SECOND HARMONIC GENERATION IN LOCALLY RESONANT NONLINEAR WEYL SEMIMETAL BASED ACOUSTIC METAMATERIALS: FINITE ELEMENT ANALYSIS

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Weyl semimetal based locally resonant metamaterials, which have applications such as imaging and sound / vibration isolation, wave focusing, are high performance materials with superior properties that are obtained artificially.
In this study, second harmonic generation due to the energy induced in nonlinear locally resonant metamaterials is demonstrated by the finite element method. The second energy transfer mechanism has recently been arises from a nonlinear interaction between propagating and evanescent waves triggered by autoparametric resonance manifesting itself through the appearance of a subharmonic transmission attenuation zone.

Keywords: Second Harmonic Generation, Acoustic Metamaterials, Metamaterials
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INTRODUCTION

Metamaterials are artificial materials designed to achieve unusual properties not found in natural materials [1,2]. These materials, which have band gaps in which the propagation of acoustic/elastic waves at certain frequencies are prevented, have attracted the attention of researchers because of their potential application areas [3-5]. These materials have potential applications such as wide band gap at low frequencies, noise and vibration reduction [5], wave reduction, and high-resolution acoustic imaging.

In these periodic structures band gaps at lower wavelengths can be obtained by using locally resonant unit cells where band gaps are formed due to multiple scattering [6-8]. Locally resonant metamaterials have enabled new applications such as acoustic diodes [14], logic gates [15] due to their non-linear operating region [9-13].

Despite the great interest of researchers, there are few studies on nonlinear metamaterials due to the difficulty of modeling [16-20]. In some of these limited studies, resonator structures with irreversible energy transfer mechanism triggered by non-linear energy sinks have been studied [21-24].

Also, few papers have examined wave propagation in materials with nonlinear periodic resonators. In such structures, two types of mechanisms, called intermodal and modal inner tunneling, can occur. This mechanism [25-30], which is based on the energy transition between modes in propagating waves, which was first put forward by Lazarov and Jensen [31], has been theoretically studied [25] and confirmed experimentally [26].

The other theoretically predicted [32] energy transfer mechanism arises from the formation of a sub-harmonic transmission attenuation region, which provides new, advanced tools for wave attenuation and control, resulting from the non-linear interaction between propagated and vanishing waves generated by the resonance mechanism. Nonlinear locally resonant metamaterials consisting of rubber between the resonator and the base material were investigated theoretically and numerically [32].

In this study, a locally resonant structure made of $ZrTe_5$ – Weyl semimetal was designed, inspired by the locally resonant designs commonly used in micromechanical systems [43-48], and the existence of second harmonic generation was investigated by the finite element method.

MATERIAL METHOD

The designed metamaterial with spring-mass mechanism with local resonance consists of the periodic arrangement of the unit cells in figure 1.a. In this structure, the resonator springs must be non-linear in order for the sub-harmonic weakening region to occur. Silva et al. demonstrated by [32]. It can be accomplished by different methods [39] such as contact dynamics [36-38], electrostatic actuation [33-35].

Examples of studies on second-order non-linear resonators are micromechanical cantilever system [40], hanging cables [41], M-shaped resonator [42] arc resonators [43-48] not connected with metamaterials.

According to the Bloch-Floquet periodicity condition [49], the displacement field u can be expressed as:

$$u(x,k,t) = Ue^{i(k.x-\omega t)} \quad (1)$$

where U denotes a periodic Bloch displacement vector, x position vector, k wave vector, ω frequency. Since the longitudinal waves propagating in the chain of unit

