TWO- AND THREE-PARTICLE DECAY CHANNELS OF SUPERSYMMETRIC HIGGS BOSONS

S.K. ABDULLAYEV, E.Sh. OMAROVA
23, Z. Khalilova str., Baku State University,
Baku, Azerbaijan, AZ 1148
[a]sabdullayev@bsu.edu.az, [b]emiliya.abdullayeva@inbox.ru

In the framework of the Minimal Supersymmetric Standard Model we investigated the two and three- particle decay channels of Higgs bosons: $H \to W^- W^+$, $H \to Z^0 Z^0$, $A \to hZ^0$, $H^+ \to hW^\pm$, $H \to hh$, $H \to hh$, $H \to hb\bar{b}$, $H \to Z^0 Ah$, $A \to Z^0 hh$, $H^\pm \to W^\pm hh$, $H(A) \to t\bar{b}W^-$. Analytical expressions for the amplitudes and probabilities of the corresponding decays are obtained and the dependence of the widths of the decays on the mass of the Higgs bosons are studied. The calculation results are illustrated by graphs, which the widths of the decays to the Higgs boson mass are very sensitive. In some processes, the width of the decay increases with the increasing mass of the Higgs boson, while in others, on the contrary, it decreases.

Keywords: Standard Model, Minimal Supersymmetric Standard Model, Higgs boson, decay width, coupling constant.
PACS: 12.15-y, 12.15 Mm, 14.70 Hp, 14.80 Bn

INTRODUCTION

The Standard Model (SM) based on local gauge symmetry $SU_C(3) \times SU_L(2) \times U_Y(1)$ and well describes the strong and electroweak interactions of the physics of quarks, leptons and gauge bosons [1-4]. In this case, quarks are triplets and leptons are singlets of the color group $SU_C(3)$, the left components of quarks and leptons are doublets of the group $SU_L(2)$, and the right components are singlets, and they all have a hypercharge according to the group $U_Y(1)$. The discovery of the Higgs boson in 2012 by the ATLAS and CMS collaborations [5, 6] (see also reviews [7–10]) has begun a new stage in the study of the properties of fundamental interactions.The mechanism of fundamental particles of mass generation – the mechanism of spontaneous breaking of the local gauge symmetry of Brout - Englert - Higgs [11, 12] was experimentally confirmed. Thus, the SM received a logical conclusion and acquired the status of a standard theory.

A doublet of scalar complex fields $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$ is introduced in the SM, the neutral component has a nonzero vacuum value. As a result, the electroweak group $SU_L(2) \times U_Y(1)$ is spontaneously broken up to the electromagnetic group $U_Q(1)$. In this case, three of the four components of the scalar field φ are absorbed by the $W^\pm$- and Z^0-vector bosons, and the remaining fourth neutral component of the scalar field is the Higgs boson $H_{\rm SM}$. In the first experiments conducted at the Large Hadron Collider (LHC), the main properties of this particle were established. The Higgs boson is a scalar particle with spin zero, possessing positive parity, mass about 125 GeV, interacting with $W^\pm$- and Z^0-bosons, and also quarks and leptons with a constant proportional to their masses. With the discovery of the Higgs boson, interest in various channels of production and decay has greatly increased. The various properties of the Higgs boson have been studied in a number of papers [4,13-19].

Note that along with the SM, other alternative models are widely discussed in the literature. One of the extensions of the SM is the two-doublet Higgs model 2 HDM [20, 21]. Another alternative model is the Minimal Supersymmetric Standard Model (MSSM) [22-26]. In this model, two doublets of the scalar field are introduced with opposite hypercharges –1 and +1:

$$\varphi_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

To obtain the Higgs boson physical fields, φ_1 and φ_2 are written as

$$\varphi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \upsilon_1 + H_1^0 + iP_1^0 \\ H_1^- \end{pmatrix},$$

$$\varphi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} H_2^+ \\ \upsilon_2 + H_2^0 + iP_2^0 \end{pmatrix}$$

Here H_1^0, P_1^0, H_2^0 and P_2^0 are the fields describing the system excitations with respect to the vacuum states $\langle \varphi_1 \rangle = \frac{1}{\sqrt{2}} \upsilon_1$ and $\langle \varphi_2 \rangle = \frac{1}{\sqrt{2}} \upsilon_2$. The CP-even Higgs bosons of the H - and h - bosons are obtained by mixing the fields H_1^0 and H_2^0 (mixing angle α):

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}$$

131, H. Javid ave., AZ-1143, Baku
ANAS, G.M. Abdullayev Institute of Physics
E-mail: jophphysics@gmail.com

Similarly, the fields P_1^0 and P_2^0 are mixed, and also $H_1^\pm$ and $H_2^\pm$, get a CP-odd Higgs boson A and charged Higgs bosons H^+ and H^- (mixing angle β):

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} P_1^0 \\ P_2^0 \end{pmatrix},$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix}$$

where G^0 and $G^\pm$ are neutral and charged massless bosons. Therefore, after spontaneous symmetry breaking, five Higgs particles appear in the MSSM: CP-even h- and H-bosons, CP-odd A-boson, and charged $H^\pm$-bosons.

In the MSSM, the Higgs sector is characterized by six parameters $M_h, M_H, M_A, M_{H^\pm}, \alpha$ and β. Of these, only two parameters are free, and $tg\beta$ and M_A are usually chosen as such parameters. The parameter $tg\beta$ is equal to the ratio $\frac{\upsilon_2}{\upsilon_1}$ and varies within

$$1 \le \tan\beta \le \frac{m_t}{m_b} = 35{,}5,$$

where $m_t = 173{,}2$ GeV and $m_b = 4{,}88$ GeV are the masses of t- and b-quarks.

The masses of neutral h- and H-(charged $H^\pm$-) bosons are expressed in masses M_A and M_Z (M_A and M_W):

$$M^2_{H(h)} = \\ = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta} \right],$$

$$M^2_{H^\pm} = M_A^2 + M_W^2.$$

It follows that between the masses of supersymmetric Higgs bosons there is a relationship

$$M_h < M_A < M_H < M_{H^\pm}$$

The mixing angles of fields α and β are related by:

$$\tan 2\alpha = \tan 2\beta \, \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad \left(-\frac{\pi}{2} \le \alpha < 0 \right).$$

The Higgs bosons of the MSSM can decay through various channels (see [25,27-29] and there are references to primary sources). In previous works [30–32], we considered the Higgs boson decay channels of the MSSM into an arbitrarily polarized fermion pair $H(h,A) \to f + \bar{f}$, $H^+ \to f + \bar{f}'$ and also three-particle $H(A) \to t + \bar{b} + W^-$, $H^\pm \to b + \bar{b} + W^\pm$, $H \to Z^0 + f + \bar{f}$, $H \to W^\pm + f + \bar{f}'$ decays.

In this paper, we study some two and three-particle decay channels for supersymmetric Higgs bosons:

$$H \to W^- + W^+, \; H \to Z^0 + Z^0 \tag{1}$$

$$A \to Z^0 + h, \; H^\pm \to W^\pm + h \tag{2}$$

$$H \to h + h, \; H \to h + b + \bar{b} \tag{3}$$

$$A \to Z^0 + h + h, \; H^\pm \to W^\pm + h + h \tag{4}$$

$$H(A) \to H^{+*} + W^- \to t + \bar{b} + W^-, \quad H \to W^{+*} + W^- \to t + \bar{b} + W^- \tag{5}$$

Analytical expressions are obtained for the amplitudes and probabilities of the corresponding decays, and the dependence of the decay width on the Higgs boson mass is studied.

