

LONGITUDINAL SPIN ASYMMETRIES IN SEMI-INCLUSIVE DEEP-INELASTIC SCATTERING OF POLARIZED LEPTONS BY POLARIZED NUCLEONS

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Within the framework of the Standard Model, the processes of semi-inclusive deep inelastic scattering of longitudinally polarized leptons (antileptons) by polarized nucleons are considered: $\ell^-(\ell^+)N \rightarrow \ell^-(\ell^+)h^\pm X$, here $\ell^-(\ell^+)$ – an electron or muon (positron or antimuon), h^\pm – charged π^\pm - or K^\pm -meson, X – a system of undetected hadrons. By introducing non-polarization and polarization structure functions of hadrons and taking into account the longitudinal polarizations of the lepton and target nucleon, analytical expressions are obtained for the differential cross sections of the processes. All structure functions of hadrons are found in the quark-parton model; they depend on the distribution and fragmentation functions of quarks and antiquarks. The longitudinal spin asymmetries $A_N^{h^+-h^-}$, $A_N^{h^+}$, $A_N^{h^-}$ and others are determined, the dependence of the asymmetries on the invariant variables x , y and z is studied. The longitudinal spin asymmetry $A_d^{\pi^+-\pi^-}$ is compared with the experimental data of the COMPASS collaboration.

Keywords: Semi-inclusive deep inelastic scattering, structure functions, longitudinal spin asymmetries, spiral amplitudes, distribution functions, fragmentation functions.

PACS: 12.15.-Y, 13.60.-R, 13.85.NI, 13.87.FH

1. INTRODUCTION

The study of the quark structure of nucleons is one of the main tasks of modern high-energy physics. In this direction, the processes of deep inelastic scattering (DIS) of longitudinally polarized leptons (muons, electrons) by polarized nucleons are intensively investigated in various laboratories around the world (COMPASS, EIC, EMC, DEZY, HERMES) [1-8]. In the first experiments, the polarization structure functions (SF) of the proton $F_2^p(x)$ and neutron $F_2^n(x)$ were measured in a wide range of variation of the Bjorken variable x and it was established that the integral

$$\int_0^1 \frac{dx}{x} [F_2^p(x) - F_2^n(x)] = 0.235 \pm 0.0026.$$

However, this result contradicts the Gottfried sum rules, according to which this integral is equal to 1/3. The experimental results show that the distribution densities of sea u - and d -quarks in the proton $\bar{u}(x)$ and $\bar{d}(x)$ are different:

$$\int_0^1 dx [\bar{d}(x) - \bar{u}(x)] = 0.148 \pm 0.039 \neq 0.$$

Thus, in the DIS processes of unpolarized leptons in unpolarized nucleons $\ell^\mp + N \rightarrow \ell^\mp + X$, we determine the distribution of sea (\bar{q}) and valence ($q_v = q - \bar{q}$) quarks in a nucleon. The question arises, what new information does the investigation of the DIS of polarized leptons on polarized nucleons provide? Here it becomes possible to determine the density of distributions of polarized sea and valence quarks in polarized nucleons:

$$\Delta q(x, Q^2) = q^\uparrow(x, Q^2) - q^\downarrow(x, Q^2),$$

$$\Delta \bar{q}(x, Q^2) = \bar{q}^\uparrow(x, Q^2) - \bar{q}^\downarrow(x, Q^2),$$

$$\Delta q_v(x, Q^2) = \Delta q(x, Q^2) - \Delta \bar{q}(x, Q^2),$$

where q^\uparrow and q^\downarrow (\bar{q}^\uparrow and \bar{q}^\downarrow) is the distribution density of a quark (antiquark) whose spin is parallel and antiparallel to the nucleon spin. This arises a very interesting question: is the difference between the distributions of polarized sea \bar{u} - and \bar{d} -quarks $\Delta \bar{u}(x, Q^2) - \Delta \bar{d}(x, Q^2)$ equal to zero or is it different from zero? We can give a positive answer to this question by studying the processes of semi-inclusive deep inelastic scattering (SIDIS) of polarized leptons by polarized nucleons

$$\ell^-(\lambda) + N(h_N) \rightarrow \ell^-(\lambda) + h^\pm + X, \quad (1)$$

$$\ell^+(\lambda) + N(h_N) \rightarrow \ell^+(\lambda) + h^\pm + X, \quad (2)$$

where λ – is the helicity of the lepton (antilepton), h_N – is the longitudinal polarization of the target nucleon, $h^\pm(\pi^\pm, K^\pm)$ – is the recorded hadron in the final state together with the lepton at coincidence, X – is the system of nondetectable hadrons.

In [9, 10], the spin asymmetries in reactions (1) and (2) were studied within the framework of the quark-parton model (QPM). However, these works did not consider the non-polarization and polarization structure functions of hadrons. In this paper, within the framework of the Standard Model (SM), by introducing the SF of hadrons, analytical expressions are obtained for the differential cross sections of processes (1) and (2), a number of longitudinal spin asymmetries are determined, and the dependence of these asymmetries on invariant variables is studied in detail, x , y and z .

2. KINEMATIC VARIABLES OF REACTION

The SIDIS process of a lepton on a nucleon is described by the Feynman diagrams shown in Fig. 1.

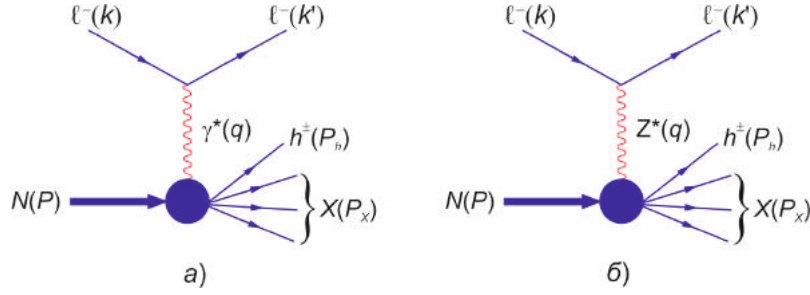


Fig. 1. Feynman diagrams of process $\ell^- N \rightarrow \ell^- h^+ X$.

Here k and k' – are the 4-momenta of the incident and scattered leptons, P and P_h – are the 4-momenta of a nucleon and an inclusive hadron h^+ , P_X – is the total 4-momentum of undetected hadrons X . The exchange between a lepton and a nucleon occurs by a virtual photon γ^* and a neutral Z^* -boson, these particles transfer a 4-momentum $q = k - k'$ from a lepton to a nucleon. A number of invariant variables are introduced to describe the SIDIS of a lepton on nucleons:

1) energy transfer from lepton to nucleon

$$\nu = \frac{(P \cdot q)}{M} = E - E', \quad (3)$$

where M – is the mass of the nucleon, $E(E')$ – is the energy of the initial (final) lepton;

2) square of transmitted momentum

$$Q^2 = -q^2 = -(k - k')^2 = 4EE' \sin^2 \frac{\theta}{2} - 2m_\ell^2,$$

where m_ℓ and θ – are the mass and scattering angle of the lepton; for the lepton mass $EE' \sin^2 \frac{\theta}{2} \gg m_\ell^2$ we can neglect and

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2}; \quad (4)$$

3) ordinary Bjorken variables DIS

$$x = \frac{Q^2}{2M\nu}, \quad y = \frac{(P \cdot q)}{(P \cdot k)} = \frac{\nu}{E}, \quad (5)$$

which range from 0 to 1;

4) variable z that determines the fraction of the energy transferred to the nucleon carried away by the hadron h

$$z = \frac{(P \cdot P_h)}{(P \cdot q)} = \frac{E_h}{\nu}; \quad (6)$$

The shaded area in the diagrams shows that the nucleon has an internal structure, which is taken into account by introducing the SF.

5) the square of the total energy of the lepton-nucleon system in the center of mass system

$$s = (k + P)^2 = \frac{Q^2}{xy} + M^2 + m_\ell^2,$$

when $Q^2 \gg M^2$ we get

$$s = \frac{Q^2}{xy}. \quad (7)$$

Shown in Fig. 1, the process is called SIDIS of a lepton on a nucleon if $Q^2 \gg M^2$ and $\nu \gg M$. In this case, we can neglect the masses of the nucleon and lepton.