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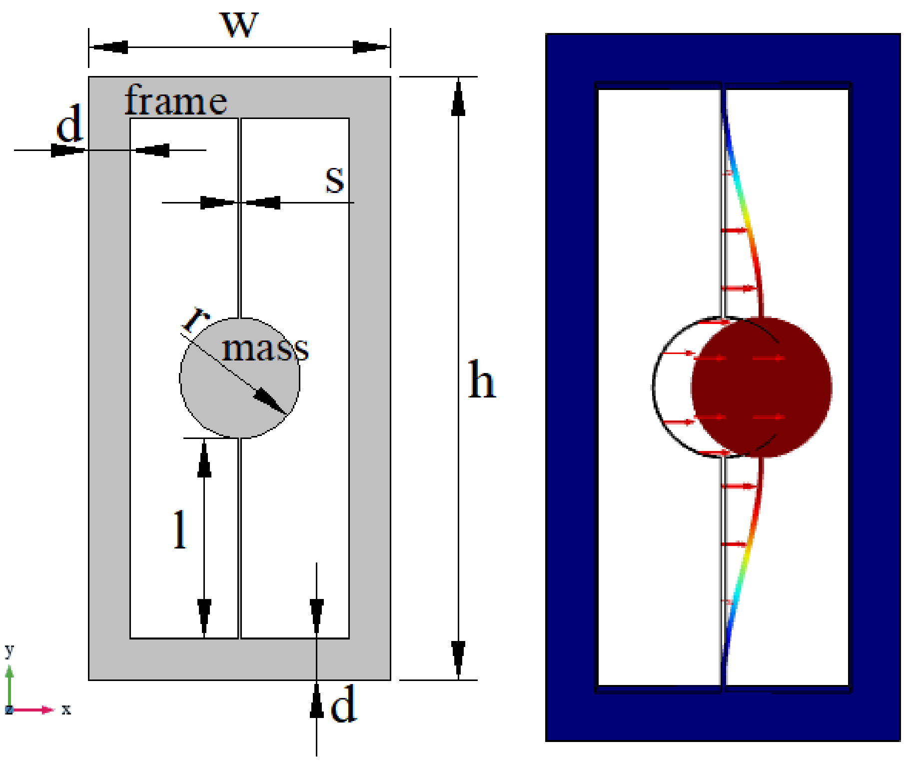
cells are taken into account, the band structure of the Brillouin region along the Γ-X path was obtained using the commercial software COMSOL Multiphysics. Bloch-Floquet periodic boundary conditions are applied along the *x*-direction on the left and right sides of the 3D unit cell, while all other surfaces are released.

The unit cell of the locally resonant structure seen in Figure 1.a consists of an internal mass of Weyl semimetal – $ZrTe_5$ (Young modulus E = 63 GPa and density ρ = 3.400 kg/m^3) connected to the main frame with a single beam. In the unit cell, h=40 mm, w=20 mm, d=2.75 mm, r=4 mm, s=0.25 mm, l=13.25 mm and its thickness in the z direction is 5 mm.



a) b)

Fig. 1. a) Unit cell of a metamaterial with nonlinear local resonance b) resonance mode of the 3D unit cell (416.89 MHz)

Due to the design of the unit cell, the properly excited structure transmits the wave in the *x*-direction. The source of the nonlinear terms in force-displacement relations is the beams holding the mass in the unit cell.

As seen in Figure 1.b, the back-and-forth movements of the beam-bound mass in the unit cell explain the nonlinear behavior of the structure required to provide the sub-harmonic resonance energy exchange mechanism.

The degree of freedom in the unit cell is important for designing nonlinear locally resonant metamaterials. By changing the parameter h, which determines the preliminary deflection of the beam in the unit cell, the magnitude of the quadratic term relative to the cubic term can be controlled and thus the desired design can be obtained. The second harmonic is obtained by changing the desired *h* parameter in the proposed structure.

