DEPENDENCE OF ELECTRON MOBILITY ON THEIR SURFACE DENSITY IN A SEMICONDUCTOR QUANTUM WELL WITH THE MODIFIED PÖSCHL -TELLER CONFINING POTENTIAL

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The dependence of electron mobility on their surface density in a semiconductor quantum well is studied. The modified Poschl-Teller potential is used as the confining potential in the quantum well. It is shown that the dependence of electron mobility on their surface density is close to linear. Comparison of the obtained theoretical results with experimental data shows that the modified Poschl-Teller potential represents the confining potential in $GaAs/Al_xGa_{1-x}As$ quantum well properly.

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Semiconductor quantum well is the quantum film in which the motion of the charge carriers is not restricted in two directions (let us assume the Cartesian coordinates in these directions *x* and *y*) but in one direction (the z -axis). Theoretical study of transport phenomena in electron gas in semiconductor quantum wells requires to choose the shape of the confining potential in the z -direction. Since the actual shape of this potential is unknown [1], different models are used in theoretical researches. In scientific literature, the most used models are the square well potential [1] and the parabolic well potential [2]. In real crystals the energy of electrons depends on *z* and height of potential is finite quantity. The flaw of square well potential is that this model does not consider *z* dependence; parabolic model considers this dependence but its value tends to infinity at large distances.

In recent years the modified Poschl-Teller potential has been used as the confining potential in quantum wells [3-5]. This potential has a shape between square well and parabolic well models, depends on *z* and gets finite value at large z (fig. 1).

In the present paper dependence of the mobility of electrons on their surface density in a quantum well with the modified Poschl-Teller potential is studied. This potential can be written as [6]:

$$U(z) = \frac{\hbar^2\alpha^2}{2m}\lambda(\lambda+1)\tanh^2\alpha z, \lambda > 0 \quad (1)$$

Since in x and y directions motion is not restricted, dispersion law of electrons with potential (1) is as follows:

$$\varepsilon(k_x, k_y) = E_{\lambda,N} + \frac{\hbar^2(k_x^2 + k_y^2)}{2m}$$

$$E_{\lambda,N} = \frac{\hbar^2\alpha^2}{2m}[\lambda(\lambda+1) - (\lambda - N)^2] \quad (2)$$

Here m is the effective mass of conduction electrons, k_x and k_y are the projections of wave vector of electrons, respectively in *x* and *y* directions, *N*=0,1,2, ... is the number of energy levels ($N \leq \lambda$).

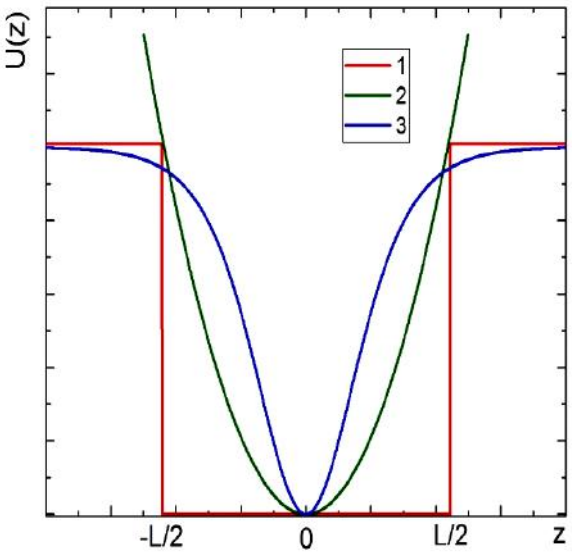

Fig. 1. Different models of confining potential: 1- square well, 2- parabolic, 3- modified Poschl-Teller potential (*L* - width of the quantum well).

Wave function of the electrons with energy spectrum (2) is given as:

$$\psi_{N,\lambda,k_x,k_y}(\vec{r}) = \frac{e^{i(k_x x + k_y y)}}{\sqrt{L_x L_y L_z}}\left[\frac{\alpha(\lambda-1)\Gamma(2\lambda-N+1)}{N!}\right]^{\frac{1}{2}} P_\lambda^{N-\lambda}(\tanh\alpha z) \quad (3)$$

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Here $\vec{r}(x, y, z)$ – coordinates of the electron, L_x, L_y, L_z –dimensions of the sample in the respective directions, $P_\lambda^{N-\lambda}(\tanh \alpha z)$ are the Legendre functions [6].