3. NON-POLARIZING AND POLARIZING SF

According to the SM, the Lagrangian of the interaction of an Z -boson with a fermion pair is:

$$L_{Zff} = -\frac{g}{2 \cos \theta_W} \gamma_\alpha [g_L(f)(1 + \gamma_5) + g_R(f)(1 - \gamma_5)] \cdot Z_\alpha$$

where $g_L(f)$ ($g_R(f)$) – is the left (right) constant of the interaction of the fermion with the Z -boson:

$$\begin{aligned} g_L(\ell) &= -\frac{1}{2} + x_W, & g_R(\ell) &= x_W, \\ g_L(u) &= \frac{1}{2} - \frac{2}{3}x_W, & g_R(u) &= -\frac{2}{3}x_W, \\ g_L(d) &= -\frac{1}{2} + \frac{1}{3}x_W, & g_R(d) &= \frac{1}{3}x_W, \end{aligned} \quad (8)$$

$x_W = \sin^2 \theta_W$ – Weinberg parameter (θ_W – Weinberg angle).

The amplitude corresponding to the diagrams in Fig. 1 is written as:

$$\begin{aligned} M_{i \rightarrow f} &= i \frac{e^2}{Q^2} \{ [\bar{u}(k') \gamma_\alpha u(k)] \langle P_X, P_h | J_\alpha^{(\gamma)} | P, S \rangle + \\ &+ G_{eff} \{ \bar{u}(k') \gamma_\alpha [g_L(\ell)(1 + \gamma_5) + g_R(\ell)(1 - \gamma_5)] u(k) \} \langle P_X, P_h | J_\alpha^{(Z)} | P, S \rangle, \end{aligned} \quad (9)$$

where

$$G_{eff} = -\frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha} \cdot \frac{Q^2}{Q^2 + M_Z^2}, \quad (10)$$

M_Z – Z -boson mass, $J_\alpha^{(\gamma)}$ and $J_\alpha^{(Z)}$ – electromagnetic and weak neutral hadron currents, G_F – Fermi constant of weak interactions, S – 4-vector of target nucleon polarization.

The differential cross section of the SIDIS of polarized leptons on polarized nucleons with registration of the lepton and hadron h in the final state is expressed by the product of the lepton and hadron tensors:

$$\frac{d\sigma^{(-)}}{dx dy dz} = \frac{2\pi y \alpha^2}{Q^4} [L_{\mu\nu}^{(\gamma)} H_{\mu\nu}^{(\gamma)} + G_{eff} L_{\mu\nu}^{(I)} H_{\mu\nu}^{(I)} + G_{eff}^2 L_{\mu\nu}^{(Z)} H_{\mu\nu}^{(Z)}]. \quad (11)$$

Here $L_{\mu\nu}^{(\gamma)}$ and $H_{\mu\nu}^{(\gamma)}$ ($L_{\mu\nu}^{(Z)}$ and $H_{\mu\nu}^{(Z)}$) – are the tensors corresponding to the photon (Z -boson) exchange, $L_{\mu\nu}^{(I)}$ and $H_{\mu\nu}^{(I)}$ – are the interference of the photon and boson mechanisms. Lepton tensors $L_{\mu\nu}^{(\gamma)}$, $L_{\mu\nu}^{(I)}$ and $L_{\mu\nu}^{(Z)}$ are easily calculated based on the amplitude (9):

$$\begin{aligned} L_{\mu\nu}^{(\gamma)} &= 2[k_\mu k'_\nu + k'_\mu k_\nu - (k \cdot k') g_{\mu\nu} + i\lambda \varepsilon_{\mu\nu\rho\sigma} k_\rho k'_\sigma], \\ L_{\mu\nu}^{(I)} &= [g_L(\ell) + g_R(\ell) - \lambda(g_L(\ell) - g_R(\ell))] L_{\mu\nu}^{(\gamma)}, \\ L_{\mu\nu}^{(Z)} &= 2[g_L^2(\ell) + g_R^2(\ell) - \lambda(g_L^2(\ell) - g_R^2(\ell))] L_{\mu\nu}^{(\gamma)}, \end{aligned} \quad (12)$$

where $\lambda = +1$ (-1) – corresponds to the right (left) -polarized lepton.

The hadron tensor in $H_{\mu\nu}^{(i)}$ ($i = \gamma, I, Z$) the general case contains three non-polarization ($F_1^{(i)}, F_2^{(i)}, F_3^{(i)}$) and five polarization ($G_1^{(i)}, G_2^{(i)}, G_3^{(i)}, G_4^{(i)}, G_5^{(i)}$) hadron SF [11, 12]:

$$\begin{aligned} H_{\mu\nu}^{(i)} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1^{(i)} + \frac{\tilde{P}_\mu \tilde{P}_\nu}{(P \cdot q)} F_2^{(i)} - i\varepsilon_{\mu\nu\alpha\beta} \frac{q_\alpha P_\beta}{2(P \cdot q)} F_3^{(i)} + \\ &+ i\varepsilon_{\mu\nu\alpha\beta} \frac{q_\alpha}{(P \cdot q)} \left[S_\beta G_1^{(i)} + \left(S_\beta - \frac{(S \cdot q)}{(P \cdot q)} P_\beta \right) G_2^{(i)} \right] + \frac{1}{(P \cdot q)} \left[\frac{1}{2} (\tilde{P}_\mu \tilde{S}_\nu + \tilde{S}_\mu \tilde{P}_\nu) - \frac{(S \cdot q)}{(P \cdot q)} \tilde{P}_\mu \tilde{P}_\nu \right] G_3^{(i)} + \\ &+ \frac{(S \cdot q)}{(P \cdot q)} \left[\frac{\tilde{P}_\mu \tilde{P}_\nu}{(P \cdot q)} G_4^{(i)} + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) G_5^{(i)} \right]. \end{aligned} \quad (13)$$

Here

$$\tilde{P}_\mu = P_\mu - \frac{(P \cdot q)}{q^2} q_\mu, \quad \tilde{S}_\mu = S_\mu - \frac{(S \cdot q)}{q^2} q_\mu,$$

all structure functions depend on three invariant variables x, z, Q^2 :

$$F_n = F_n(x, z, Q^2) \quad (n=1, 2, 3); \quad G_m = G_m(x, z, Q^2) \quad (m=1 \div 5).$$

The differential cross section of the SIDIS of polarized leptons on unpolarized nucleons depends on the SF $F_1^{(i)}, F_2^{(i)}, F_3^{(i)}$:

$$\frac{d\sigma^{(-)}(\lambda)}{dx dy dz} = \frac{4\pi\alpha^2}{xyQ^2} \eta_i \left[xy^2 F_1^{(i)} + \left(1 - y - x^2 y^2 \frac{M^2}{Q^2} \right) F_2^{(i)} \pm xy \left(1 - \frac{y}{2} \right) F_3^{(i)} \right], \quad (14)$$

$i = \gamma, Z$ and γZ correspond to the exchange of a photon, Z -boson and their interference, the factor η_i is determined as follows:

$$\eta_\gamma = 1, \quad \eta_{\gamma Z} = -G_{eff}, \quad \eta_Z = G_{eff}^2,$$

lepton (antilepton) scattering corresponds to the upper (lower) sign.

SF for the processes of DIS leptons (antileptons) are equal:

$$\begin{aligned} F_{1,2}^{(i)} &= F_{1,2}^\gamma + [\pm \lambda g_A(\ell) - g_V(\ell)] F_{1,2}^{\gamma Z} + [g_V^2(\ell) + g_A^2(\ell) \mp 2\lambda g_V(\ell) g_A(\ell)] F_{1,2}^Z, \\ F_3^{(i)} &= [\pm \lambda g_V(\ell) - g_A(\ell)] F_3^{\gamma Z} + [2g_V(\ell) g_A(\ell) \mp \lambda [g_V^2(\ell) + g_A^2(\ell)]] F_3^Z. \end{aligned} \quad (15)$$

Here $g_V(\ell) = g_L(\ell) + g_R(\ell)$ and $g_A(\ell) = g_L(\ell) - g_R(\ell)$ – vector and axial constants of interaction of a lepton with a Z -boson.