LINEAR BEHAVIOR

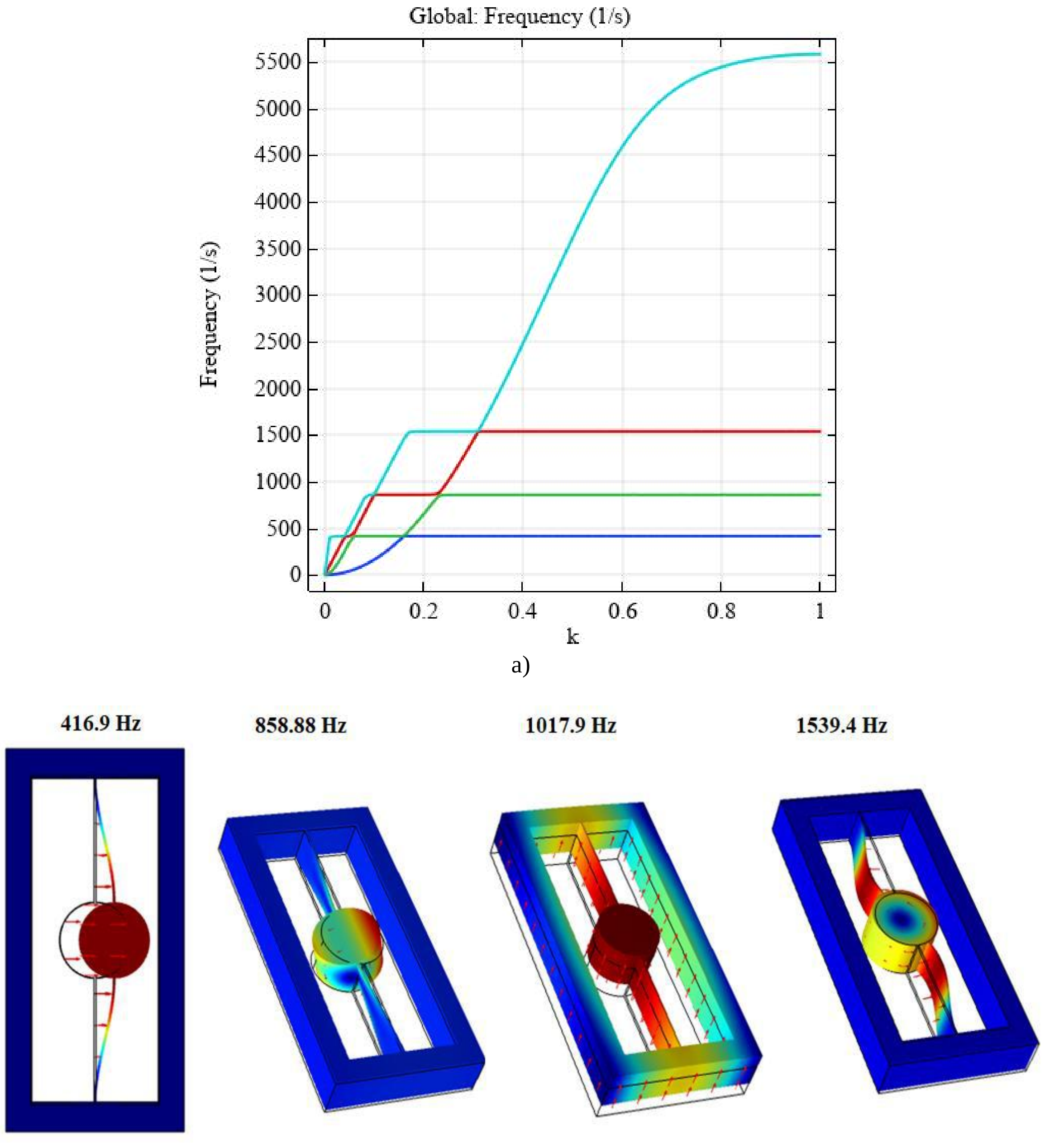
The locally resonant unit cell shown in Figure 1.a was designed and simulated in COMSOL. Floquet Periodicity Boundary conditions were applied to the *x*-direction walls of the unit cell, while the other surfaces were released.

In Figure 2.a, the band structure of the unit cell obtained by using COMSOL Multiphysics software [50-54] is shown, it is seen that the band gap occurs around the natural frequency of the in-plane bending mode according to the band structure. Figure 2.b shows the mode shapes of the unit cell at different frequencies.

A finite structure in the form of a chain consisting of 50-unit cells was designed to ensure the interaction between the propagating and disappearing waves and to represent the dynamic behavior of the resonators.

The finite structure would be excited with a prescribed displacement from the left end with a displacement of 10^{-6} m in the *x* direction. Parametric scanning was performed between 200-900 Hz to obtain the transmission diagram. By obtaining the displacements on the left and right sides of the finite structure in Figure 2, the transmission diagram in Figure 3 was obtained according to the relation $20.\log10(U_2/U_1)$.

Here, U_1 denotes the displacement at the left margin, and U_2 the displacement at the right margin. As seen in Figure 3, there are two modes that result in an increase in the input signal around the frequency at which the transmission decreases.



b)

Fig. 2 a) The band structure of the unit cell in the Γ − X direction, b) the mode shapes of the unit cell

