## ATTRACTORS IN NANO-STRUCTURED LAYERED CRYSTALS

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The impurities accumulate and the quantitative changes take place under influence of intercalation in different  $A^{V}_{2}B_{3}^{VI}$  layers: the old bonds are broken and new ones are appeared between its separate layers, even some quintets in Bi<sub>2</sub>Te<sub>3</sub> <Cu> structures are destroyed and the several nano-objects of fractal character form on their places. These changes lead to instable states, i.e. to bifurcation points. The appeared fluctuations are chaotic ones, some of them damp, only those which form new structures, i.e. attractors attracting the many trajectories of system development and forming the special cone, stay. The nano-islands between T<sub>e</sub><sup>(1)</sup> – T<sub>e</sub><sup>(1)</sup> Bi<sub>2</sub>Te<sub>3</sub> <Cu> layers can be shown in the capacity of its appearance model. If unstable microstructure (curprum in vacancies, between layers and etc) will be in attractor cone, it will evolve to stable state with appearance of many development variants: cluster formation, nano-steps at edge of which the nano-islands form.

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### FRACTALS AND CHAOS IN DYNAMIC SYSTEMS

The determinate equations in systems with dynamic chaos lead to chaotic solutions, i.e. the firstly neighboring trajectories exponentially rapidly diverge in them and instability appears. Moreover, the fractals as geometrical objects can be successfully applied at description of chaotic system trajectories. One can said, that the fractals are the chaos geometrical images.

The structures, changing in time, are defined as dynamic systems. It is clear that chaos is the dynamic fit of the fractal. The chaos describes the states of extreme unpredictability appearing in dynamic system whereas the fractality describes the extreme irregularity inherent to geometric configuration.

Probably, there is no possibility to obtain the math expressions for solution in closed species even the special functions is used.

The dynamic system attractors, in particular, socalled "the strange attractors" are study objects in this direction.

The example that visually demonstrates what is the "chaotic dynamics" is considered in [1]. The following system of differential equations is studied by E.Lorentz with the help of convection flow modeling [1]:

$$\begin{cases} \dot{x} = \frac{dx}{dt} = \sigma(-x+y), \\ \dot{y} = \frac{dy}{dt} = rx - y - xz, \\ \dot{z} = \frac{dz}{dt} = -bz + xy. \end{cases}$$
(1)

where  $\sigma$ ,r,b are constant positive parameters. The essential dependence on initial conditions that is the main character of chaotic dynamics is inherent to the given system. Here the examples are illustrated what differential equation attractor is. The fractal dimension of concrete differential equation attractors is obtained. Let's consider two examples. The first one is connected with equation system:

$$\begin{cases} \dot{x_1} = \frac{dx_1}{dt} = -x_1 ,\\ \dot{x_2} = \frac{dx_2}{dt} = -2x_2 \end{cases}$$
(2)

The general solution of this system is given by formulas:

$$x_1(t) = c_1 * e^{-t}, x_2(t) = c_2 * e^{-2t}, c_1, c_2 = const.$$
 (3)

If the initial conditions  $x_i(0)$  and  $x_i(0)$  are given so the trajectory outgoing from this point (i.e. (2) system solution) is given by following formulas:

$$x_1(t) = e^{-t} * x_1(0)$$
,  $x_2(t) = e^{-2t} * x_2(0)$ 

Probably, any trajectory at  $t \rightarrow +\infty$  strives to (0,0) point. In this simple case we can say that point (0,0) is A system attractor (2) (or solution of (2) system): A= (0,0).

The attractor second example being the limit cycle is: let's the system oft wo differential equations is given and it has the following form in  $p, \varphi$ ) polar coordinates on  $\mathbb{R}^2$  plane:

$$\dot{p} = p(1-p), \tag{4}$$

$$\dot{\varphi}=1.$$
 (5)

From equation (5) it is followed that:

$$\varphi(t) = t + c_1 \tag{6}$$

It is easy to see that the circle is the solution of equation (4).

$$p(t) = l \tag{7}$$

Indeed  $\dot{p} = 0$  and right part of equation (4) at p=1 is transformed in zero:

$$p(1-p^2)/p=1=0$$

The circles (6), (7) are the solution of systems (4) and (5) and moreover, periodic one with period  $2\pi$ . Indeed  $\varphi$  polar coordinate has this property: the points with polar coordinates  $(1,\phi)$  and  $(1, \phi+2\pi)$  show the one and the same point in the circle p=1.

Let's note that if (p(t),  $\phi(t)$ ) are solutions of (4), (5) and 0 < p(t) < 1 systems then from equation (4) it is followed:  $\dot{p}(t) = p(t) * (1-(p(t))^2) > 0$ , i.e. function p(t) increases approximating to value p==1 and (p(t),  $\phi(t))=(p(t),$ t+c<sub>1</sub>) trajectory does the spiral motion inside circle p=1.