The addition of polarized leptons (antileptons) by a polarized target to the differential cross section of the

SIDIS is proportional to the polarization of the nucleon h_N and depends on the polarization SF $G_1^{(i)} - G_5^{(i)}$:

$$\frac{d\sigma^{(-)}(\lambda; h_N)}{dx dy dz} = \frac{4\pi\alpha^2}{xyQ^2} \eta_i h_N \left\{ \lambda xy \left(2 - y - 2x^2 y^2 \frac{M^2}{Q^2} \right) G_1^{(i)} - 4\lambda x^3 y^2 \frac{M^2}{Q^2} G_2^{(i)} - 2x^2 y \frac{M^2}{Q^2} \times \right. \\ \left. \times \left(1 - y - x^2 y^2 \frac{M^2}{Q^2} \right) G_3^{(i)} + \left(1 + 2x^2 y \frac{M^2}{Q^2} \right) \left[\left(1 - y - x^2 y^2 \frac{M^2}{Q^2} \right) G_4^{(i)} + xy^2 G_5^{(i)} \right] \right\}. \quad (16)$$

It follows from this expression that the contributions to the cross section of polarization SF $G_2^{(i)}$ and $G_3^{(i)}$ are proportional to the ratio M^2/Q^2 , and therefore are suppressed, since in the DIS region $Q^2 \gg M^2$.

In the deep inelastic region of lepton-nucleon scattering for differential cross sections of processes $\ell^\mp + N \rightarrow \ell^\mp + h + X$, we obtain the expressions (it is taken into account that $M^2/Q^2 \rightarrow 0$):

$$\frac{d\sigma^{(\mp)}(\lambda; h_N)}{dx dy dz} = \frac{4\pi\alpha^2}{xyQ^2} \eta_i \left\{ [1 + (1-y)^2] x F_1^{(i)} \pm \frac{1}{2} [1 - (1-y)^2] x F_3^{(i)} + (1-y) F_L^{(i)} - \right. \\ \left. - h_N [[1 + (1-y)^2] x G_5^{(i)} \mp \lambda [1 - (1-y)^2] x G_1^{(i)} + 2(1-y) G_L^{(i)}] \right\}, \quad (17)$$

here we have introduced longitudinal SF by means of the relations

$$F_L^{(i)} = F_2^{(i)} - 2x F_1^{(i)}, \quad G_L^{(i)} = G_4^{(i)} - 2x G_5^{(i)}. \quad (18)$$

Polarizing SF are determined by the expressions:

$$G_1^{(i)} = \mp \lambda G_1^\gamma - [g_A(\ell) \mp \lambda g_V(\ell)] \cdot G_1^{\gamma Z} + [2g_V(\ell) g_A(\ell) \mp \lambda (g_V^2(\ell) + g_A^2(\ell))] G_1^Z, \\ G_4^{(i)} = 2x G_5^{(i)} = 2x \{ -[g_V(\ell) \mp g_A(\ell)] \cdot G_5^{\gamma Z} + [g_V^2(\ell) + g_A^2(\ell) \mp 2\lambda g_V(\ell) g_A(\ell)] G_5^Z \}. \quad (19)$$

4. SF IN THE QUARK-PATRON MODEL

Let's calculate the SF of hadrons in the QPM. The electromagnetic and weak neutral quark currents are determined by the formulas

$$J_\mu^{(\gamma)} = \sum_q Q_q (\bar{q} \gamma_\mu q), \\ J_\mu^{(Z)} = \sum_q \bar{q} \gamma_\mu [g_V(q) + \gamma_5 g_A(q)] q, \quad (20)$$

Q_q – where is the quark charge.

On the basis of these currents, the following expressions were obtained in the QPM for the SF of hadrons (the contribution of valence quarks and sea quark-antiquark pairs is taken into account):

$$F_1^\gamma = \frac{1}{2} \sum_q Q_q^2 [q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z)], \\ F_2^\gamma = x \sum_q Q_q^2 [q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z)], \\ F_3^\gamma = 0, \\ F_1^{\gamma Z} = \sum_q Q_q g_V(q) [q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z)], \\ F_2^{\gamma Z} = x \sum_q 2Q_q g_V(q) [q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z)], \\ F_3^{\gamma Z} = \sum_q 2Q_q g_A(q) [q(x) D_q^h(z) - \bar{q}(x) D_{\bar{q}}^h(z)], \quad (21)$$

$$F_1^Z = \frac{1}{2} \sum_q [g_V^2(q) + g_A^2(q)] [q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z)], \\ F_2^Z = x \sum_q [g_V^2(q) + g_A^2(q)] [q(x) D_q^h(z) + \bar{q}(x) D_{\bar{q}}^h(z)], \\ F_3^Z = \sum_q 2g_V(q) g_A(q) [q(x) D_q^h(z) - \bar{q}(x) D_{\bar{q}}^h(z)], \\ G_1^\gamma = \frac{1}{2} \sum_q Q_q^2 [\Delta q(x) D_q^h(z) + \Delta \bar{q}(x) D_{\bar{q}}^h(z)],$$

$$\begin{aligned}
G_4^\gamma &= G_5^\gamma = 0, \\
G_1^{\gamma Z} &= \sum_q Q_q g_V(q) [\Delta q(x) D_q^h(z) + \Delta \bar{q}(x) D_{\bar{q}}^h(z)], \\
G_4^{\gamma Z} &= x \sum_q 2 Q_q g_A(q) [\Delta q(x) D_q^h(z) - \Delta \bar{q}(x) D_{\bar{q}}^h(z)], \\
G_5^{\gamma Z} &= \sum_q Q_q g_A(q) [\Delta q(x) D_q^h(z) - \Delta \bar{q}(x) D_{\bar{q}}^h(z)], \\
G_1^Z &= \frac{1}{2} \sum_q [g_V^2(q) + g_A^2(q)] [\Delta q(x) D_q^h(z) + \Delta \bar{q}(x) D_{\bar{q}}^h(z)], \\
G_4^Z &= x \sum_q 2 g_V(q) g_A(q) [\Delta q(x) D_q^h(z) - \Delta \bar{q}(x) D_{\bar{q}}^h(z)], \\
G_5^Z &= \sum_q g_V(q) g_A(q) [\Delta q(x) D_q^h(z) - \Delta \bar{q}(x) D_{\bar{q}}^h(z)].
\end{aligned}$$

Here $q(x)$ ($\bar{q}(x)$) – are the variable distributions of the number x of quarks (antiquarks) in a nucleon, $\Delta q(x) = q^\uparrow(x) - q^\downarrow(x)$ ($\Delta \bar{q}(x) = \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x)$) – is the difference between the distributions of a quark (antiquark) whose spin is parallel and antiparallel to the spin of a nucleon, $D_q^h(z)$ ($D_{\bar{q}}^h(z)$) – is the function of fragmentation of a q quark (\bar{q} antiquark) into a hadron h . Summation over q propagates over all quarks and antiquarks, which are present in a nucleon at a given value of the variable x .

As can be seen from expressions (21), in the QPM, the SF of deep inelastic scattering of leptons by nucleons depend on the scaling variables x and z , and satisfy the relations

$$F_2^i = 2x F_1^i, \quad G_4^i = 2x G_5^i,$$

which leads to the vanishing of the longitudinal SF: $F_L^i = 0$, $G_L^i = 0$. The reason for this is the lack of interaction between quarks. In reality, however, quarks interact by exchanging gluons with each other. Quark-gluon interaction is described by quantum chromody-

namics. According to quantum chromodynamics, the emission of hard gluons by quarks leads to logarithmic breaking of scaling. With an increase in the square of the transmitted momentum Q^2 , the number of gluons emitted by quarks increases sharply, gluons give rise to a quark-antiquark pair $q\bar{q}$. These processes lead to an increase in the density of gluons and sea $q\bar{q}$ -pairs in the nucleon at large x . Thus, taking into account quark-gluon interactions leads to the dependence of the distribution functions and quark fragmentation on the square of the momentum transfer Q^2 :

$$q(x, Q^2), \quad \bar{q}(x, Q^2), \quad D_q^h(z, Q^2), \quad D_{\bar{q}}^h(z, Q^2).$$

Let us now substitute the hadron SF obtained in the QPM (21) into the general expressions for the cross sections (17). In the quark-parton approximation for the cross section of the process $\ell^- + N \rightarrow \ell^- + h + X$, we obtain [9]:

$$\begin{aligned}
\frac{d\sigma^{(-)}(\lambda; h_N)}{dx dy dz} &= \pi \alpha^2 s x \times \\
&\times \sum_q \{ q(x) D_q^h(z) [(1+\lambda)(F_{RR}^2 + (1-y)^2 F_{RL}^2) + (1-\lambda)(F_{LL}^2 + (1-y)^2 F_{LR}^2)] + \\
&+ \bar{q}(x) D_{\bar{q}}^h(z) [(1+\lambda)(F_{RL}^2 + (1-y)^2 F_{RR}^2) + (1-\lambda)(F_{LR}^2 + (1-y)^2 F_{LL}^2)] + \\
&+ h_N \Delta q(x) D_q^h(z) [(1+\lambda)(F_{RR}^2 - (1-y)^2 F_{RL}^2) - (1-\lambda)(F_{LL}^2 - (1-y)^2 F_{LR}^2)] + \\
&+ h_N \Delta \bar{q}(x) D_{\bar{q}}^h(z) [(1+\lambda)(F_{RL}^2 - (1-y)^2 F_{RR}^2) - (1-\lambda)(F_{LR}^2 - (1-y)^2 F_{LL}^2)] \}. \quad (22)
\end{aligned}$$