2. The decays of $H \to W^- W^+$ and $H \to Z^0 Z^0$

First, consider the decay of the heavy Higgs H boson into gauge bosons $W^- W^+$ ($Z^0 Z^0$). This process is described by the Feynman diagram shown in fig. 1 a (4-particle momenta are written in brackets). To this diagram corresponds the amplitude:

$$M(H \to VV) = i g_{HVV} H(p) \cdot U^*_\mu(p_1) \cdot U^*_\mu(p_2) \tag{6}$$

where g_{HVV} is the Higgs boson coupling constant H with gauge VV bosons (expressions of various coupling constants are given in [25]), $H(p)$ is the H-boson wave function normalized to unity, $U^*_\mu(p_1)$ and $U^*_\mu(p_2)$ are 4 polarization vectors of gauge $W^- W^+$ ($Z^0 Z^0$)-bosons.

For the decay width $H \to V + V$, summed over the polarization states of the vector bosons, the expression is obtained:

$$\Gamma(H \to VV) = \frac{G_F M_H^3}{8\sqrt{2}\pi} \cdot \cos^2(\beta - \alpha)(1 - 4r_V + 12 r_V^2)\sqrt{1 - 4r_V} \cdot \delta_V. \tag{7}$$

The $r_V = \left(\frac{M_V}{M_H} \right)^2$ designation is entered here and it is taken into account

$$g^2_{HVV} = 4\sqrt{2} G_F M_V^4 \cos^2(\beta - \alpha),$$

G_F is the Fermi constant of weak interactions a $\delta_W = 1$ in the case of the production of charged $W^- W^+$ bosons, $\delta_Z = \frac{1}{2}$ at the production of neutral Z^0-bosons.

Figure 2 shows the dependence of the decay widths $H \to W^- W^+$ and $H \to Z^0 Z^0$ on the Higgs mass of the M_H boson at $tg\beta = 3$ and $M_W = 80{,}425$

GeV, and $M_Z = 91{,}1875$ GeV. As can be seen, with increasing Higgs boson mass, the decay width monotonously increases, the decay width $\Gamma(H \to W^- W^+)$ prevails over the decay width $\Gamma(H \to Z^0 Z^0)$.

As is known, a massive vector particle is characterized by three independent polarization vectors: two transverse (with respect to the k 3-momentum of the particle) and one longitudinal:

$$U_\mu^{(1)} = (0,1,0,0)\ ,\ U_\mu^{(2)} = (0,0,1,0)\ ,\ U_\mu^{(L)} = \left(\frac{|\vec{k}|}{M_W}, 0, 0, \frac{E}{M_W} \right) ,$$

$$U_\mu^{(i)} \cdot k_\mu = 0\ ,\ k_\mu = (E, 0, 0, |\vec{k}|)\ ,\ (U_\mu^{(i)})^2 = -1\ , \tag{8}$$

where E and $\vec{k}$ are the energy and the 3-momentum of the vector boson, the Z axis is chosen along the momentum $\vec{k}$. It can be seen from (8) that the components of the 4 -vector of longitudinal polarization $U_\mu^{(L)}$ turn out to grow with an increase in the energy of the vector boson. Therefore, the decay width of the formation of longitudinally polarized $W^- W^+$ ($Z^0 Z^0$) -bosons will increase with increasing mass M_H.

The width of the Higgs boson decay into a pair of longitudinally polarized vector bosons is given by the formula:

$$\Gamma(H \to V_L V_L) = \frac{G_F M_H^3}{8\sqrt{2}\pi} \cos^2(\beta - \alpha)(1 - 4r_V + 4r_V^2)\sqrt{1 - 4r_V}\,\delta_V \tag{9}$$

This means that for large Higgs masses of the M_H boson, vector bosons are predominantly polarized longitudinally. Consider the decay width ratio

$$\frac{\Gamma(H \to V_L V_L)}{\Gamma(H \to VV)} = \frac{1 - 4r_V + 4r_V^2}{1 - 4r_V + 12r_V^2}. \tag{10}$$

Figure 3 illustrates the dependence of the $\Gamma(H \to V_L V_L)/\Gamma(H \to VV)$ ratio on the mass of M_H. As can be seen, with increasing M_H mass, this ratio increases and at large M_H approaches 1.

3. The decays of $A \to Z^0 h$ and $H^\pm \to W^\pm h$

The Feynman diagram of the decay of the pseudoscalar A -boson into the vector Z^0 -boson and the scalar h -boson is shown in fig. 1 b) and to this diagram corresponds the amplitude

$$M(A \to Z^0 h) = i g_{AZh} A(p) \cdot h^*(p_1) \cdot U_\mu^*(p_2) R_\mu \tag{11}$$

where g_{AZh} is the coupling constant, $A(p)$ and $h^*(p_1)$ are the wave functions of A and h -bosons normalized to unity, $U_\mu^*(p_2)$ is the 4-polarization vector of the Z^0 boson, $R_\mu = (p + p_1)_\mu$ is the sum of 4-impulses of A and h -bosons. Based on the matrix element (11), the expression for the decay width $A \to Z^0 h$ is obtained:

$$\Gamma(A \to Z^0 h) = \frac{G_F}{8\sqrt{2}\pi M_A^3} \cos^2(\beta - \alpha)[(M_A^2 - M_Z^2 - M_h^2)^2 - 4M_Z^2 M_h^2]^{3/2} \tag{12}$$

The Feynman decay diagram of the $H^\pm \to W^\pm h$ is similar to the diagram in fig. 1 (c) and we write the corresponding matrix element as follows:

$$M(H^\pm \to W^\pm h) = \pm i g_{H^\pm W^\pm h} H^\pm(p) \cdot h^*(p_1) \cdot U_\mu^*(p_2) R_\mu \tag{13}$$

Having performed the calculations according to the usual MSSM rules, for the decay width $H^\pm \to W^\pm h$ we have the following expression

$$\Gamma(H^\pm \to W^\pm h) = \frac{G_F}{8\sqrt{2}\pi M_{H^\pm}^3} \cos^2(\beta - \alpha)[(M_{H^+}^2 - M_h^2 - M_W^2)^2 - 4M_W^2 M_h^2]^{3/2}\ . \tag{14}$$

Figure 4 shows the dependence of the decay width $A \to Z^0 h$ and $H^\pm \to W^\pm h$ on the mass M_A and $M_{H^\pm}$ at $tg\beta = 3$, $M_Z = 91{,}1875$ GeV and $M_W = 80{,}425$ GeV. As the mass of the Higgs boson $M_A(M_{H^\pm})$ increases, the width of the decay $\Gamma(A \to Z^0 h)(\Gamma(H^\pm \to W^\pm h))$ increases.

