transforms. The results elucidate axial magnon propagation and how lattice symmetry shapes the spectrum.

Keywords: hexagonal nanotubes, antiferromagnet, spin waves, Green's function
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Nanostructured antiferromagnets are drawing strong interest for both basic science and applications thanks to their ultrafast response to high-frequency fields, robustness from zero net magnetization, and promise for information transport [1]. We investigate spin-wave dynamics in a hexagonal antiferromagnetic nanotube, where exchange interactions and spatial symmetry jointly govern magnon propagation and quantum-mode splitting. The hexagonal geometry fixes lattice periodicity and drives symmetry-dependent degeneracies, motivating a dispersion construction and symmetry analysis [2]. Using a Green's-function framework, we compute spin–spin correlations and energy spectra, derive magnon dispersions for the model nanotube, and resolve mode degeneracies [3]. Our aim is to clarify the energetic landscape of spin waves, explain the origins of dispersion minima, and quantify how interaction parameters reshape the spectrum.

The system under study is a hexagonal antiferromagnetic nanotube composed of six-atom rings stacked along the tube axis. The lattice comprises two antiferromagnetically coupled sublattices, a and b. As illustrated in Fig. 1, we include exchange interactions within each ring (parameter J) and between adjacent rings along the axis (parameter Y).

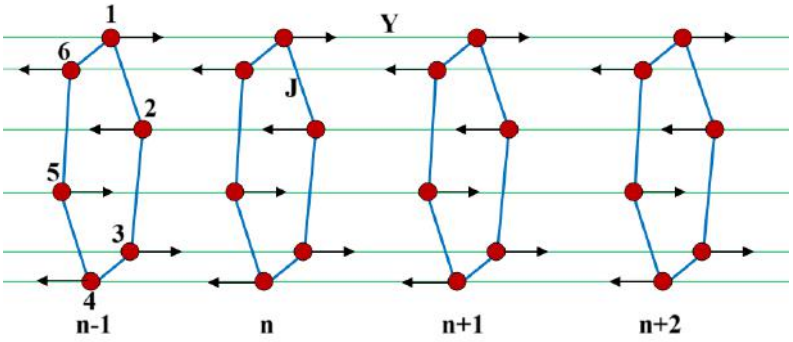

Fig. 1. Schematic representation of hexagonal antiferromagnetic nanotube. Each red dot denotes a spin site. The intrachain coupling along the horizontal direction is denoted by Y, and the interchain coupling along the rungs of the ladder is denoted by J. The spins alternate directions, indicating antiferromagnetic ordering.

The system's Hamiltonian includes only nearest-neighbor interactions and the crystal field and is given as follows [4]:

$$H = \sum_{i,j} J_{ij}\left(S_i S_j\right) - \sum_i h^{(A)}_{AFM,i} S^z_{i,a} + \sum_j h^{(A)}_{AFM,i} S^z_{j,b} + D\left(\sum_i \left(S^z_{i,a}\right)^2 + \sum_j \left(S^z_{j,b}\right)^2\right) \quad (1)$$

The first term captures exchange coupling between spins; the second and third terms describe internal (effective) fields acting on the z-components, enforcing opposite spin orientations on the two sublattices; the final term is a uniaxial anisotropy that sets the preferred alignment direction. The parameter D indicates the strength of anisotropy.

The alternating spin order governs the system's dynamics, making the dispersion relation central to understanding its behavior. We analyze these dynamics with Green's functions, which quantify how a local spin flip influences another site—i.e., how collective spin excitations propagate.

Within this framework, a convenient choice is the retarded double-time Green's function

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$$G_{i,j}^{(a)}(\omega) = << S_{i,(a)}^{+}(t); S_{j,(a)}^{-}(t') >>_{\omega}, \qquad G_{i,j}^{(b)}(\omega) = << S_{i,(b)}^{+}(t); S_{j,(a)}^{-}(t') >>_{\omega} \quad (2)$$

The resulting system of equations for these functions is:

$$\begin{cases} (\omega - \Omega) G_{n,n'}^{(a)}(\omega) - \langle S_{n,a}^{z} \rangle \left[2J\cos(qa) G_{n,n'}^{(b)}(\omega) + Y G_{n-1,n'}^{(b)}(\omega) + Y G_{n+1,n'}^{(b)}(\omega) \right] = 2\langle S_{n,a}^{z} \rangle \delta_{n,n'} \\ (\omega + \Omega) G_{n,n'}^{(b)}(\omega) - \langle S_{n,b}^{z} \rangle \left[2J\cos(qa) G_{n,n'}^{(a)}(\omega) + Y G_{n-1,n'}^{(a)}(\omega) + Y G_{n+1,n'}^{(a)}(\omega) \right] = 2\langle S_{n,b}^{z} \rangle \delta_{n,n'} \end{cases}$$