Here, $F_{RR}(q)$, $F_{RL}(q)$, $F_{LL}(q)$ and $F_{LR}(q)$ are the spiral amplitudes of the quark-parton subprocesses

$$\begin{aligned}
\ell_R + q_R &\rightarrow \ell_R + q_R, \quad \ell_R + q_L \rightarrow \ell_R + q_L, \\
\ell_L + q_R &\rightarrow \ell_L + q_R, \quad \ell_L + q_L \rightarrow \ell_L + q_L,
\end{aligned}$$

respectively (the first and second indices correspond to the helicities of the lepton and quark). The spiral amplitudes depend on the square of the momentum transfer Q^2 and on the charges of the neutral weak currents of the lepton and quarks

$$F_{\alpha\beta}(q) = \frac{Q_\ell Q_q}{Q^2} + \frac{g_\alpha(\ell)g_\beta(q)}{Q^2 + M_Z^2} \quad (\alpha; \beta = L, R). \quad (23)$$

The differential cross section of the SIDIS of a polarized antilepton on a polarized nucleon $\ell^+ + N \rightarrow \ell^+ + h + X$ is obtained from the differential cross section (22), replacing the indices of the spiral amplitudes $F_{RR}(q) \leftrightarrow F_{LR}(q)$, $F_{LL}(q) \leftrightarrow F_{RL}(q)$:

$$\begin{aligned} \frac{d\sigma^{(+)}(\lambda; h_N)}{dx dy dz} &= \pi\alpha^2 s x \times \\ &\times \sum_q \{ q(x) D_q^h(z) [(1+\lambda)(F_{LR}^2 + (1-y)^2 F_{LL}^2) + (1-\lambda)(F_{RL}^2 + (1-y)^2 F_{RR}^2)] + \\ &+ \bar{q}(x) D_{\bar{q}}^h(z) [(1+\lambda)(F_{LL}^2 + (1-y)^2 F_{LR}^2) + (1-\lambda)(F_{RR}^2 + (1-y)^2 F_{RL}^2)] + \\ &+ h_N \Delta q(x) D_q^h(z) [(1+\lambda)(F_{LR}^2 - (1-y)^2 F_{LL}^2) - (1-\lambda)(F_{RL}^2 - (1-y)^2 F_{RR}^2)] + \\ &+ h_N \Delta \bar{q}(x) D_{\bar{q}}^h(z) [(1+\lambda)(F_{LL}^2 - (1-y)^2 F_{LR}^2) - (1-\lambda)(F_{RR}^2 - (1-y)^2 F_{RL}^2)] \}. \end{aligned} \quad (24)$$

5. LONGITUDINAL SPIN ASYMMETRIES

The set of spiral amplitudes $F_{\alpha\beta}(q)$ ($\alpha; \beta = L, R$) introduced above also determine the longitudinal spin asymmetries in semi-inclusive reactions (1) and (2). The study of longitudinal spin asymmetries in semi-inclusive reactions $\ell^\mp + N \rightarrow \ell^\mp + h^\mp + X$ is an important source of information on the distribution functions of polarized quarks, antiquarks and gluons in polarized nucleons. The longitudinal spin asymmetries include the following expressions:

$$\begin{aligned} A_{N1}^{h^+ - h^-} &= \frac{[\sigma_{LL}^{(\mp)}(h^+) - \sigma_{LL}^{(\mp)}(h^-)] - [\sigma_{LR}^{(\mp)}(h^+) - \sigma_{LR}^{(\mp)}(h^-)]}{[\sigma_{LL}^{(\mp)}(h^+) - \sigma_{LL}^{(\mp)}(h^-)] + [\sigma_{LR}^{(\mp)}(h^+) - \sigma_{LR}^{(\mp)}(h^-)]}, \\ A_{N2}^{h^+ - h^-} &= \frac{[\sigma_{RR}^{(\mp)}(h^+) - \sigma_{RR}^{(\mp)}(h^-)] - [\sigma_{RL}^{(\mp)}(h^+) - \sigma_{RL}^{(\mp)}(h^-)]}{[\sigma_{RR}^{(\mp)}(h^+) - \sigma_{RR}^{(\mp)}(h^-)] + [\sigma_{RL}^{(\mp)}(h^+) - \sigma_{RL}^{(\mp)}(h^-)]}, \end{aligned} \quad (25)$$

$$\begin{aligned} A_{N1}^{h^\pm} &= \frac{\sigma_{LL}^{(\mp)}(h^\pm) - \sigma_{LR}^{(\mp)}(h^\pm)}{\sigma_{LL}^{(\mp)}(h^\pm) + \sigma_{LR}^{(\mp)}(h^\pm)}, \\ A_{N2}^{h^\pm} &= \frac{\sigma_{RR}^{(\mp)}(h^\pm) - \sigma_{RL}^{(\mp)}(h^\pm)}{\sigma_{RR}^{(\mp)}(h^\pm) + \sigma_{RL}^{(\mp)}(h^\pm)}; \end{aligned} \quad (26)$$

$$\begin{aligned} A_1(e_R^- - e_L^+) &= \frac{\sigma_R^{(-)}(h^\pm) - \sigma_L^{(+)}(h^\pm)}{\sigma_R^{(-)}(h^\pm) + \sigma_L^{(+)}(h^\pm)}, \\ A_2(e_L^- - e_R^+) &= \frac{\sigma_L^{(-)}(h^\pm) - \sigma_R^{(+)}(h^\pm)}{\sigma_L^{(-)}(h^\pm) + \sigma_R^{(+)}(h^\pm)}; \end{aligned} \quad (27)$$

$$\begin{aligned} A_1 &= \frac{\sigma_{RL}^{(-)}(h^\pm) - \sigma_{LR}^{(+)}(h^\pm)}{\sigma_{RL}^{(-)}(h^\pm) + \sigma_{LR}^{(+)}(h^\pm)}, \\ A_2 &= \frac{\sigma_{RR}^{(-)}(h^\pm) - \sigma_{LR}^{(+)}(h^\pm)}{\sigma_{RR}^{(-)}(h^\pm) + \sigma_{LR}^{(+)}(h^\pm)}. \end{aligned} \quad (28)$$

Here $\sigma_{LL}^{(-)}(h^\pm)$ ($\sigma_{LL}^{(+)}(h^\pm)$), $\sigma_{LR}^{(-)}(h^\pm)$ ($\sigma_{LR}^{(+)}(h^\pm)$), $\sigma_{RR}^{(-)}(h^\pm)$ ($\sigma_{RR}^{(+)}(h^\pm)$) and $\sigma_{RL}^{(-)}(h^\pm)$ ($\sigma_{RL}^{(+)}(h^\pm)$) are the differential cross sections of semi-inclusive reactions

$\ell^- + N \rightarrow \ell^- + h^\mp + X$
($\ell^+ + N \rightarrow \ell^+ + h^\mp + X$) at helicities of the lepton (antilepton), $\lambda = -1$, $h_N = -1$, $\lambda = -1$, $h_N = +1$, $\lambda = 1$, $h_N = 1$ and $\lambda = 1$, $h_N = -1$, $\sigma_L^{(-)}(h^\pm)$ and $\sigma_R^{(-)}(h^\pm)$, $(\sigma_L^{(+)}(h^\pm))$ and $\sigma_R^{(+)}(h^\pm)$ – are the differential cross sections of the SIDIS of the left and right polarized lepton (antilepton) on an unpolarized nucleon target. It should be noted that it is precisely this kind of longitudinal spin asymmetries that are experimentally investigated in a number of laboratories around the world (COMPASS, HERMES, EMC, SLAC, SMC, etc. [1-8]).

When determining the «difference» longitudinal spin asymmetry $A_N^{h^+ - h^-}$ the polarization states of the target lepton and nucleon are shown in the diagram in Fig. 2. First, left-handed leptons (ℓ_L^-) are scattered by a left-handed nucleon (N_L) (the nucleon spin vector is oriented against the motion of the lepton), and then the direction of the nucleon spin vector is reversed, i.e. left-handed leptons are already scattered by a right-handed nucleon (N_R).