The analysis of the results of most experiments performed in quantum films shows that electrons occupy the states with the lowest energy (N = 0), namely the average energy satisfies the relation $\bar{\varepsilon} < \hbar^2\alpha^2/2m$. So we can take parameters of the potential as $\lambda = 1,\ N = 0$.

Let us assume the case that the electric field is directed along the quantum film. Then in order to find the mobility, Boltzmann kinetic equation can be used. We are going to use the relaxation time approximation. In the absence of magnetic field and temperature gradient the distribution function of electrons can be written as follows [7]:

$$f(k_x, k_y) = f_0(\varepsilon) + \frac{e}{\hbar}\tau(\varepsilon)\vec{E}\nabla_{\vec{k}} f_0(\varepsilon) \quad (4)$$

Here $f_0(\varepsilon)$ is the isotropic part of the distribution function, $\vec{E}$ -the intensity of the electric field, $\tau(\varepsilon)=(\nu_{ak}+\ \nu_{pz}+\ \nu_i)^{-1}$ is the total relaxation time due to different scattering mechanisms, ν_{ak}, ν_{pz}, and ν_i are scattering rates due to acoustic and piezoacoustic phonons and impurity ions, respectively. These rates were calculated in [4] and [5]. When the distribution function of electrons is known (4) mobility can be derived from the expression for current density along the quantum film:

$$\vec{j} = -2e\sum_{k_x,k_y}\vec{v}\, f(k_x, k_y) \quad (5)$$

Here $\vec{v} = \hbar\vec{k}/m$ is the electron velocity. If we take into account (4) in equation (5) and convert the summation over k_x and k_y into an integral, we get:

$$\vec{j} = \frac{e^2\hbar^2}{2\,\pi^2 m^2}\int_{-\infty}^{\infty}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\tau(\varepsilon)\vec{k}\left(\vec{k}\vec{E}\right)dk_x dk_y \quad (6)$$

We can write the last expression in polar coordinates (k, φ) and integrate over angle φ:

$$\vec{j} = \frac{e^2\hbar^2}{2\,\pi\, m^2}\vec{E}\int_0^{\infty}\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\tau(\varepsilon)k^3 dk = \sigma\,\vec{E} \quad (7)$$

Here σ is conductivity. When electrons are located in parabola with the lowest energy (N =0) for the chemical potential of electrons we get:

$$\zeta = \frac{\hbar^2\alpha^2}{2m} + k_0 T\ln\left[\exp\left(\frac{\pi\hbar^2 n}{m k_0 T}\right) - 1\right] \quad (8)$$

Here k_0 –Boltzmann constant, T - the temperature of the crystal, n – the surface density of electrons. Note that $E_{1,0} = (\hbar^2\alpha^2/2\,m)$ is the minimum of the parabola with the lowest energy. Let us take origin as the minimum of the parabola and consider new variables as below:

$$x = \frac{\varepsilon}{k_0 T} - \frac{\hbar^2\alpha^2}{2m\ k_0 T} = \frac{\hbar^2 k^2}{2m\ k_0 T}, \eta = \frac{\zeta}{k_0 T} - \frac{\hbar^2\alpha^2}{2m\ k_0 T} \quad (9)$$

Then electron mobility $\mu = \sigma/ne$ can be written as:

$$\mu = \frac{e\ k_0 T}{\pi\ \hbar^2 n}\int_0^{\infty}\frac{e^{x-\eta}}{(e^{x-\eta}+1)^2}\tau(x)\ x dx \quad (10)$$

At low temperatures, the main mechanism of the scattering is the scattering by impurity ions. In this case mobility of electrons becomes:

$$\mu = \frac{\chi^2(k_0 T)^2}{2\pi^2\hbar\, e^3\alpha^2\, n\, n_i}\int_0^{\infty}\frac{e^{x-\eta}}{(1+e^{x-\eta})^2}\left[\int_0^1\frac{I^2(k,t)\,dt}{\epsilon^2(k,t)\sqrt{1-t^2}}\right]^{-1} x^2 dx \quad (11)$$

Here, $k = \left(\sqrt{2\,mk_0\,T}/\hbar\right)x^{1/2}, I(k,t) = \int_0^{\infty} e^{-2kzt}\left(1 - \mathrm{Tanh}[\alpha z]^2\right)dz, t = \frac{q}{2k}$.