If the solution  $(p_1(t), \phi(t))=(p_1(t), t+c_1)$  in some time moment t has  $p_1(t)>1$  then according to equation (4)  $\dot{p}_1(t) = p_1(t) * (1 - (p_1(t))^2) < 0$  and  $p_1(t)$  decrease with t increase. In this case  $(p_1(t), t+c_1)$  curve spirally approaches to p=1 circle outside at t $\rightarrow +\infty$ .

In [1,2] it is shown that all trajectories of equation system (besides the origin of coordinates p=0 which is the fixed point of systems (4) and (5)) strive for  $\{p=1\}=A$  circle at  $t \rightarrow +\infty$ . This is called the limit cycle of systems (4) and (5). Such limit cycle (attracting) is called attractor of systems (4) and (5).

The formation of ordered metallic and semiconductor structures with definite density is understood as self-organization process. The nano-fragments in interlaminar space of  $A_2^VB_3^{VI}$  <impurity> [1-3] crystal can be the example of self-organization example. The formation of interlaminar steps is the also consequence of self-organization. The self organization includes the system interaction able to spontaneous appearance of order in space and in time. It also includes the structural and time order [2].

The appearance of gradation at graphite intercalation by impurities is directly confirmed in electronmicroscopic investigations. The co-existence of steps on micro-structural level is shown.

There is necessity in study of nano-fragment morphology forming in interlaminar space of layered crystals by  $Sb_2Te_3$  and  $Bi_2Te_3$  <impurity> type. The fractal particles and nano-meter dimension surfaces are formed as a result of crystallization in interlaminars.

According to above mentioned one can solve thetas in revealing of stepped interlaminar structures, nanoislands and other nano-fragments in layered crystals.

The **aim** of the work is the nano-structurization by impurities and revealing of morphological peculiarities of interlaminar surface (0001)  $Bi_2Te_3$  <impurity> with attracting attractors. The electron-microscopic images are obtained on scanning probe microscope (SPM) by (Co-JBEP HEKCT mark). X-ray investigations of surface (0001) are carried out on diffractometer by Philips Panalytical (XRD).

The appearing steps in  $A^{V}_{2}B_{3}^{VI}$  <In> in increase process play the significant role when intralayer forces significantly exceed the interlaminar ones. The given peculiarities of interlaminar stepped surface can lead to specific dynamics of crystal lattice that should reveal in physical phenomena.

#### **RESULTS AND THEIR DISCUSSION**

The physicochemical processes of formation on  $A_2^V B_3^{VI}$  crystal surface (0001) with positions far from

thermodynamic forces and self-organization of fractal structures in medium  $T_e^{(1)} - T_e^{(1)}$  (between quintets of crystal structure).

The self-organization processes of impurities of cuprum and nickel on interlaminar surface (0001)  $A_2^V B_3^{VI}$  <impurity> are investigated with the help of scanning probe microscopy. It is shown that forming impurity surfaces inside  $T_e^{(1)} - T_e^{(1)} A_2^V B_3^{VI}$  <impurity> have the stepped-layered character with nano-islands, nano-bump height forming in process of vertical directed crystallization at step edge are 5-20nm.

The positions of layer-quintets, their atomic defect structure allows to destroy it and form the different types of nano-formations (nano-islands and steps). Moreover, the doping by impurities influences on peculiarities of such crystal electronic properties. The layered character of crystal structure of  $A^V_2B_3^{VI}$  matrix makes possible the shift of easy-diffusing atoms (Cu,Ni, B and other) from indention center positions where they are in donor states in Van-der-Waals cracks between five-layer packets; here these atoms order and become electrically neutral ones.  $T_e^{(1)} - T_e^{(1)I}$  distance in  $A^V_2B_3^{VI}$  is bigger than distance of  $B_i - T_e^{(1)}$  and  $B_i - T_e^{(2)}$ .

# THE PROCESS OF INTERCALATION AND SELF-INTERCALATION IN ${\rm AV}_2{\rm B_3}^{\rm VI}$ SYSTEM

Let's consider the atom diffusion along surface (0001)  $A_2^V B_3^{VI}$  <impurity> in direction from "C". The initial energy of atom motion is maximum one. The friction forces with real surface (0001) or collision between moving atoms themselves lead to the fact that atom motion energy transforms into heat energy and disappears. The interlaminar medium  $T_e^{(1)} - T_e^{(1)}$  is heated. Moreover, the heating can lead to chemical reactions. For example, at free movement the intercalating atoms disperse the order kinetic energy their movement and transform it in chaotic heat motion of the particles (the process of energy dispersion itself is called dissipation). The systems formed in  $T_e^{(1)} - T_e^{(1)} A^V_2 B_3^{VI}$  medium at such gradient processes can be called dissipative ones [1,4]. Here it is said on the atom bond breakage in crystal lattice  $T_e^{(1)}$  –  $T_e^{(1)} A_2^V B_3^{VI}$  and the new bonds appear leading to formation of interlaminar nano-structures on the base of impurity super-stoichiometric components. The initial stage can be instable one (bifurcation point).

