MAJORANA FERMIONS IN ONE- AND QUASI- ONE DIMENSIONAL INSULATOR WITH CHARGE-DENSITY WAVE

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Majorana fermions can be created in strongly anisotropic crystals with wave-charge density (Peierls instability) in the presence of external magnetic field. Many quasi-one dimensional crystals display Peierls phase transition due to doubling of the crystal unit cell, when a metallic phase of the crystal transforms to an insulator. In this case time reversal symmetry and the particle-hole symmetry are preserved. The formation of the topological phase of crystal is realized in the presence of the strong spin-orbit interaction and external magnetic field in the structure. The interplay between the magnetic field, spin-orbit interaction constants, and the charge density wave gap seems to support Majorana bound states under appropriate values of the external parameters.

Keywords: Majorana fermions, charge density wave, spin-orbit interaction, magnetic field
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1. INTRODUCTION

Although the high-speed computers operating on the base of quantum principles were very attractive up to last thirty years but inaccessible due to the low-level technology, nowadays theoretical and experimental knowledge supports a reality of realization of the quantum computer, and this scientific branch is an extremely perspective one of the modern sciences. According to the law proposed by one of the founders of Intel, Gordon E. Moore, the number of the transistors in the integral circuit of the modern computers increases twice every year and at present, this process seems to be going to saturation [1,2]. According to this law, which indicates the future of the computer technology, in a recent 10-15 years, there should be started the period that will be based on the quantum computing. Quantum computers are based on principles of the quantum physics that allows to increase the speed of the computations. Due to that, the energy of the carriers of the information in the quantum transistors –quasi-particles, is less than 1 eV, even any small local excitations can break the information through the transition of quasi-particles to another state. Therefore, researchers try to decode quantum information non-locally that leads to the study of topologically protected qubits.

The recently, charge density waves, observed experimentally [3,4,5] in atomic wires Au, In, Ge, have been obtained self-assembly of Si (553), Si (557), Ge (001) on vicinal surfaces. The latest development of the technology makes it possible to obtain wires nesting of single metallic atoms [2] on the dielectric substrate with an atomic width using a scanning tunneling microscope. In these quasi- one-dimensional structures distances between vicinal wires are 1.5-2 nanometers. The angle-resolved photoemission spectroscopy measurements of the electron dispersion show that these structures are one-dimensional with weak coupling between the wires. Modeling the quasi-one-dimensional structure according to the strong coupling approximation and utilizing characteristics of the Fermi contour getting in the experiment, it was obtained the value $t_{||}/t_{\perp}$ ~60 for [6] ratio of the longitudinal and transverse overlap integrals. This fact allows to neglect the coupling between the wires in structures of Au /Si (557), Au /Si (553) and Au /Ge (001). In all of these structures, Peierls instability and formation of charge density wave are observed at higher temperatures, 30K<T <270K.

Thus, in this paper I suggest that the formation of qubit can be obtained in a strong spin-orbit interaction material with the charge density wave of dielectric phase. The Majorana fermions are emerged only at the edges of a single wire by changing the external magnetic field in the quasi-one dimensional structure with a strong spin-orbit interaction.

2. DENSITY-WAVE ORDERING IN THE PRESENCE OF RASHBA AND DRESSELHAUS SPIN-ORBIT INTERACTIONS

The model considered here is essentially a 1D Hubbard model in the presence of both Rashba and Dresselhaus spin-orbit interactions and a Zeeman magnetic field. Hamiltonian of the system is given by the following form:

$$\widehat{H} = \widehat{H}_0 + \widehat{H}_{int} \tag{1}$$

where $\widehat{H}_0$ and $\widehat{H}_{int}$ are the noninteracting part of the Hamiltonian and the part expressing the correlation between electrons, correspondingly. $\widehat{H}_0$ reads in momentum space as:

$$\widehat{H}_0 = \sum_{0<k<\frac{G}{2}} \sum_{\sigma,\sigma'} \Big\{ \xi_k c^{+}_{k,\sigma} c_{k,\sigma'} \delta_{\sigma,\sigma'} + \omega_z c^{+}_{k,\sigma} (\sigma_x)_{\sigma,\sigma'} c_{k,\sigma'} + \alpha \sin(kd) c^{+}_{k,\sigma} (\sigma_z)_{\sigma,\sigma'} c_{k,\sigma'} + \beta \sin(kd) c^{+}_{k,\sigma} \left(\sigma_y\right)_{\sigma,\sigma'} c_{k,\sigma'} + (k \leftrightarrow k - \tfrac{G}{2}) \Big\} \tag{2}$$

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where α and β are constants of the Rashba and Dresselhaus spin-orbit interactions, $\omega_z = \frac{g\hbar\mu_B B}{2}$ is Zeeman energy of a magnetic field *B*, $\xi_k = -2t\cos k - \mu$ with $\epsilon_k = -2t\cos(kd)$, and μ- is the Fermi energy. $G = \frac{2\pi}{d}$ is the reciprocal lattice vector with *d* being the unit cell size. The interaction term $\widehat{H}_{int}$ in the Hamiltonian is written as:

$$\widehat{H}_{int} = \frac{1}{2N}\sum_{0<q<G}\sum_{\sigma}\{\sum_{k,k'} U(k,k',q)\, c^{+}_{k+q,\sigma}c_{k,\sigma}c^{+}_{k'-q,-\sigma}c_{k',-\sigma} + \sum_{n,n'} U(n,n',q)c^{+}_{n,\sigma}c_{n+q-G,\sigma}c^{+}_{n',-\sigma}c_{n'-q+G,-\sigma}\} \qquad (3)$$

where $k \in \left(-\frac{G}{2}, \frac{G}{2} - q\right)$, $k' \in \left(q - \frac{G}{2}, \frac{G}{2}\right)$ and $n \in \left(\frac{G}{2} - q, \frac{G}{2}\right)$, $n' \in \left(-\frac{G}{2}, q - \frac{G}{2}\right)$. *U* is a strength of the Hubbard interaction and *N* is the number of lattice sites.