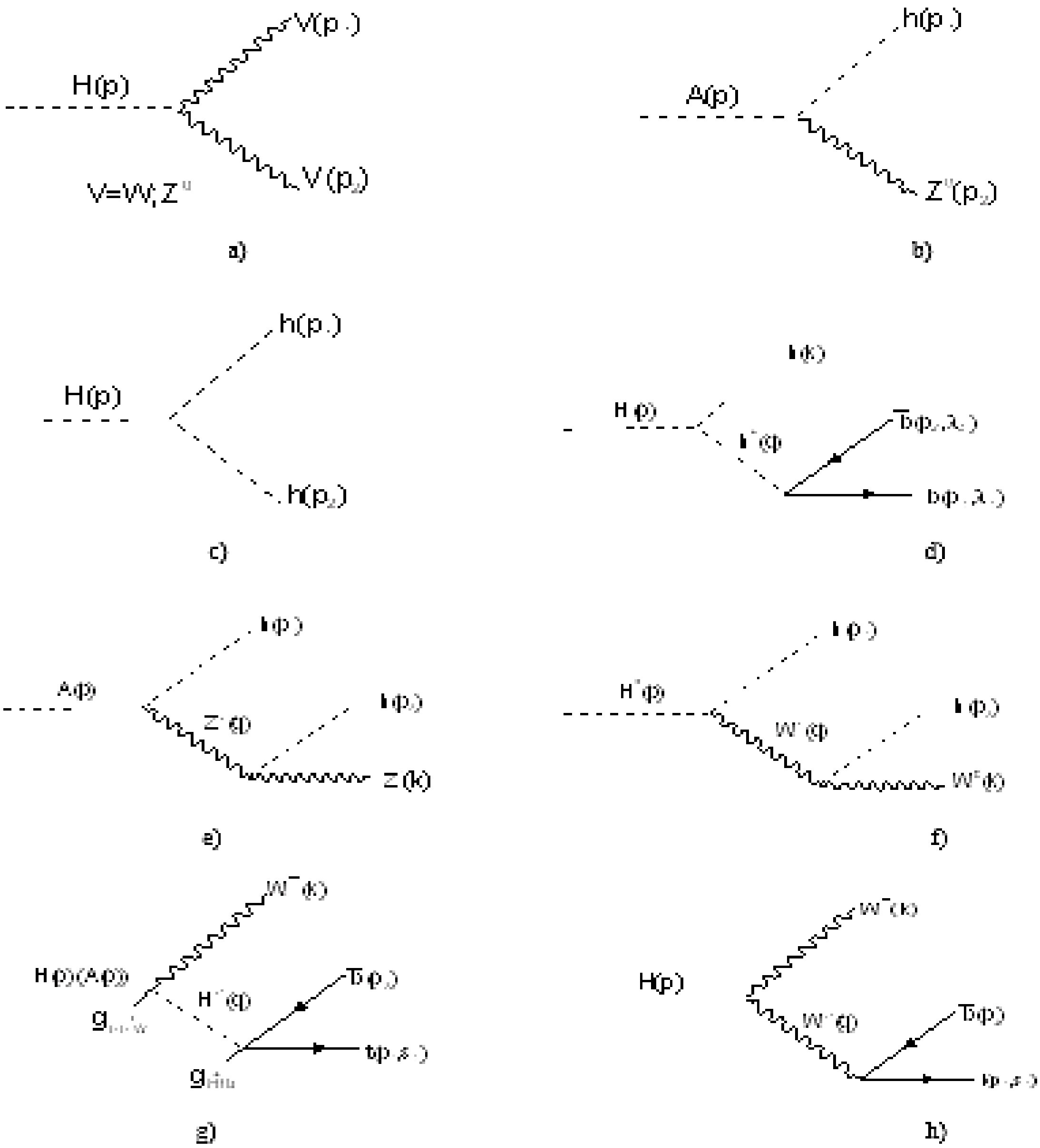

Fig. 1. Feynman diagrams for various Higgs boson decay channels

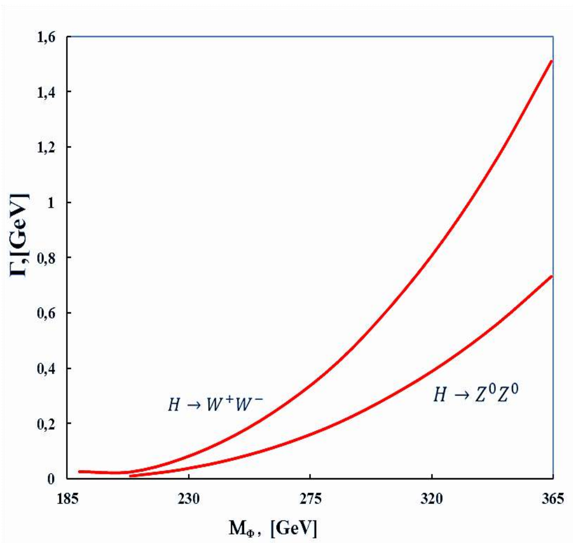

Fig. 2. Dependence of the decay width $\Gamma(H \to W^-W^+)$ and $\Gamma(H \to Z^0Z^0)$ on the mass M_H at $tg\beta = 3$

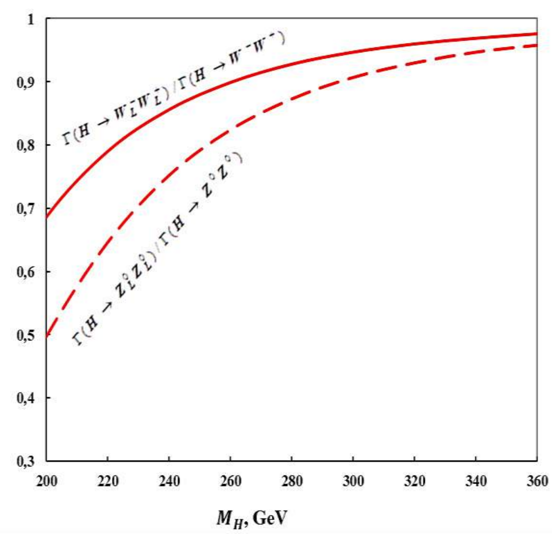

Fig.3. Dependence of $\Gamma(H \to W_L^- W_L^+)/\Gamma(H \to W^-W^+)$ and $\Gamma(H \to Z_L^0 Z_L^0)/\Gamma(H \to Z^0Z^0)$ ratios on M_H

4. The decays of $H \to hh$ and $H \to hb\bar{b}$

One of the possible channels for the decay of a heavy Higgs boson is its decay into two light h -bosons: $H \to h + h$. The diagram of this decay is shown in fig. 1 (c). The amplitude of this decay can be written as:

$$M(H \to hh) = ig_{Hhh} H(p) \cdot h^*(p_1) \cdot h^*(p_2) \qquad (15)$$

The decay width of this process is:

$$\Gamma(H \to hh) = \frac{G_F}{16\sqrt{2}\pi} \frac{M_Z^4}{M_H} \sqrt{1 - 4r_h} \cdot g_{Hhh}^2 \qquad (16)$$

where $r_h = \left(\frac{M_h}{M_H}\right)^2$. Figure 5 shows the dependence of the decay width $H \to h + h$ on the mass M_h for two values of the parameter $tg\beta$. In both cases, the decay width monotonically decreases with increasing mass of the Higgs boson M_H.