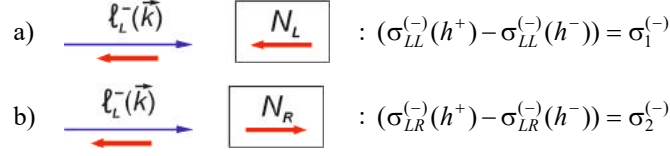


Fig. 2. Directions of the spins of the lepton and nucleon of the target.

The «difference» longitudinal spin asymmetry is equal to the ratio of the difference of the cross sections to their sum $(\sigma_1^{(-)} + \sigma_2^{(-)})$:

$$A_N^{h^+ - h^-} = \frac{[\sigma_{LL}^{(-)}(h^+) - \sigma_{LL}^{(-)}(h^-)] - [\sigma_{LR}^{(-)}(h^+) - \sigma_{LR}^{(-)}(h^-)]}{[\sigma_{LL}^{(-)}(h^+) - \sigma_{LL}^{(-)}(h^-)] + [\sigma_{LR}^{(-)}(h^+) - \sigma_{LR}^{(-)}(h^-)]}.$$

Based on the formulas for the differential cross sections of the processes $\ell^- + N \rightarrow \ell^- + h^\mp + X$ and $\ell^+ + N \rightarrow \ell^+ + h^\mp + X$ (22) and (24) for the longitudinal spin asymmetries (25) - (28), the expressions are obtained:

$$\begin{aligned} A_{N1}^{h^+ - h^-} = & \left\{ \sum_q \Delta q(x) [D_q^{h^+}(z) - D_q^{h^-}(z)] [F_{LL}^2(q) - (1-y)^2 F_{LR}^2(q)] + \right. \\ & + \sum_q \Delta \bar{q}(x) [D_{\bar{q}}^{h^+}(z) - D_{\bar{q}}^{h^-}(z)] [F_{LR}^2(q) - (1-y)^2 F_{LL}^2(q)] \} \times \\ & \times \left\{ \sum_q q(x) [D_q^{h^+}(z) - D_q^{h^-}(z)] [F_{LL}^2(q) + (1-y)^2 F_{LR}^2(q)] + \right. \\ & + \sum_q \bar{q}(x) [D_{\bar{q}}^{h^+}(z) - D_{\bar{q}}^{h^-}(z)] [F_{LR}^2(q) + (1-y)^2 F_{LL}^2(q)] \}^{-1}. \end{aligned} \quad (29)$$

$$\begin{aligned} A_{N2}^{h^+ - h^-} = & \left\{ \sum_q \Delta q(x) [D_q^{h^+}(z) - D_q^{h^-}(z)] [F_{RR}^2(q) - (1-y)^2 F_{RL}^2(q)] + \right. \\ & + \sum_q \Delta \bar{q}(x) [D_{\bar{q}}^{h^+}(z) - D_{\bar{q}}^{h^-}(z)] [F_{RL}^2(q) - (1-y)^2 F_{RR}^2(q)] \} \times \\ & \times \left\{ \sum_q q(x) [D_q^{h^+}(z) - D_q^{h^-}(z)] [F_{RR}^2(q) + (1-y)^2 F_{RL}^2(q)] + \right. \\ & + \sum_q \bar{q}(x) [D_{\bar{q}}^{h^+}(z) - D_{\bar{q}}^{h^-}(z)] [F_{RL}^2(q) + (1-y)^2 F_{RR}^2(q)] \}^{-1}. \end{aligned} \quad (30)$$

$$\begin{aligned} A_{N1}^{h^\pm} = & \left\{ \sum_q \Delta q(x) D_q^{h^\pm}(z) [F_{LL}^2(q) - (1-y)^2 F_{LR}^2(q)] + \right. \\ & + \sum_q \Delta \bar{q}(x) D_{\bar{q}}^{h^\pm}(z) [F_{LR}^2(q) - (1-y)^2 F_{LL}^2(q)] \} \times \\ & \times \left\{ \sum_q q(x) D_q^{h^\pm}(z) [F_{LL}^2(q) + (1-y)^2 F_{LR}^2(q)] + \right. \\ & + \sum_q \bar{q}(x) D_{\bar{q}}^{h^\pm}(z) [F_{LR}^2(q) + (1-y)^2 F_{LL}^2(q)] \}^{-1}. \end{aligned} \quad (31)$$

$$\begin{aligned} A_{N2}^{h^\pm} = & \left\{ \sum_q \Delta q(x) D_q^{h^\pm}(z) [F_{RR}^2(q) - (1-y)^2 F_{RL}^2(q)] + \right. \\ & + \sum_q \Delta \bar{q}(x) D_{\bar{q}}^{h^\pm}(z) [F_{RL}^2(q) - (1-y)^2 F_{RR}^2(q)] \} \times \\ & \times \left\{ \sum_q q(x) D_q^{h^\pm}(z) [F_{RR}^2(q) + (1-y)^2 F_{RL}^2(q)] + \right. \\ & + \sum_q \bar{q}(x) D_{\bar{q}}^{h^\pm}(z) [F_{RL}^2(q) + (1-y)^2 F_{RR}^2(q)] \}^{-1}. \end{aligned} \quad (32)$$

$$A_1(e_R^- - e_L^+) = f(y) \frac{\sum_q [q(x) D_q^{h^\pm}(z) - \bar{q}(x) D_{\bar{q}}^{h^\pm}(z)] [F_{RR}^2(q) - F_{RL}^2(q)]}{\sum_q [q(x) D_q^{h^\pm}(z) + \bar{q}(x) D_{\bar{q}}^{h^\pm}(z)] [F_{RR}^2(q) + F_{RL}^2(q)]}, \quad (33)$$

$$A_2(e_L^- - e_R^+) = f(y) \frac{\sum_q [q(x)D_q^{h\pm}(z) - \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{LL}^2(q) - F_{LR}^2(q)]}{\sum_q [q(x)D_q^{h\pm}(z) + \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{LL}^2(q) + F_{LR}^2(q)]}. \quad (34)$$

$$\begin{aligned} A_1 = f(y) & \left\{ \sum_q [q(x)D_q^{h\pm}(z) - \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{RR}^2(q) - F_{RL}^2(q)] - \right. \\ & - \sum_q [\Delta q(x)D_q^{h\pm}(z) + \Delta \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{RR}^2(q) + F_{RL}^2(q)] \} \times \\ & \times \left\{ \sum_q [q(x)D_q^{h\pm}(z) + \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{RR}^2(q) + F_{RL}^2(q)] + \right. \\ & \left. + \sum_q [\Delta q(x)D_q^{h\pm}(z) - \Delta \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{RR}^2(q) - F_{RL}^2(q)] \right\}^{-1}. \end{aligned} \quad (35)$$

$$\begin{aligned} A_2 = f(y) & \left\{ \sum_q [q(x)D_q^{h\pm}(z) - \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{RR}^2(q) - F_{RL}^2(q)] + \right. \\ & + \sum_q [\Delta q(x)D_q^{h\pm}(z) + \Delta \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{RR}^2(q) + F_{RL}^2(q)] \} \times \\ & \times \left\{ \sum_q [q(x)D_q^{h\pm}(z) + \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{RR}^2(q) + F_{RL}^2(q)] + \right. \\ & \left. + \sum_q [\Delta q(x)D_q^{h\pm}(z) - \Delta \bar{q}(x)D_{\bar{q}}^{h\pm}(z)][F_{RR}^2(q) - F_{RL}^2(q)] \right\}^{-1}. \end{aligned} \quad (36)$$

Introduced here function

$$f(y) = \frac{1 - (1 - y)^2}{1 + (1 - y)^2}. \quad (37)$$

Due to the charge independence of strong interactions, the quark fragmentation functions in π^\pm - and K^\pm -mesons satisfy a number of relations

$$\begin{aligned} D_u^{\pi^\pm}(z) &= D_d^{\pi^\pm}(z) = D_d^{\pi^\mp}(z) = D_u^{\pi^\mp}(z), \\ D_u^{K^\pm}(z) &= D_s^{K^\pm}(z) = D_s^{K^\mp}(z) = D_u^{K^\mp}(z). \end{aligned}$$

Calculations show that because of these relations, the «difference» longitudinal spin asymmetries