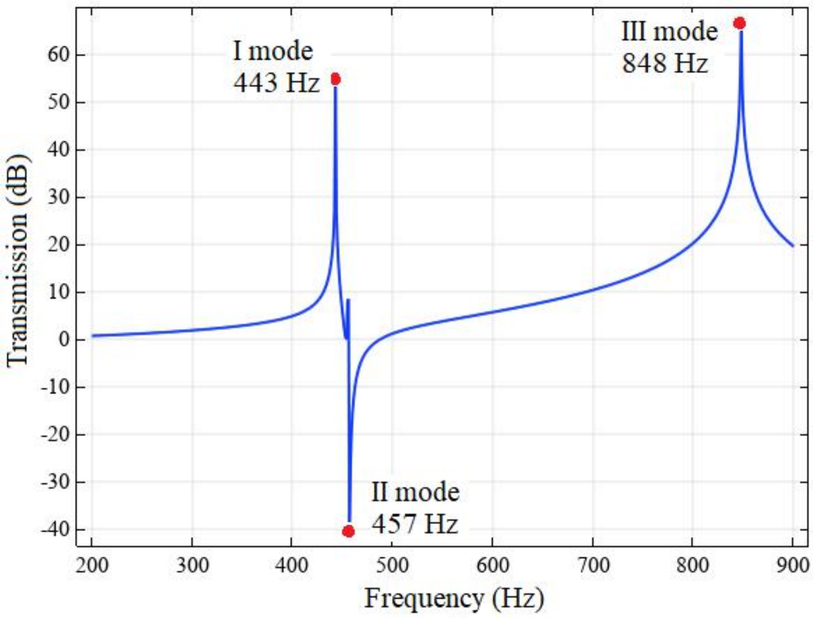
NONLINEAR BEHAVIOR

In locally resonant metamaterials, the resonator generates a band gap and a vanishing wave. Harmonic excitation at one end of the metamaterial generates the same frequency wave propagation and near-field waves [32].

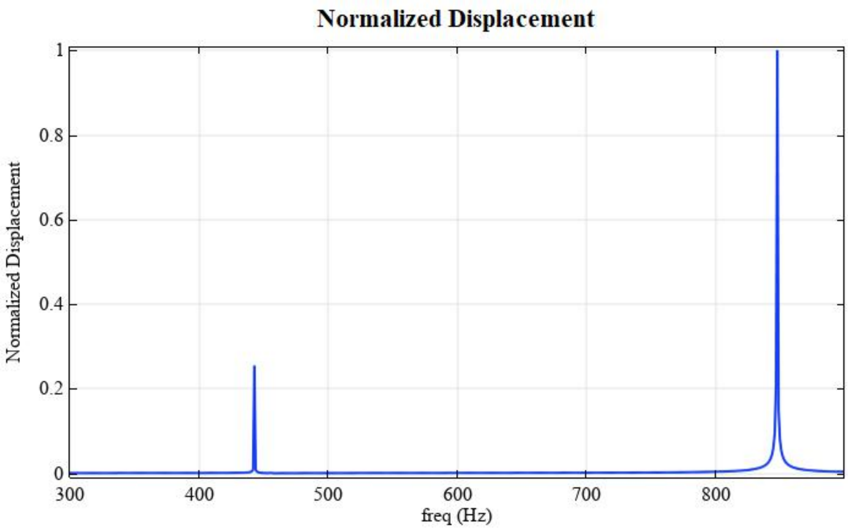
When there is a second-order non-linear interaction between the local resonator and the main frame and there is a resonance frequency around half the excitation frequency, the sub-harmonic wave and the propagating waves match, and in nonlinear systems, the energy is transferred from the advancing wave to the vanishing wave [32]. In this study, the propagating initial wave around 850 Hz was transformed into a vanishing wave at a quasi-harmonic frequency around 440 Hz in the band gap close to the local resonance frequency.

The chain-like finite structure consisting of unit cells with resonators excited from the left side reflects elasto-acoustic waves and prevents their progression in a finite structure. Thus, the energy flow to the other (right) end of the metastructure is significantly reduced.

The transmission diagram of the finite structure consisting of 50-unit cells, obtained by using COMSOL Multiphysics, is shown in figure 3. The normalized displacement obtained at the rightmost edge of the finite structure is shown in Figure 3.b.



a)



b)

Fig. 3. a) Transmission diagram of the finite structure b) normalized displacement of the right edge of the finite structure.

CONCLUSION

In conclusion, we numerically investigated nonlinear propagation and second harmonic generation in acoustic metamaterial with periodic array of elastic plates with nonlinear resonators. According to the results obtained, the energy exchange between the sub-harmonic wave and the first wave is seen with the effect of autoparametric resonance. A unit cell metastructure consisting of a nonlinear resonator frame with autoparametric resonance to provide sub-harmonic attenuation is designed. The results showed that

- High performance nonlinear locally resonant metamaterials can be developed.
- Tunable metamaterials with amplitude-dependent attenuation regions can be developed.
- New applications such as multi-harmonic tunable filters and display devices can be developed.

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