The pole of the single particle Green's function determines the energy spectrum of quasi-particle excitations:

$$G^{-1}(E,k) = E - \widehat{\mathcal{H}} \qquad (4)$$

For charge density wave state, this energy is found as solution to the following matrix:

$$\begin{vmatrix} E-\xi_k-\alpha\sin k & -\Delta & -\omega_z-\beta\sin k & 0 \\ \Delta^* & E-\xi_{k-G/2}-\alpha\sin(k-G/2) & 0 & -\omega_z-\beta\sin(k-G/2) \\ -\omega_z+\beta\sin k & 0 & E-\xi_k+\alpha\sin k & -\Delta \\ 0 & -\omega_z+\beta\sin(k-G/2) & -\Delta^* & E-\xi_{k-G/2}+\alpha\sin(k-G/2) \end{vmatrix} = 0. \qquad (5)$$

Expressions for the energy spectrum can be written as

$$E^2_{YSD} = \xi_k^2 + \gamma^2\sin^2 k + |\Delta|^2 + \omega_z^2 \pm 2\sqrt{\xi_k^2\gamma^2\sin^2 k + \omega_z^2|\Delta|^2 + \xi_k^2\omega_z^2} \qquad (6)$$

where $\gamma = \sqrt{\alpha^2+\beta^2}$ is constant of spin-orbit coupling.

The energy spectrum at the center of the Brillouin zone for the topological charge density wave with gapped "bulk" states and zero energy end states can be written as

$$E(0) = \left|\omega_z - \sqrt{\mu_t^2 + |\Delta|^2}\right| \qquad (7)$$

where $\mu_t = -2t - \mu$.

The gap at *k* = 0 vanishes under this condition, with emerging Majorana fermion states at the ends of the wire, which is plotted in Fig. for the dimensionless parameters (a) γ=0.8, Δ=0.7, μ= - 0.1, ω= √1.7; (b) γ=0.8, Δ=0.7, μ= -0.3, ω=√2.18.

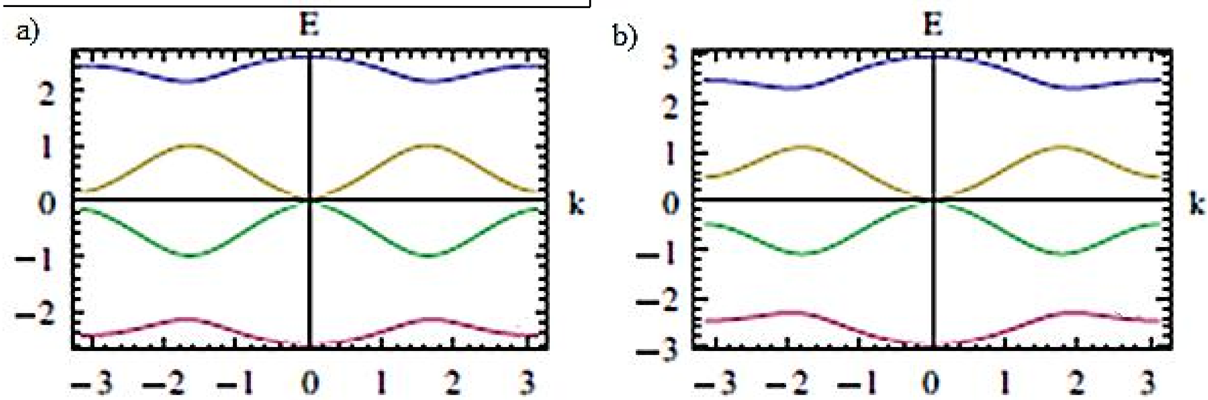

Fig.: The energy spectrum is plotted according to Eq. (6) for fixed values of *t* = 0.5 and for the following values of the dimensionless parameters: (a) γ=0.8, Δ=0.7, μ= - 0.1, ω= √1.7; (b) γ=0.8, Δ=0.7, μ= -0.3, ω=√2.18.

3. CONCLUSION

This paper demonstrates that a one-dimensional lattice in a CDW phase with strong spin-orbit interactions and a Zeeman magnetic field can support Majorana modes. The basic principle here is that the quantum topological order is realized in a 1D-wire with charge-density wave in the presence of spin-orbit interactions by tuning the external Zeeman energy. I show that for the Zeeman coupling below a critical value ($\omega_z^2 > \mu_t^2 + |\Delta|^2$), the system is a nontopological charge density wave semiconductor. However, above the critical value of the Zeeman field ($\omega_z^2 < \mu_t^2 + |\Delta|^2$), the lowest energy excited state is a zero-energy Majorana fermion state for topological CDW crystals.

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