$A_{N1}^{h^+ - h^-}$ and $A_{N2}^{h^+ - h^-}$ do not depend on the functions of quark fragmentation into hadron h . These asymmetries include only the distribution functions of quarks in a nucleon $q(x)$, $\bar{q}(x)$, $\Delta q(x)$ and $\Delta \bar{q}(x)$. For example, in the processes $\ell^- + p \rightarrow \ell^- + \pi^\mp + X$ (only the contribution of valence and sea u - and d -quarks is taken into account), $\ell^- + N \rightarrow \ell^- + K^\mp + X$ (only the contribution of valence and sea u - and s -quarks is taken into account), longitudinal spin asymmetries $A_{p1}^{\pi^+ - \pi^-}$ and $A_{p1}^{K^+ - K^-}$ are expressed by formulas [9, 10]:

$$\begin{aligned} A_{p1}^{\pi^+ - \pi^-} &= \{ \Delta u_v(x)[F_{LL}^2(u) - (1 - y)^2 F_{LR}^2(u)] - \Delta d_v(x)[F_{LL}^2(d) - (1 - y)^2 F_{LR}^2(d)] + \\ &+ [1 + (1 - y)^2] [\Delta u_s(x)(F_{LL}^2(u) - F_{LR}^2(u)) - \Delta d_s(x)(F_{LL}^2(d) - F_{LR}^2(d))] \} \times \\ &\times \{ u_v(x)[F_{LL}^2(u) + (1 - y)^2 F_{LR}^2(u)] - d_v(x)[F_{LL}^2(d) + (1 - y)^2 F_{LR}^2(d)] + \\ &+ [1 - (1 - y)^2] [u_s(x)(F_{LL}^2(u) - F_{LR}^2(u)) - d_s(x)(F_{LL}^2(d) - F_{LR}^2(d))] \}^{-1}, \end{aligned} \quad (38)$$

$$\begin{aligned} A_{p1}^{K^+ - K^-} &= \{ \Delta u_v(x)[F_{LL}^2(u) - (1 - y)^2 F_{LR}^2(u)] + [1 + (1 - y)^2] [\Delta u_s(x)(F_{LL}^2(u) - F_{LR}^2(u)) - \\ &- \Delta s(x)(F_{LL}^2(s) - F_{LR}^2(s))] \} \times \{ u_v(x)[F_{LL}^2(u) + (1 - y)^2 F_{LR}^2(u)] + \\ &+ [1 - (1 - y)^2] [u_s(x)(F_{LL}^2(u) - F_{LR}^2(u)) - s(x)(F_{LL}^2(s) - F_{LR}^2(s))] \}^{-1}. \end{aligned} \quad (39)$$

Here $u_v(x)$ and $d_v(x)$, $u_s(x)$, $d_s(x)$ and $s(x)$ are the distribution functions of valence u - and d (sea u -, d - and s)-quarks in a proton.

The expressions for longitudinal spin asymmetries (38) and (39) contain phenomenological parameters

– the distribution functions of valence and sea quarks in a polarized nucleon. There are a number of sets in the literature for the distribution functions of quarks in nucleons [11, 13-19]. For numerical estimates of asymmetries, we used the distribution functions of valence and sea polarized quarks in nucleons

given in [19].

In Fig. 3 shows the dependence of the «difference» longitudinal spin asymmetry $A_{p1}^{\pi^+-\pi^-}$ in reaction $e^- + p \rightarrow e^- + \pi^\mp + X$ on variable x at energy $\sqrt{s}=300$ GeV and various fixed $y=0.1$ (curve 1), $y=0.4$ (curve 2) and $y=0.9$ (curve 3). As follows from the figure, the longitudinal spin asymmetry $A_{p1}^{\pi^+-\pi^-}$ in reactions $e^- + p \rightarrow e^- + \pi^\mp + X$ is positive and monotonically increases with increasing variable x . A similar dependence is observed for the longitudinal spin asymmetry $A_{p1}^{K^+-K^-}$ in the processes $e^- + N \rightarrow e^- + K^\mp + X$ (see Fig. 4, which illustrates the dependence of the asymmetry $A_{p1}^{K^+-K^-}$ on the variable x for different y).

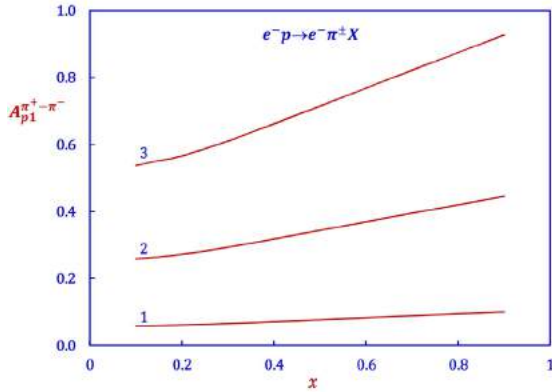


Fig. 3. Dependence of asymmetry $A_{p1}^{\pi^+-\pi^-}$ on x in reactions $e^- p \rightarrow e^- \pi^\mp X$ at $y = 0.1$; 0.4 and 0.9 (curves 1, 2, and 3, respectively).

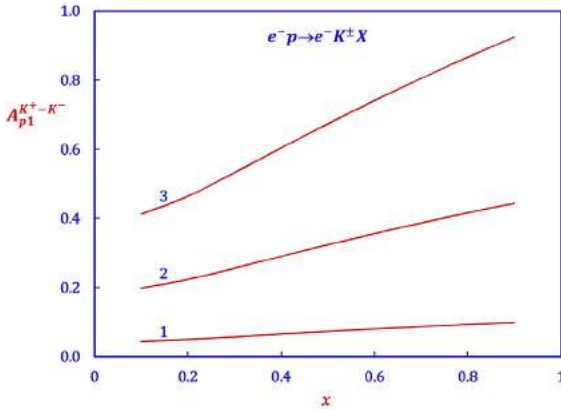


Fig. 4. Dependence of asymmetry $A_{p1}^{K^+-K^-}$ on x in reactions $e^- p \rightarrow e^- K^\mp X$ at $y = 0.1$; 0.4 and 0.9 (curves 1, 2, and 3, respectively).

For small values of the square of the momentum transfer ($Q^2 \ll M_Z^2$), the contribution of the Z -boson diagram to the cross section of the processes $e^\mp + p \rightarrow e^\mp + \pi^\pm + X$, $e^\mp + N \rightarrow e^\mp + K^\mp + X$ can be neglected, while all spiral amplitudes for a given quark subprocesses are the same

$$F_{LL}(q) = F_{LR}(q) = F_{RL}(q) = F_{RR}(q) = \frac{Q_e Q_q}{Q^2},$$

then the «difference» longitudinal two-spin asymmetry $A_N^{h^+-h^-}$ will depend only on the distribution functions of valence quarks:

$$\begin{aligned} A_{p1}^{\pi^+-\pi^-} &= \frac{4\Delta u_v(x) - \Delta d_v(x)}{4u_v(x) - d_v(x)} \cdot f(y), \\ A_{n1}^{\pi^+-\pi^-} &= \frac{4\Delta d_v(x) - \Delta u_v(x)}{4d_v(x) - u_v(x)} \cdot f(y), \\ A_{p1}^{K^+-K^-} &= \frac{\Delta u_v(x)}{u_v(x)} \cdot f(y), \\ A_{n1}^{K^+-K^-} &= \frac{\Delta d_v(x)}{d_v(x)} \cdot f(y), \\ A_{d1}^{\pi^+-\pi^-} &= A_{d1}^{K^+-K^-} = \frac{\Delta u_v(x) + \Delta d_v(x)}{u_v(x) + d_v(x)} \cdot f(y). \end{aligned} \quad (40)$$

As can be seen, for a polarized isoscalar deuteron target, the «difference» longitudinal spin asymmetries coincide:

$$A_d^{\pi^+-\pi^-} = A_d^{K^+-K^-} = \frac{\Delta u_v(x) + \Delta d_v(x)}{u_v(x) + d_v(x)}, \quad (41)$$

where $\Delta q_v(x) = \Delta q(x) - \Delta \bar{q}(x)$.