When $M_h < M_H \le 2M_h$, the Higgs boson H decays through the $H \to h + h^* \to h + b + \bar{b}$ channel, the Feynman diagram of which is shown in fig. 1 d) (h^* -virtual Higgs boson). The amplitude of this decay is

$$M(H \to hb\bar{b}) = ig_{Hhh} \cdot g_{hbb} \frac{H(p) \cdot h^*(k)}{(p_1 + p_2)^2 - M_h^2 + iM_h \Gamma_h} \bar{u}(p_1, \lambda_1) \cdot \upsilon(p_2, \lambda_2) \qquad (17)$$

here g_{Hhh} and g_{hbb} are the corresponding coupling constants, λ_1 and λ_2 are the helicities of the b -quark and $\bar{b}$ -antiquark, M_h and Γ_h are the mass and width of the decay of the h -boson.

We introduce the scaling energies of the b -quark $x_1 = \frac{2E_1}{M_H}$, $\bar{b}$ - antiquark $x_2 = \frac{2E_2}{M_H}$ and h - boson $x_h = \frac{2E_h}{M_H} = 2 - x_1 - x_2$, as well as the notation $r_b = \left(\frac{m_b}{M_H}\right)^2$, $r_h = \left(\frac{m_h}{M_H}\right)^2$, $\gamma_h = \left(\frac{\Gamma_h}{M_H}\right)^2$. Then for the decay width $H \to h + b + \bar{b}$ we have the expression:

$$\frac{d\Gamma(H \to hb\bar{b})}{dx_1 dx_2} = \frac{N_C G_F^2 M_Z^4 m_b^2}{64\pi^3 M_H} \cdot \frac{\sin^2\alpha}{\cos^2\beta} \cdot \lambda_{Hhh}^2 \cdot (1 + \lambda_1 \lambda_2) \frac{1 - x_h + r_h - 2r_b}{(1 - x_h)^2 + r_h \gamma_h} \qquad (18)$$

Here it is taken into account

$$g_{Hhh}^2 = 4\sqrt{2} G_F M_Z^4 \lambda_{Hhh}^2, \quad \lambda_{Hhh} = 2\sin 2\alpha \sin(\beta + \alpha) - \cos 2\alpha \cos(\beta + \alpha), \quad g_{hbb}^2 = \sqrt{2} G_F m_b^2 \cdot \frac{\sin^2\alpha}{\cos^2\beta}$$

From the decay width (18) it follows that b -quark and $\bar{b}$ -antiquark must have the same helicity: $\lambda_1 = \lambda_2 = \pm 1$, or they are polarized right ($b_R \bar{b}_R$), or left ($b_L \bar{b}_L$). This is due to the law of conservation of the total moment in the transition $h^* \to b + \bar{b}$ .The decay width $H \to h + b + \bar{b}$, summed over the spin states of quarks, has the form (Dalitz distribution):

$$\frac{d\Gamma(H \to hb\bar{b})}{dx_1 dx_2} = \frac{N_C G_F^2 M_Z^4 m_b^2}{16\pi^3 M_H} \cdot \frac{\sin^2\alpha}{\cos^2\beta} \cdot \lambda_{Hhh}^2 \cdot \frac{x_1 + x_2 - 1 + r_h - 2r_b}{(1 - x_1 - x_2)^2 + r_h \gamma_h} \qquad (19)$$

The quark scaling energies vary within $1 - x_2 - r_h < x_1 < 1 - \frac{r_h}{1 - x_2}$, $0 < x_2 < 1 - r_h$, then the integration of the Dalitz distribution density (19) over these variables can be performed analytically. As a result, the expression for the decay width $H \to h + h^* \to h + b + \bar{b}$ is obtained (with $r_b = \gamma_b = 0$):

$$\Gamma(H \to hb\bar{b}) = \frac{3G_F^2}{16\pi^3} \cdot \frac{M_Z^4}{M_H} m_b^2 \lambda_{Hhh}^2 \frac{\sin^2\alpha}{\cos^2\beta} \times \left[(1 - r_h)\left(2 - \frac{1}{2}\ln r_h\right) - \frac{1 - 5r_h}{\sqrt{1 - 4r_h}} \left(arctg \frac{2r_h - 1}{\sqrt{1 - 4r_h}} - arctg \frac{1}{\sqrt{1 - 4r_h}} \right) \right]. \qquad (20)$$

Figure 6 illustrates the dependence of the width of the decay $\Gamma(H \to hb\bar{b})$ on the mass of the Higgs boson M_H at $tg\beta = 3$. As can be seen, with an increase in the Higgs boson mass, the width of the decay $H \to h + b + \bar{b}$ first increases and reaches a maximum at $M_H = 144{,}62$ GeV, and then the width of this decay slowly decreases.

5. The decays of $A \to Z^0 hh$ and $H^{\pm} \to W^{\pm} hh$

We note that not only with the two-particle Higgs boson decays $A \to Z^0 + h$ and $H^{\pm} \to W^{\pm} + h$, but also three-particle $A \to Z^0 + h + h$ and $H^{\pm} \to W^{\pm} + h + h$ decays are possible. In this case, the additional h-boson is emitted by the vector $Z^0 (W^{\pm})$-boson (see fig. 1 (e) and f)), where the Feynman diagrams of these decays are presented.).

According to the MSSM rules, the decay of $A \to Z^0 + h + h$ corresponds to the amplitude

$$M(A \to Zhh) = ig_{AZh} \cdot g_{ZZh} \cdot R_{\mu} \left(-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{M_Z^2} \right) \cdot \frac{U_{\nu}^*(k)}{q^2 - M_Z^2 + iM_Z \Gamma_Z} \qquad (21)$$

where g_{AZh} and g_{ZZh} are the corresponding coupling constants, $U_{\nu}^*(k)$ is the 4-vector of the Z^0-boson polarization.

On the basis of (21) for the square of the matrix element of the decay $A \to Z^0 + h + h$ we get the expression:

$$\left| M(A \to Zhh) \right|^2 = \frac{g_{AZh}^2 \cdot g_{ZZh}^2}{(q^2 - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \cdot R_{\mu} R_{\alpha} \left(-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{M_Z^2} \right) \cdot \left(-g_{\alpha\beta} + \frac{q_{\alpha} q_{\beta}}{M_Z^2} \right) \cdot \left(-g_{\beta\nu} + \frac{k_{\beta} k_{\nu}}{M_Z^2} \right) =$$

$$= 2G_F^2 M_Z^4 \sin^2(\beta - \alpha) \cos^2(\beta - \alpha) \cdot \frac{f_Z}{(1 - x_1 + r_h - r_Z)^2 + r_Z \gamma_Z} \qquad (22)$$

Here is the function

$$f_Z = -4r_Z (1 + x_1 + r_h)^2 + 4(1 - r_h)^2 \left[2 - \frac{1}{r_Z}(1 - x_1 + r_h) + \frac{1}{4r_Z^2}(1 - x_1 + r_Z)^2 \right] +$$
$$+ (1 + x_Z - x_2 - r_Z)^2 - \frac{2}{r_Z}(1 - r_h)(1 + x_Z - x_2 - r_Z)(1 - x_1 - r_Z) \qquad (23)$$

$x_1 = \frac{2E_1}{M_A}$, $x_2 = \frac{2E_2}{M_A}$ and $x_Z = \frac{2E_Z}{M_A} = 2 - x_1 - x_2$ are the scaling energies of the h-bosons and the Z^0-boson, $r_h = \left(\frac{M_h}{M_A} \right)^2$, $r_Z = \left(\frac{M_Z}{M_A} \right)^2$, $\gamma_Z = \left(\frac{\Gamma_Z}{M_A} \right)^2$ and it is taken into account that according to the MSSM $g_{AZh}^2 \cdot g_{ZZh}^2 = 2G_F^2 M_Z^4 \sin^2(\beta - \alpha) \cos^2(\beta - \alpha)$.