Thus, cuprum atom motion from vacant sites of tellurium and bismuth in  $T_e^{(1)} - T_e^{(1)}$  Bi<sub>2</sub>Te<sub>3</sub> <Cu-In> layers forms the effect of self-intercalation [1] for consideration of their aggregation ways on plane (0001) with nanoisland formation. The model of cuprum atom transition from instable states in bifurcation field (fig.1) is given.

In state far from equilibrium, the very weak fluctuations can step-wise transform the previous structure  $A^{V}_{2}B_{3}^{VI}$  into structure with interlaminar elements with complex destroying structure in whole. Such complex morphological data change in cuprum atom selfintercalation process from other layers in  $T_{e}^{(1)} - T_{e}^{(1)}$  layer of very few quantity of cuprum. Aggregating they form the bigger nano-islands. The considered effects can be considered as the peculiarities of self-organization processes in  $A^{V}_{2}B_{3}^{VI}$  <Cu> systems with formation only interlaminar dissipative nano-islands.

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Considering the different fields we see that reconstruction at the presence between the quintets of thermal field take place on surfaces (0001). It is connected with significant inclination and cuprum transition from vacancies and interlaminars in  $T_e^{(1)} - T_e^{(1)}$  region. Here the nano-islands (NI), steps, corrugated structures form. Occupying the significant field of this medium, these nanoobjects self-organize and transform into different patterns. The self-organization and pattern formation in the form of nano-islands in pattern formation in the form of nanoislands in Bi<sub>2</sub>Te<sub>3</sub> <In-Cu> are connected with dissipation conception associating with damping of Cu atom motion from accumulated layers and vacancies (see the arms in fig.1, which show the field "pool") from which the process of attracting trajectory shows on definite cycle and saves its stability in it in the form of different nanoformations. These cycles can be related to so-called limit ones [4]. According to [4] the any dynamic system has its extraordinary flow, its limit cycle and stay there forever. The own limit cycle in the form of regularly distributed nano-formations in  $T_e^{(1)} - T_e^{(1)}$  medium is observed in Bi<sub>2</sub>Te<sub>3</sub> <In-Cu> system. And these trajectories can be called not limit cycles but attractors because they like limit cycles attract the trajectories from instable fields (5) and form the geometric fractals in dissipative medium  $T_e^{(1)} - T_e^{(1)} A^V_2 B_3^{VI}$  <impurity>.

It is considered that dynamic system has the attractor if some right subset A of phase space  $R^E$  having the following property: at any point  $\sigma(0)$  and enough big t the point  $\sigma(t)$  is in small neighborhood of any point belonging to A [1-3].

Each realistic dynamic system achieves the attractor after outflow "enough big" t. The selection result will always coincide with attractors in homogeneous space with concentration gradient. The final result of selection process will correspond to either stable stationary state or periodically changing state family. In some especially occasional situations the non-periodic changes in limits of definite multiple states. The common term that is "attractor" of dynamic system where the stable points can be included, the closed orbits and aperiodic curves [1], is used for characteristics of all these stable and quasi-stable final situations in differential topology. The achievement of the one and the same attractor independently on concrete initial conditions inside the given "pool" is the result of going process.

The density of distribution of such defects as Koch figures, nano-islands, plane breakage (0001) which are steps, is the parameter controlling the achievement of bifurcation point when the dissipative structure with special peculiarities forms that is revealed at processes of intercalation and self-intercalation of Ni, Cu and B impurities.



*Fig.1.* The model of inter-crystalline structure demonstrating the cuprum atom drop-out from  $B_i$  layer in which Cu is accumulated. Designations: 1 is Cu diffusion direction at self-intercalation; 2 and 3 are "pool" fields from which Cu atoms diffuse into interlaminar space; 3 are  $B_i$  vacancy fields with accumulated Cu atoms; 4 are vacancy fields with cuprum; 5 is interlaminar medium  $T_e^{(1)} - T_e^{(1)}$  which is limit cycle field. The places "pools" from which Cu atoms in field (5) are shown by two arms (right at the top) in field (5). The trajectories of atom drop-out into limit cycle fields are shown by arms in left.

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