Longitudinal spin asymmetries $A_N^{h^+}$ and $A_N^{h^-}$ contain the functions of fragmentation of quarks (anti-quarks) into a hadron h^+ and h^- :

$$A_N^{h^\pm} = \frac{\sum_q Q_q^2 [\Delta q(x) D_q^{h^\pm} + \Delta \bar{q}(x) D_{\bar{q}}^{h^\pm}]}{\sum_q Q_q^2 [q(x) D_q^{h^\pm} + \bar{q}(x) D_{\bar{q}}^{h^\pm}]}, \quad (42)$$

where

$$D_q^{h^\pm} = \int_0^1 D_q^{h^\pm}(z) dz.$$

Using $SU(2)$ and C -charge symmetries, at the production of π^\pm -mesons, we have three fragmentation functions:

$$\begin{aligned} D_1 &= D_u^{\pi^+} \stackrel{SU(2)}{=} D_d^{\pi^-} \stackrel{C}{=} D_{\bar{d}}^{\pi^+} \stackrel{SU(2)}{=} D_{\bar{u}}^{\pi^-}, \\ D_2 &= D_{\bar{u}}^{\pi^+} \stackrel{SU(2)}{=} D_{\bar{d}}^{\pi^-} \stackrel{C}{=} D_d^{\pi^+} \stackrel{SU(2)}{=} D_u^{\pi^-}, \\ D_3 &= D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-}. \end{aligned}$$

Experiments conducted by the NMC, E866 and HERMES collaborations revealed that the unpolarized sea is not symmetrical.:

$$\int_0^1 [\bar{d}(x) - \bar{u}(x)] dx = \begin{cases} 0.148 \pm 0.039 \text{ (NMC)}, \\ 0.118 \pm 0.012 \text{ (E866)}, \\ 0.16 \pm 0.03 \text{ (HERMES)}. \end{cases}$$

What we know about the polarized sea $\int_0^1 [\Delta \bar{u}(x) + \Delta \bar{d}(x)] dx = ?$

To answer this question, consider the flavor symmetry of quarks $SU_f(3)$ ($m_u \cong m_d \cong m_s \cong 0$). We introduce the following notation

$$\begin{aligned} a_0 &= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}, \\ a_3 &= \Delta u + \Delta \bar{u} - (\Delta d + \Delta \bar{d}), \\ a_8 &= \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2(\Delta s + \Delta \bar{s}), \end{aligned} \quad (43)$$

where

$$\Delta q = \int_0^1 \Delta q(x) dx, \quad \Delta \bar{q} = \int_0^1 \Delta \bar{q}(x) dx.$$

From (43) we find the sum $\Delta \bar{u} + \Delta \bar{d}$:

$$\Delta \bar{u} + \Delta \bar{d} = (\Delta s + \Delta \bar{s}) + \frac{1}{2}(a_8 - \Gamma_v), \quad (44)$$

here Γ_v – is the first moment of the valence polarized quarks in the proton:

$$\Gamma_v = \Delta u_v + \Delta d_v = \int_0^1 [\Delta u_v(x) + \Delta d_v(x)] dx. \quad (45)$$

Thus, knowing the distribution of valence quarks in a nucleon $\Delta u_v(x) + \Delta d_v(x)$, we can find the sum $\Delta \bar{u} + \Delta \bar{d}$. The dependence of this sum on is a straight line (see Fig. 5, which shows the dependence of $\Delta \bar{u} + \Delta \bar{d}$ on Γ_v).

From experiments carried out by the COMPASS collaboration on the DIS of leptons on nucleons, the value a_0 :

$$a_0 = 0.35 \pm 0.03 \pm 0.15.$$

By the width of the decays of hyperons found a_8 :

$$a_8 = 0.59 \pm 0.03.$$

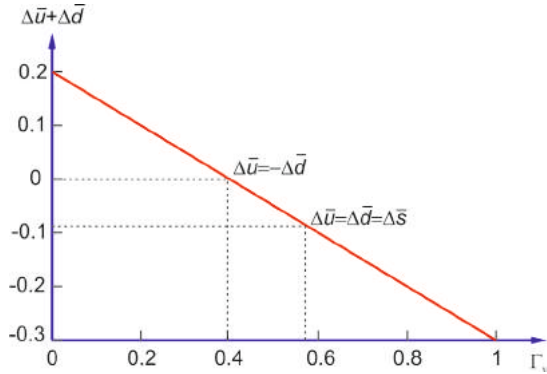


Fig. 5. Dependence of the amount $\Delta \bar{u} + \Delta \bar{d}$ on Γ_v .

Knowing (a_0) and (a_8), from formulas (43) we find the sum ($\Delta s + \Delta \bar{s}$):

$$\Delta s + \Delta \bar{s} = \frac{1}{3}(a_0 - a_8) = -0.09 \pm 0.01 \pm 0.02.$$

For the sum of the polarized sea $\Delta \bar{u} + \Delta \bar{d}$, the following linear dependence is obtained:

$$\Delta \bar{u}_v + \Delta \bar{d}_v = 0.2 - 0.5\Gamma_v. \quad (46)$$

With a symmetric distribution of quarks in a nucleon, we have:

$$\Gamma_v = a_8 = 0.59 \text{ and } \Delta \bar{u} = \Delta \bar{d} = -0.075.$$

If $\Gamma_v = 0.4$, then the distribution of polarized quarks in the proton is asymmetric:

$$\Delta \bar{u} + \Delta \bar{d} = 0 \rightarrow \Delta \bar{u} = -\Delta \bar{d}.$$

From formula (41), we determine the distribution of polarized valence quarks in a nucleon:

$$\begin{aligned} \Delta u_v(x) + \Delta d_v(x) &= A_d^{\pi^+ - \pi^-}(x) \cdot (u_v(x) + d_v(x)) \\ \Delta \bar{u} &= \Delta \bar{d} = \Delta \bar{s} \end{aligned} \quad (47)$$

where $A_d^{\pi^+ - \pi^-}(x)$ is the «difference» longitudinal spin asymmetry, $(u_v(x) + d_v(x))$ – is the distribution of valence unpolarized quarks in a proton.

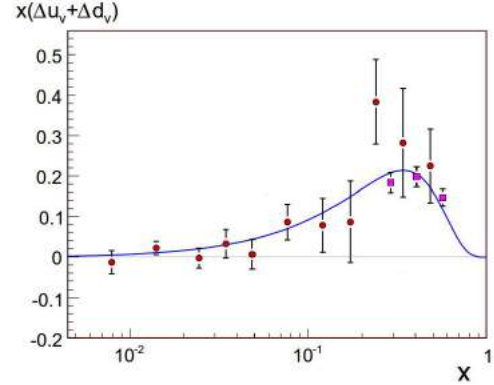


Fig. 6. Dependence of polarized valence quark Distributions $x(\Delta u_v + \Delta d_v)$ from a variable x .

The x distribution of polarized valence quarks in the nucleon was studied by the COMPASS collaboration in the SIDIS processes of polarized muons with an energy of 160 GeV on a polarized deuteron target at $Q^2 = 10 \text{ GeV}^2$ and $0.006 < x < 0.7$ [20]. In Fig. 6 shows the dependence of the valence polarized quark distributions $x(\Delta u_v + \Delta d_v)$ on the variable x . The experimental results of COMPASS with the corresponding errors are also noted there. The solid line corresponds to the distribution functions of polarized quarks from [17]. The results of the COMPASS experiments show that the first moment of polarized quark distributions is

$$\int_{0.006}^{0.7} [\Delta u_v(x) + \Delta d_v(x)] dx = 0.40 \pm 0.08 \pm 0.05.$$

This result is consistent with the fact that the asymmetric distribution of quarks ($\Delta \bar{u} = -\Delta \bar{d}$) prevails over the symmetric distribution ($\Delta \bar{u} = \Delta \bar{d}$).

It should be noted that the longitudinal spin asymmetries $A_{N1}^{h\pm}$, $A_{N2}^{h\pm}$, $A_1(e_R^- - e_L^+)$, $A_2(e_L^- - e_R^+)$, A_1 and A_2 , in contrast to the «difference» spin asymmetries, depend on the functions of quark fragmentation into a hadron h^\pm . For example, the asymmetries $A_{p1}^{\pi^+}$ and A_1 in the reactions $e^\mp + p \rightarrow e^\mp + \pi^+ + X$ are determined by the formulas (only the contribution of the diagram with the γ -quant exchange is taken into account):

$$\begin{aligned}
 A_{p1}^{\pi^+} = -A_1 = f(y) \{ & Q_u^2 [\Delta u_v(x) D_u^{\pi^+}(z) + \Delta u_s(x) (D_u^{\pi^+}(z) + D_{\bar{u}}^{\pi^+}(z))] + \\
 & + Q_d^2 [\Delta d_v(x) D_d^{\pi^+}(z) + \Delta d_s(x) (D_d^{\pi^+}(z) + D_{\bar{d}}^{\pi^+}(z) + 2D_s^{\pi^+}(z))] \} \times \\
 & \times \{ Q_u^2 [u_v(x) D_u^{\pi^+}(z) + u_s(x) (D_u^{\pi^+}(z) + D_{\bar{u}}^{\pi^+}(z))] + \\
 & + Q_d^2 [d_v(x) D_d^{\pi^+}(z) + d_s(x) (D_d^{\pi^+}(z) + D_{\bar{d}}^{\pi^+}(z) + 2D_s^{\pi^+}(z))] \}^{-1}. \quad (48)
 \end{aligned}$$