χ - the static permittivity, n_i - surface density of ions in the quantum well, $q = \left|\vec{k}_2 - \vec{k}_1\right|$ - the magnitude of the difference of two-dimensional wave vectors of the interacting electrons, $\epsilon(k,t)$ is dielectric function [4,5]:

$$\epsilon(q) = 1 + \frac{2me^2}{\hbar^2\chi q}\left\{\frac{1}{2}\left(\frac{q}{\alpha}\right)^2\Psi^{(1)}\left(-1+\frac{q}{2\alpha}\right) - \frac{4+\left(\frac{q}{\alpha}-1\right)\left(\frac{q}{\alpha}\right)^2}{\left(\frac{q}{\alpha}-2\right)^2}\right\} f_0\left(\frac{\hbar^2\alpha^2}{2m}\right) \quad (12)$$

Here $\Psi^{(1)}(z) = \frac{d^2}{dz^2}ln\Gamma(z)$ – trigamma function, $\Gamma(z)$ -gamma function, $f_0\left(\frac{\hbar^2\alpha^2}{2m}\right)$ - occupation probability of the lowest energy state ($k = 0$).

From (11) dependences of mobility on the temperature, the surface density of impurity ions and electrons can be investigated. In quantum wells dependence of the mobility of electrons on their surface density is sharply different from the bulk crystals. The reason is that in bulk impurity semiconductors concentrations of electrons and ions have the same order of magnitude, while in the quantum film it is possible to increase the surface density of electrons compared to surface ion density sufficiently. For example, in heterostructure $Al_xGa_{1-x}As/GaAs$ number of electrons in quantum well (GaAs) increases by one or two orders of magnitude because of the electrons coming from the environment ($Al_xGa_{1-x}As$) [8,9]. Since the ions in the $Al_xGa_{1-x}As$ part of the heterostructure are far from the electrons in

GaAs quantum well, their influence on scattering of electrons is insignificant, consequently increasing of surface density of electrons causes significant increase of mobility.

Let us compare obtained theoretical results with experimental ones [8] in n –type GaAs/$Al_xGa_{1-x}As$ quantum well. In this experiment, the electron mobility is measured in the interval of the surface density of electrons $n = (1 \div 8)\cdot 1015\, m^{-2}$. At these values of electron surface density at low temperatures (T<20K) electrons are strongly degenerate, then η (reduced chemical potential) and k can be written as $\eta = 2\pi\hbar n/mk_0T$ and $k = \sqrt{2\pi n}$, respectively.

Let us calculate mobility numerically at T =12 K in which experiment is performed. Values of used parameters [10]: m =0.067m_0 (here m_0 –free electron mass), ρ=5.3·10^3 kg/m^3, χ=12.9, E_1=7.4eV, e_{14}=0.16C/m^2.

In order to find parameter α of the modified Poschl-Teller potential we use the fact that the factor before $\tanh^2 \alpha z$ function (1) is equivalent to the depth Δ of the quantum well [5]; when λ =1:

$$\frac{\hbar^2\alpha^2}{m} = \Delta \qquad (13)$$

In the experiment [8] the proportion of GaAs in $Al_xGa_{1-x}As$ film is x=0.3. If we take into account that band gaps of the AlAs and GaAs are 2.23eV and 1.52eV, respectively [10], at x=0.3 for the band gap of the $Al_xGa_{1-x}As$ we get $Eg\cong$ 1.52(1-x) +2.23 $x\cong$ 1.73 eV. Then $Al_xGa_{1-x}As$ and GaAs films have band gap difference of $\Delta Eg\cong$0.21eV. The depth of n -type quantum well is equal to the difference of the minimums of the conduction bands in heterojunction. From scientific literature [11] it is known that this difference can approximately be found from the condition $\Delta \cong 0.7\cdot\Delta Eg$. Then from expression (13) for the parameter of the potential, we get α=3.6 · 10^{8}m^{-1}. We will use this value of α in numerical calculations.