(3)

Here $\Omega = h_{AFM}^{(A)} + (2J + 2Y + D)\langle S^{z} \rangle$, $\langle S_a^z \rangle = -\langle S_b^z \rangle = \langle S^z \rangle$

To solve these equations, Fourier transformation is applied, taking into account the periodicity of the structure [5]:

$$G_{n,m}^{(a)} = \frac{1}{6}\sum_{r=o}^{5} G_{n,m}^{(b)} \exp[\pm iqa]; \quad G_{n,m}^{(b)} = \frac{1}{6}\sum_{r=o}^{5} G_{n,m}^{(a)} \exp[\pm iqa] \quad (4)$$

$$q = \frac{2\pi r}{6a} = \frac{\pi r}{3a}, \quad r = 0,1,2,...,5$$

Additionally, considering periodic boundary conditions and translational symmetry, the Green's function for neighboring spins follows a phase shift rule [6]:

$$G_{n\pm1,m}^{(a,b)} = \exp[\pm ika] G_{n,m}^{(a,b)} \quad (5)$$

This approach allows for the analysis of spin wave propagation, phase transitions, and symmetry in the crystal. Thus, the following dispersion relation for spin waves is obtained:

$$\omega = \pm\sqrt{\Omega^2 + 4\langle S_{n,a}^{z} \rangle^2 \left[J\cos(qa) + Y\cos(ka) \right]^2} \quad (6)$$

Now, let us examine how the dispersion relation $\omega(k)$ varies as a function of the wave vector k. Calculations are performed for different values of the interlayer spin interaction parameter Y, while other parameters remain constant.

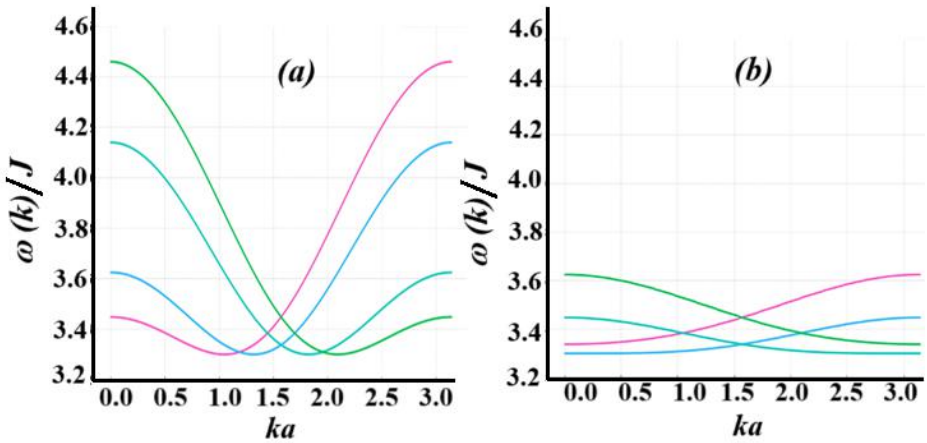

Fig. 2. Dispersion curves in a hexagonal antiferromagnetic nanobore. The parameters are as follows: $h_{AFM}^{(A)}/J = 0.2, \quad \langle S_a^z \rangle = 0.5 \quad D/J = 0.1, \quad a) Y/J = 2, \quad b) Y/J = 0.5.$

Figure 2(a): Strong interlayer coupling. The branches are nearly parabolic with minima near k=π/2. The enlarged bandwidth and steeper slopes indicate freer magnon propagation and more efficient energy transfer across the nanotube.

Figure 2(b): Weak interlayer coupling. The branches lie closer together and exhibit reduced curvature. Diminished bandwidth and flatter slopes point to weaker energy variability, with more localized modes and less efficient spreading of excitations.

Interlayer spin coupling is a primary control knob for the dispersion. Increasing this coupling enlarges band separation and broadens the spectrum, while the symmetric placement of minima around k=π/2 reflects the lattice symmetry and periodicity. The observed trends imply that quantum excitations can be engineered via the parameters J (intra-ring) and Y (inter-ring), enabling tunable behavior relevant to spintronics, magnetic nanostructures, and quantum information applications.

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