The width of the decay $A \to Z^0 + h + h$ is directly proportional to the square of the amplitude (22)

$$\frac{d\Gamma(A \to Z^0 hh)}{dx_1 dx_2} = \frac{\left| M(A \to Z^0 hh) \right|^2}{2^8 \pi^3} M_A = \frac{G_F^2 M_Z^4 M_A}{2^7 \pi^3} \cdot \sin^2(\beta - \alpha) \cos^2(\beta - \alpha) \cdot \frac{f_Z}{(1 - x_1 + r_h - r_Z)^2 + r_Z \gamma_Z}. \qquad (24)$$

Consider the width of the decay $A \to Z^0 + h + h$, normalized to the width of the usual decay $A \to Z^0 + h$:

$$\frac{1}{\Gamma(A \to Z^0 h)} \cdot \frac{d\Gamma(A \to Z^0 hh)}{dx_1 dx_2} = \frac{G_F M_Z^4}{8\sqrt{2}\pi^2 M_A^2} \sin^2(\beta - \alpha) \cdot \frac{f_Z}{(1 - x_1 + r_h - r_Z)^2 + r_Z \gamma_Z} \cdot [(1 - r_Z - r_h)^2 - 4r_Z r_h]^{-3/2} \qquad (25)$$

Figure 7 shows the dependence of the normalized decay width (25) on the scaling energy x_1 at a fixed $x_2 = 0{,}5$, $M_A = 220$ GeV, $tg\beta = 3$, $M_Z = 91{,}1875$ GeV. As can be seen, with an increase in the scaling energy x_1, the normalized decay width $\frac{1}{\Gamma(A \to Z^0 h)} \cdot \frac{d\Gamma(A \to Z^0 hh)}{dx_1 dx_2}$ decreases.

The decay of $H^{\pm} \to W^{\pm} + h + h$ corresponds to the diagram shown in fig. 1 (f) and the amplitude of this decay can be written as:

$$M(H^{\pm} \to W^{\pm} hh) =$$
$$ig_{H^{\pm}W^{\pm}h} \cdot g_{WWh} \cdot R_{\mu} \left(-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{M_W^2} \right) \cdot \frac{U_{\nu}^*(k)}{q^2 - M_W^2 + iM_W \Gamma_W} \qquad (26)$$

where $g_{H^{\pm}W^{\pm}h}$ and g_{WWh} are the corresponding coupling constants, $U_{\nu}^*(k)$ is the 4-polarization vector of the W-boson.

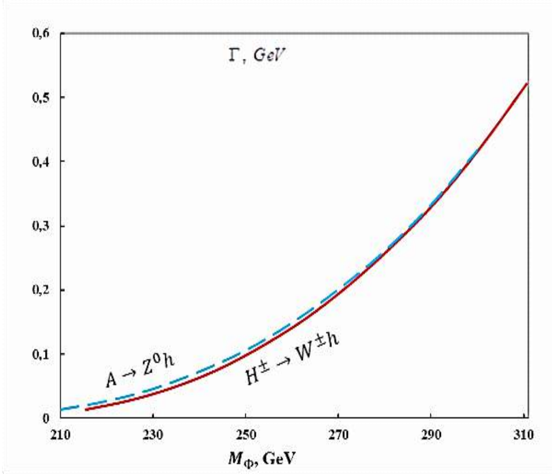

Fig.4. The dependence of the width of the decay $A \to Z^0 h$ $(H^{\pm} \to W^{\pm} h)$ on the mass $M_A(M_{H^{\pm}})$

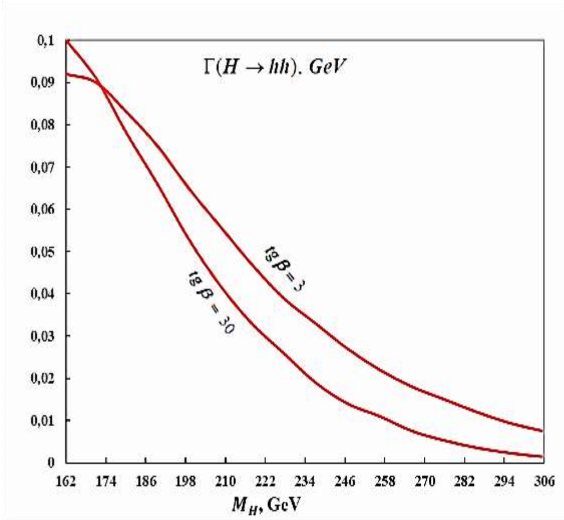

Fig. 5. Dependence of the decay width $H \to hh$ on the mass M_H

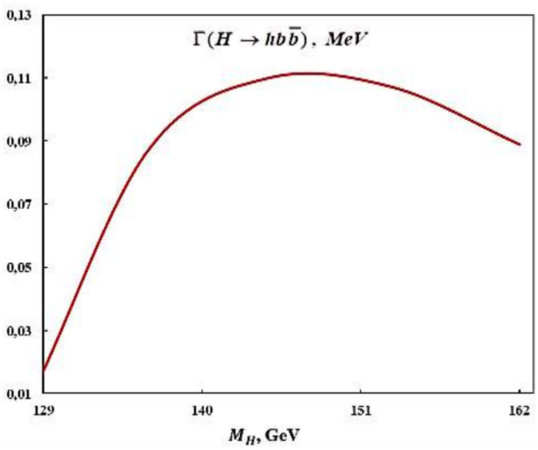

Fig. 6. Dependence of the decay width $H \to hb\bar{b}$ on the mass M_H

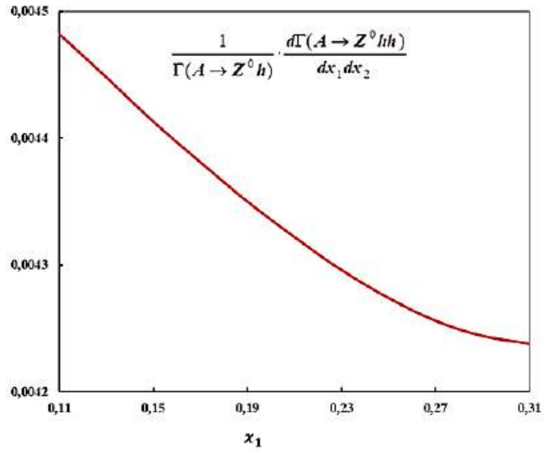

Fig .7. Dependence of the ratio $\frac{1}{\Gamma(A \to Z^0 h)} \cdot \frac{d\Gamma(A \to Z^0 hh)}{dx_1 dx_2}$ on x_1