As for the asymmetry $A_1(e_R^- - e_L^+)$, we note that, in contrast to other longitudinal two-spin asymmetries, this asymmetry is due to the longitudinal polarizations of the lepton and antilepton. This one-spin asymmetry P – is odd and is expressed by the formula:

$$\begin{aligned}
 A_1(e_R^- - e_L^+) = f(y) \{ & [F_{RR}^2(u) - F_{RL}^2(u)] [u_v(x) D_u^{\pi^+}(z) + u_s(x) (D_u^{\pi^+}(z) - D_{\bar{u}}^{\pi^+}(z))] + \\
 & + [F_{RR}^2(d) - F_{RL}^2(d)] [d_v(x) D_d^{\pi^+}(z) + d_s(x) (D_d^{\pi^+}(z) - D_{\bar{d}}^{\pi^+}(z))] \} \times \\
 & \times \{ [F_{RR}^2(u) + F_{RL}^2(u)] [u_v(x) D_u^{\pi^+}(z) + u_s(x) (D_u^{\pi^+}(z) + D_{\bar{u}}^{\pi^+}(z))] + \\
 & + [F_{RR}^2(d) + F_{RL}^2(d)] [d_v(x) D_d^{\pi^+}(z) + d_s(x) (D_d^{\pi^+}(z) + D_{\bar{d}}^{\pi^+}(z) + 2D_s^{\pi^+}(z))] \}^{-1}. \quad (49)
 \end{aligned}$$

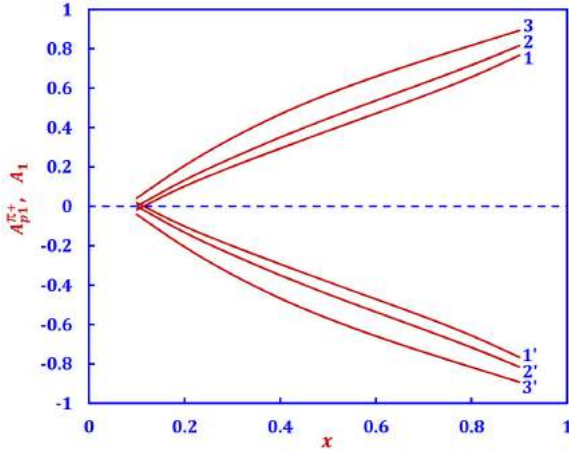


Fig. 7. Dependence of asymmetries $A_{p1}^{\pi^+}$ (curves 1, 2 and 3) and A_1 (curves 1', 2' and 3') on the variable x at $z = 0.2, 0.5$ and 0.8 respectively.

In Fig. 7 shows the dependence of the longitudinal spin asymmetries $A_{p1}^{\pi^+}$ and A_1 on the variable x at $y=1$ and a fixed value $z = 0.2, z = 0.5$ and $z = 0.8$. The asymmetry A_{p1} (A_1) is positive (negative) and with an increase in the variable x it increases (decreases), an increase in the variable z leads to an increase (decrease) in this asymmetry.

Fig. 8 illustrates the dependence of asymmetries $A_{p1}^{\pi^+}$ and A_1 on variable z at $y=1$ and a fixed value $x = 0.2, x = 0.5$ and $x = 0.8$. Asymmetry A_{p1} (A_1) is positive (negative) and with increasing variable x slowly increases (decreases). In Fig. 9 shows the dependence of the P -odd longitudinal spin asymmetry $A_1(e_R^- - e_L^+)$ on the variable x at a fixed value of $z = 0.2, 0.5$ and 0.8 (curves 1, 2, and 3). It is observed here that with an increase in the variable x , the asymmetry $A_1(e_R^- - e_L^+)$ increases and reaches a maximum at $x = 0.6$, and a further increase in the variable leads to a decrease in this asymmetry.

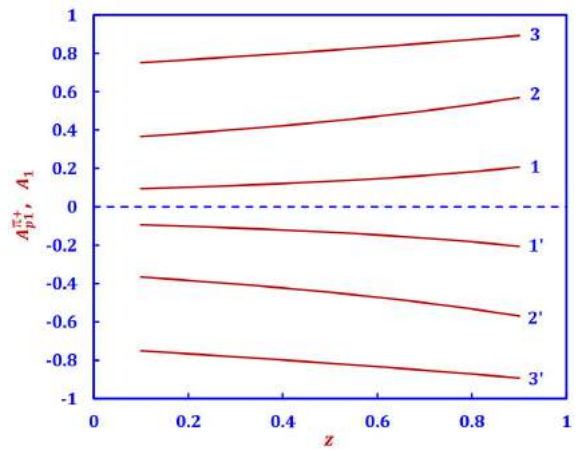


Fig. 8. Dependence of asymmetries $A_{p1}^{\pi^+}$ (curves 1, 2 and 3) and A_1 (curves 1', 2' and 3') on the variable z at $x = 0.2, 0.5$ and 0.8 , respectively.

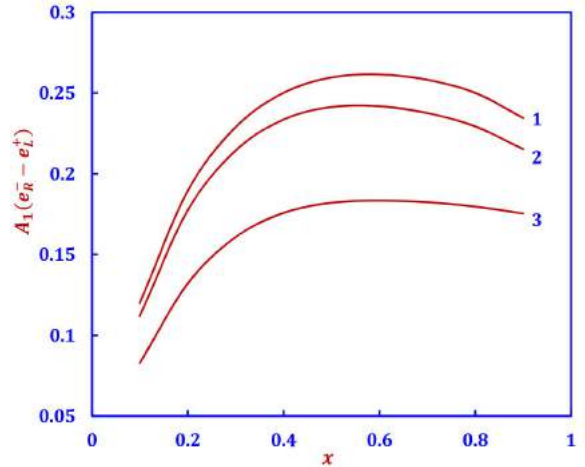


Fig. 9. Dependence of the P -odd longitudinal spin asymmetry $A_1(e_R^- - e_L^+)$ on the variable x at fixed $z = 0.2$ (curve 1), 0.5 (curve 2) and 0.8 (curve 3).

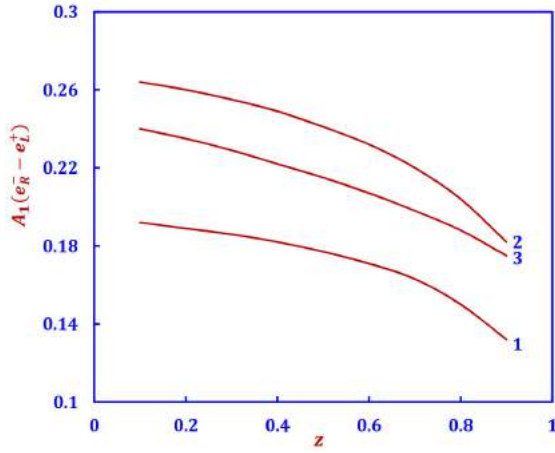


Fig. 10. Dependence of the P -odd longitudinal spin asymmetry $A_1(e_R^- - e_L^+)$ on the variable z at fixed $x = 0.2$ (curve 1), 0.5 (curve 2) and 0.8 (curve 3).

In Fig. 10 illustrates the dependence of the

asymmetry $A_1(e_R^- - e_L^+)$ on the variable z at fixed $x = 0.2$, 0.5 and 0.8 (curves 1, 2 and 3). The asymmetry is positive and decreases with increasing z .

CONCLUSION

Here we discussed the longitudinal spin asymmetries $A_N^{h^+h^-}$, $A_N^{h^+}$, $A_N^{h^-}$, $A_1(e_R^- - e_L^+)$, A_1 in semi-inclusive deep inelastic scattering of polarized leptons (antileptons) by polarized nucleons: $\ell^\pm + N \rightarrow \ell^\pm + h^\pm + X$. By introducing the hadron structure functions, analytical expressions for the differential cross sections of the processes are obtained, the hadron structure functions are calculated in the quark-parton model, the longitudinal spin asymmetries are determined, and the asymmetry behavior depending on the variables x , y and z is studied in detail.

The longitudinal spin asymmetry $A_d^{\pi^+\pi^-}$ is compared with the results of the COMPASS experiments.

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Received:17.11.2020