The dependence of mobility on the electron surface density has been calculated at the different values of the surface density of the ions and given in fig. 2(a). As seen from the figure the dependence of electron mobility on their surface density is close to linear dependence.

In fig. 2(b) the comparison of the obtained theoretical results with experimental ones [8] is given at n_i=6×10^{14}m^{-2}. From this figure, we can see that obtained theoretical results fit well with experimental results. Based on this, we can conclude that the modified Poschl-Teller potential represents confining potential in GaAs/$Al_xGa_{1-x}A$ quantum well properly.

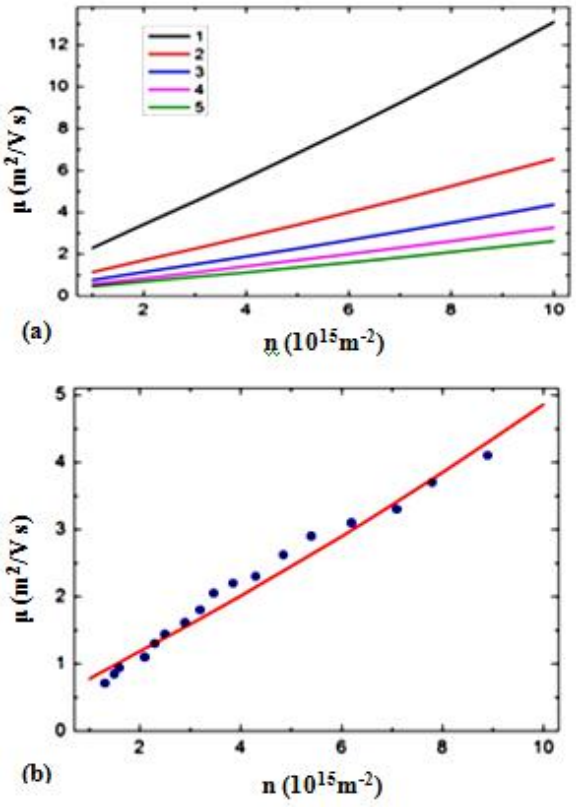

Fig. 2. (a) Dependence of electron mobility on their surface density at: 1-n_i= 2×10^{14}m^{-2}; 2- n_i=4×10^{14}m^{-2}; 3- n_i=6×10^{14}m^{-2}; 4- n_i =8×10^{14} m^{-2}; 5- n_i =10^{15} m^{-2}; (b) Comparison of experimental and theoretical results at n_i=6×10^{14}m^{-2}; line - theoretical, dots - experimental results.

[1] *T.Ando, A.B.Fowler, F.Stern*, Rev. Mod. Phys., 54, 1982, 437.

[2] *F.M. Hashimzade, Kh.A. Hasanov, M.M. Babayev*. Phys. Rev. B,73, 235349, 2006.

[3] *P.P.Kostrobiy, I.A.Ryzha*. Journal of National University"Lvivska Politechnika", Phys. Math. Sci., 718, 89, 2011.

[4] *M.M.Babayev, X.B.Sultanova, N.B.Mustafayev*, AJP Fizika, XXII, 19, 2016.

[5] *M.M.Babayev, Kh.B.Sultanova, N.B. Mustafayev*. Chinese J. Phys., 56, 2977, 2018.

[6] *S.Cruz y Cruz, S. Kuru, J. Negro*, Phys. Lett. A, 372,1391, 2008.

[7] *B.M. Askerov*. The electron phenomena of transfer in semiconductors. Moscow,Science, p.318, 1985.

[8] *K. Hirakawa* and *H. Sakaki*, Phys. Rev. B, 33, 8291, 1986.

[9] *R.Fletcher, J.C.Maan, G.Weimann*. Phys. Rev. B, 32, 8477, 1985.

[10] *V.F.Gantmakher, V.B.Levinson*. Carrier Scattering in Metals and Semiconductors, North-Holland, Amsterdam, 1987.

[11] *S.V. Evstigneyev, R.M. Imamov, A.A. Lomov, U.G. Sadofyev, U.V. Khavarov, M.A. Chuyev, D.S. Shipizin.* Semiconductors, 34, 719, 2000.

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