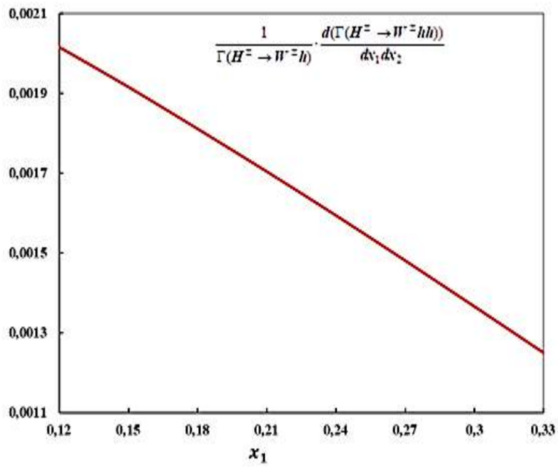

Fig.8. Dependence of the ratio $\frac{1}{\Gamma(H^{\pm} \to W^{\pm} + h)} \cdot \frac{d\Gamma(H^{\pm} \to W^{\pm} hh)}{dx_1 dx_2}$ on x_1.

The distribution density of Dalits in the decay of $H^{\pm} \to W^{\pm} + h + h$ is expressed by the formula

$$\frac{d\Gamma(H^{\pm} \to W^{\pm} hh)}{dx_1 dx_2} = \frac{G_F^2 M_W^4 M_{H^{\pm}}}{2^7 \pi^3} \cdot \sin^2(\beta - \alpha) \cdot \frac{f_W}{(1 - x_1 + r_h - r_W)^2 + r_W \gamma_W}, \quad (27)$$

where the function f_W is obtained from the function f_Z by replacing $r_Z \Rightarrow r_W$.

The width of the decay $H^{\pm} \to W^{\pm} + h + h$ is normalized to the width of the usual decay $H^{\pm} \to W^{\pm} + h$:

$$\frac{1}{\Gamma(H^{\pm} \to W^{\pm} + h)} \cdot \frac{d\Gamma(H^{\pm} \to W^{\pm} hh)}{dx_1 dx_2} = \frac{G_F M_W^4}{8\sqrt{2}\pi^2 M_{H^{\pm}}^2} \sin^2(\beta - \alpha) \times$$

$$\times \frac{f_W}{(1 - x_1 + r_h - r_W)^2 + r_W \gamma_W} \cdot [(1 - r_W - r_h)^2 - 4 r_W r_h]^{-3/2} \quad (28)$$

Figure 8 illustrates the dependence of the normalized decay width (28) on the scaling energy x_1 at a fixed $x_2 = 0{,}5$, $M_{H^\pm} = 200$ GeV, $tg\beta = 3$, $M_W = 80{,}425$ GeV. Here, a monotonous decrease of the normalized decay width is observed with increasing scaling energy x_1 .

6. The decays of $H(A) \to t\bar{b}W^-$

One of the interesting channels for the Higgs boson decay is $H(A) \to t + \bar{b} + W^-$. This decay can occur through various virtual states. The decay channel according to the scheme $H(A) \to t + \bar{t}^* \to t + \bar{b} + W^-$ was investigated by us in [32]. These decays can also occur through the virtual charged Higgs boson $H(A) \to H^{+*} + W^- \to t + \bar{b} + W^-$. In addition, the scalar Higgs boson H can also decay according to the scheme $H(A) \to W^{+*} + W^- \to t + \bar{b} + W^-$. We note that the decay of the pseudoscalar A-boson according to the scheme $A \to W^{+*} + W^-$ is forbidden by the law of conservation of CP parity.

Now let us consider the decay of the $H(A)$-boson via the $H(A) \to H^{+*} + W^- \to t + \bar{b} + W^-$ channel, the Feynman diagram which is shown in fig. 1 (g). The amplitude of this decay is written as follows:

$$M(H \to W^- H^{+*}) = i g_{HH^+W} \cdot U^*_\mu(k) \cdot g_{H^+tb} \cdot R_\mu \cdot \frac{U^*_{tb}}{q^2 - M^2_{H^+} + iM_{H^+}\Gamma_{H^+}} \times$$
$$\times \bar{u}(p_1, s_t)[m_b tg\beta(1-\gamma_5) + m_t ctg\beta(1+\gamma_5)]\upsilon(p_2) \quad (29)$$

where U_{tb} is an element of the Kobayashi-Maskawa matrix, g_{HH^+W} and g_{H^+tb} are the corresponding coupling constants, s_t is the 4-polarization vector of the t-quark. For the square of the amplitude (29) the expression is obtained

$$\left|M(H \to W^- H^{+*})\right|^2 = 4 g^2_{HH^+W} g^2_{H^+tb} \frac{|U_{tb}|^2}{(q^2 - M^2_{H^+})^2 + M^2_{H^+}\Gamma^2_{H^+}} \cdot \left(-R^2 + \frac{(R\cdot k)^2}{M_W^2}\right) \cdot [(p_1 \cdot p_2)(m_b^2 tg^2\beta + m_t^2 ctg^2\beta) -$$
$$-2m_t^2 m_b^2 - m_t(p_2 \cdot s_t)(m_b^2 tg^2\beta - m_t^2 ctg^2\beta)] \quad (30)$$

We carry the scaling energies of the t-quark $x_t = \frac{2E_t}{M_H}$, $\bar{b}$-antiquark $x_b = \frac{2E_b}{M_H}$ and W^--boson $x_W = \frac{2E_W}{M_H} = 2 - x_t - x_b$, the scaling masses $r_b = \left(\frac{m_b}{M_H}\right)^2$, $r_t = \left(\frac{m_t}{M_H}\right)^2$ and $r_{H^+} = \left(\frac{M_{H^+}}{M_H}\right)^2$.

The width of the Higgs boson decay in the $H \to t + \bar{b} + W^-$ channel is expressed by the formula

$$d\Gamma(H \to tbW^-) = \frac{(2\pi)^4}{2M_H} \cdot \left|M(H \to tbW^-)\right|^2 N_C \frac{d\vec{k}}{(2\pi)^3 2E_W} \cdot \frac{d\vec{p}_1}{(2\pi)^3 2E_t} \cdot \frac{d\vec{p}_2}{(2\pi)^3 2E_b} \cdot \delta(p - p_1 - p_2 - k) \quad (31)$$

We define the spectrum of quarks in the case of a longitudinally polarized t-quark (the helicity of the t-quark is denoted by λ_t). For this, we need to take the integral over the phase volume of the W^--boson. Then the expression for the decay width will be:

$$d\Gamma(H \to tbW^-) = \frac{N_C}{16M_H(2\pi)^5} \cdot \int \frac{\overline{\left|M(H \to tbW^-)\right|^2}}{E_W E_t E_b} d\vec{p}_1 \, d\vec{p}_2 \cdot \delta(M_H - E_t - E_b - E_W) \quad (32)$$

where the bar above the square of the matrix element means that, by the polarizations of the antiquark $\bar{b}$ is summed. Now, by integrating the angles of departure of the t and $\bar{b}$ quarks, for the decay width $H \to t + \bar{b} + W^-$ we obtain the expression

$$\frac{d\Gamma(H \to t\bar{b}W^-)}{dx_t dx_b} = \frac{N_C G_F^2 M_H^5}{128\pi^3} \cdot \frac{|U_{tb}|^2 (x_W^2 - 4r_W)}{(1 - x_W + r_W - r_{H^+})^2 + r_{H^+}\gamma_{H^+}} \cdot \sin^2(\beta - \alpha) \cdot (f_1 + \lambda_t f_2), \quad (33)$$

Here $\gamma_{H^+} = \left(\frac{M_{H^+}}{M_H}\right)^2$ and functions are entered

$$f_1 = (r_t ctg^2\beta + r_b tg^2\beta)(1 - x_W + r_W - r_t - r_b) - 4r_t r_b,$$

$$f_2 = \frac{1}{4}(r_t ctg^2\beta - r_b tg^2\beta)\left[x_b\sqrt{x_t^2 - 4r_t} - \frac{x_t}{\sqrt{x_t^2 - 4r_t}}(2(1 - x_t - x_b - r_W + r_t + r_b) + x_t x_b)\right]$$

If we neglect the antiquark mass m_b, then the decay width $H \to t + \bar{b} + W^-$ is greatly simplified ($\gamma_{H^+} = 0$):

$$\frac{d\Gamma(H \to t\bar{b}W^-)}{dx_t dx_b} = \frac{1}{2}\frac{d\Gamma_0(H \to t\bar{b}W^-)}{dx_t dx_b} \cdot (1 + \lambda_t P_t). \qquad (34)$$

Here

$$\frac{d\Gamma_0(H \to t\bar{b}W^-)}{dx_t dx_b} = \frac{N_C G_F^2 m_t^2 M_H^3}{64\pi^3} \sin^2(\beta - \alpha) ctg^2\beta \cdot \frac{|U_{tb}|^2 (x_W^2 - 4r_W)}{(1 - x_W + r_W - r_{H^+})^2} [1 - x_W + r_W - r_t] \qquad (35)$$

is the decay width of $H \to t + \bar{b} + W^-$ at the production of non-polarized quarks, and P_t is the degree of longitudinal polarization of the t-quark

$$P_t = \frac{x_b\sqrt{x_t^2 - 4r_t} - \dfrac{x_t}{\sqrt{x_t^2 - 4r_t}}(2(1 - x_t - x_b - r_W + r_t) + x_t x_b)}{4(1 - x_W + r_W - r_t)}. \qquad (36)$$

For the decay width of the pseudoscalar boson in the $A \to H^{+*} + W^- \to t + \bar{b} + W^-$ channel, a similar expression is obtained

$$\frac{d\Gamma(A \to t\bar{b}W^-)}{dx_t dx_b} = \frac{1}{2}\frac{d\Gamma_0(A \to t\bar{b}W^-)}{dx_t dx_b} \cdot (1 + \lambda_t P_t), \qquad (37)$$

where

$$\frac{d\Gamma_0(A \to t\bar{b}W^-)}{dx_t dx_b} = \frac{N_C G_F^2 m_t^2 M_A^3}{64\pi^3} ctg^2\beta \cdot \frac{|U_{tb}|^2 (x_W^2 - 4r_W)}{(1 - x_W + r_W - r_{H^+})^2} (1 - x_W + r_W - r_t), \qquad (38)$$

and the degree of longitudinal polarization of P_t is expressed by the same formula (36) as in the decay of $H \to t + \bar{b} + W^-$.

Figure 9 shows the dependence of the degree of longitudinal polarization of the t-quark on the scaling energy x_t at $M_H(M_A) = 300$ GeV, $x_b = 0{,}3$, $m_t = 173{,}2$ GeV, $M_W = 80{,}425$ GeV. As follows from the figure, with an increase in the scaling energy of t-quark, the degree of its longitudinal polarization first decreases sharply and then slightly increases.

One of the possible Higgs boson decays is the $H \to W^{+*} + W^- \to t + \bar{b} + W^-$ decay, Feynman diagram, which is shown in fig. 1 (h). The matrix element of this decay is written as

$$M(H \to W^- W^{+*}) = i g_{WWH} \cdot \frac{g_W}{2\sqrt{2}} U_\mu^*(k) \cdot \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}\right) \cdot \frac{U_{tb}^*}{q^2 - M_W^2 + iM_W\Gamma_W} \cdot [\bar{u}(p_1, s_t)\gamma_\nu(1 + \gamma_5)\upsilon(p_2)] \qquad (39)$$

The square of the matrix element (39) is expressed by the formula (with $r_b = 0$)

$$\left|M(H \to W^- W^{+*})\right|^2 = 8G_F^2 M_W^4 \frac{|U_{tb}|^2 \cos^2(\beta - \alpha)}{(1 - x_t - x_b)^2 + r_W\gamma_W} \cdot [F_1 + \lambda_t F_2] \qquad (40)$$

Here, it is taken into account that $g_{WWH}^2 = 8G_F^2 M_W^4 \cos^2(\beta - \alpha)$ and functions are introduced

$$F_1 = (1 - x_W + r_W - r_t)\left[r_W + r_t\left(2 - \frac{1}{r_W} + \frac{x_W^2}{4r_W^2}\right)\right] + (1 - x_t + r_t - r_W)\left(1 - x_b + r_t - r_W - \frac{r_t}{r_W}x_W\right),$$

$$F_2 = \frac{1}{2}\left[-r_W + r_t\left(2-\frac{1}{r_W}+\frac{x_W^2}{4r_W^2}\right)+\left(1-\frac{x_W}{2r_W}\right)(1-x_b-r_t-r_W)\right]f_3 -$$

$$-\frac{1}{2}\left[1-x_t+r_t-r_W-\left(1-\frac{x_W}{2r_W}\right)(1-x_W+r_W-r_t)\right]f_4 \quad (41)$$

$$f_3 = x_b\sqrt{x_t^2-4r_t} - \frac{x_t}{\sqrt{x_t^2-4r_t}}[2(1-x_t-x_b-r_W+r_t)+x_t x_b],$$

$$f_4 = x_W\sqrt{x_t^2-4r_t} - \frac{x_t}{\sqrt{x_t^2-4r_t}}[2(1-x_t-x_W-r_W+r_t)+x_t x_W].$$

The decay width of the $H \to W^{+*} + W^- \to t + \bar{b} + W^-$ can be written as

$$\frac{d\Gamma(H\to t\bar{b}W^-)}{dx_t dx_b} = \frac{1}{2}\frac{d\Gamma_0(H\to t\bar{b}W^-)}{dx_t dx_b}\cdot(1+\lambda_t P_t), \quad (42)$$

in here

$$\frac{d\Gamma_0(H\to t\bar{b}W^-)}{dx_t dx_b} = \frac{N_C G_F^2 M_W^4 M_H}{64\pi^3}\cos^2(\beta-\alpha)\cdot\frac{|U_{tb}|^2}{(1-x_t+x_b)^2+r_W\gamma_W}\cdot F_1 \quad (43)$$

is the width of this decay with unpolarized particles, and

$$P_t = \frac{F_2}{F_1} \quad (44)$$

is the degree of longitudinal polarization of the t-quark.

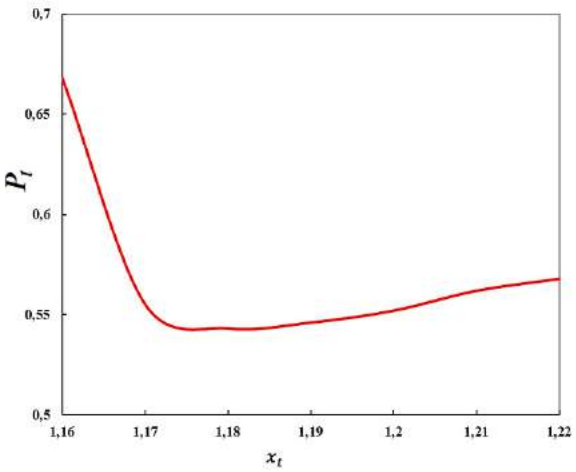

Fig.9. Dependence of the degree of longitudinal polarization of the t-quark on the scaling energy x_t.

CONCLUSION

In the framework of the MSSM, we discussed the decay channels of the supersymmetric Higgs bosons $H \to W^- + W^+$, $H \to Z^0 + Z^0$, $A \to h + Z^0$, $H \to h + h$, $H \to h + b + \bar{b}$, $H \to Z^0 + A + h$, $H^+ \to h + W^{\pm}$, $A \to Z^0 + h + h$, $H^{\pm} \to W^{\pm} + h + h$, $H(A) \to t + \bar{b} + W^-$. Analytical expressions for the amplitudes and decay widths are obtained, the dependence of the decay widths on the Higgs boson mass is studied. It was established that the width of all decays is highly sensitive to the Higgs boson mass. With an increase in the Higgs boson mass, the decay width in some processes increases, while in others it decreases.

[1] *S.Q. Abdullayev*. Standard Model, lepton and quark properties, "Zaka print", Baku, 2017, p.284.

[2] *S.Q. Abdullayev*. The general properties of fundamental interactions, "Zaka print", 2018, p. 332.

[3] *V.M. Yemelyanov*. Standard model and its extension, Mocsow, "Fizmatlit", 2007, p. 584.

[4] *A. Djouadi.* The Anatomy of Electro-Weak Symmetry Breaking. Tome I: The Higgs bosons in the Standard Model. arXiv: hep-ph/0503172v. 2, 3 May, 2005.

[5] ATLAS Collaboration. Phys. Letters, 2012, B 716, p. 1-29.
[6] CMS Collaboration. Phys. Letters, 2012, B 716, p. 30-60.
[7] *V.A. Rubakov*. The discovery of new particle with Higgs boson properties on Big Hadron Collider. UFN, 2012, vol.182, №10, pp.1017-1025.
[8] *A.V. Lanev*. CMS collaboration results. UFN, 2014, vol. 184, № 9, pp. 996-1004.
[9] *D.I. Kazakov.* Higgs boson is discovered. What's next? UFN, 2014, v. 184, № 9, pp.1004-1017.
[10] *E.E. Boos.* Standard Model and predictions for Higgs boson, UFN, 2014, vol. 184, № 9, pp.986-996.
[11] *P.W. Higgs.* Broken Symmetries and the Masses of gauge Bosons. Phys. Rev. Letters, 1964, v. 13, № 16, p. 508.
[12] *F. Englert, R. Brout*. Broken Symmetry and the mass of gauge vector Mesons. Phys. Rev. Letters, 1964, v. 13, № 9, p. 321.
[13] *S.K Abdullayev., M.Sh. Gojayev., F.A. Saddigh.* Moscow University Physics Bulletin, 2017, v.72, № 4. P. 329-339; published in Vestnik Moscovskoqo Universiteta, seriya 3: Physics, Astronomy, 2017, № 4, s.3-11.
[14] *S.K. Abdullayev, L.A. Agamaliyeva, M.Sh. Gocayev, F. Saddigh.* The investigation of Higgs boson creation in lepton-antilepton collisions. GESJ: Physics, 2015, № 1(13), pp.36-55.
[15] *S.K. Abdullayev, L.A. Agamaliyeva, M.Sh.Gocayev*. The investigation of Higgs boson creation in deep inelastic scattering. GESJ: Physics, 2015, №2(14), pp. 28-40.
[16] *R. Akbar, I. Ahmed, M.J. Aslam.* Lepton polarization asymmetries of $H \to \gamma\tau^{+}\tau^{-}$ decay in the Standard Model. Prog. Theor. Exp. Phys., 2014, p. 093B03.
[17] *F.T. Khalil-zade.* AJP Fizika, 2018, v. XXIV, № 1, p. 8-17.
[18] *S.K.Abdullayev, M.Sh.Gojayev, N.A. Nasibova*, *G.A. Soltanova*. AJP Fizika, 2018, v. XXIV, № 1, p. 32-39.
[19] *S.K.Abdullayev, M.Sh.Gojayev, N.A.Nasibova*. Russian Physics Journal, 2018, v. 61, № 1, p. 94-101.
[20] *J.F. Gunion, H.E. Haber.* The CP-conserving two-Higgs-doublet model: the approach to the decoupling limit. Phys. Rev. D, 2003, v. 67, p.075019.
[21] *G.C. Branco* et al. Phys. Rep., 2012, v.516, p.1.
[22] *H.E. Haber, G.L. Kane.* The search for supersymmetry: Probing physics beyond the standard model. Phys. Rep., 1985, v. 117, p.75.
[23] *D.I. Kazakov*. Beyond the Standard Model (in search of supersymmetry) hep-ph / 0012288.
[24] *D.I. Kazakov.* Supersymmetry on the Run: LHC and Dark Matter. Nucl. Phys. B Proc. Suppl.,2010,v.203-204,p.118;arXiv:1010.5419.
[25] *A. Djouadi.* The Anatomy of Electro-Weak Symmetry Breaking. Tome II: The Higgs in Minimal Supersymmetric Standard Model. arXiv: hep-ph/0503173v2, 2003.
[26] *K. Peters.* Prospects for beyond Standart Model Higgs Boson searches at future LHC runs and other machines. arXiv: 1701.05124v2 [hep-ex] 21, Feb. 20.
[27] *A. Djouadi. J. Kalinovski, P.M. Zerwas*. Two- and Three-Body Decay Modes of SUSY Higgs Particles. arXiv: hep-ph / 9511342V1, 16 Nov., 1995.
[28] *E. Barradas, J.L. Diaz-Cruz, A. Gutierrez, A. Rosado.* Three-body decays of Higgs bosons in the MSSM.
[29] *R.K. Barman* et. al. Current status of MSSM Higgs sector with LHC 13 TeV data. arXiv: 1608.02573v3 [hep-ph] 23 May, 2017.
[30] *S.K. Abdullayev, E.Sh. Omarova.* The decay of Higgs boson on fermion-anti-fermion couple. İzv. Vıssix Uçebnıx Zavedeniy, Fizika, 2018.
[31] *S.K. Abdullayev, E.Sh. Omarova.* The decay of Higgs boson on calibrating boson and fermion-anti-fermion couple. İzv. VUZ, Fizika, 2018.
[32] *S.K. Abdullayev, E.Sh. Omarova.* Decays of Supersymmetric Higgs bosons into fermions. AJP Fizika, 2018 (in press).

Received: 21.10.2019