# NEUTRALINO PAIR PRODUCTION IN POLARIZED LEPTON-ANTILEPTON COLLISIONS

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The process of producing a neutralino pair in arbitrarily polarized lepton-antilepton (electron-positron or muon-antimuon) collisions has been studied within the Minimal Supersymmetric Standard Model: $\ell^- + \ell^+ \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0$. We consider s-channel diagrams with neutral Z-boson and Higgs-boson $H$ ($h$ or $A$) exchanges, and t-channel diagrams with scalar $\tilde{\ell}_L^-$ and $\tilde{\ell}_R^+$-lepton exchanges. General expressions for the differential and integral cross sections of the process are obtained, transverse and longitudinal spin asymmetries due to lepton-antilepton pair polarizations, and degrees of longitudinal and transverse neutralino polarization are determined. Angular and energy dependences of cross sections and polarization characteristics of the process are studied in detail.

**Keywords**: Standard Model, Minimal Supersymmetric Standard Model, lepton-antilepton pair, neutralino, Higgs boson, effective cross section.
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## 1. INTRODUCTION

The discovery of a scalar Higgs boson with characteristics corresponding to the Standard Model (SM) predictions was made at the Large Hadron Collider (LHC) by the ATLAS and CMS collaborations in 2012. [1, 2] (see also reviews [3-5]). With its discovery the missing brick in the CM building was found. The way to the discovery was long and the fact of the discovery itself meant the beginning of great work to verify the validity of the detected signal and clarify its nature, determining the properties of the new particle.

Higgs boson $H_{\rm SM}$ is unstable particle and can decay through different channels. It was discovered at the LHC by studying decays to two photons ( $H_{\rm SM} \to \gamma + \gamma$), decays to two vector bosons $ZZ^*$ and $WW^*$ (here $Z^*$ and $W^*$-virtual bosons). Neutral bosons $Z$ were identified by decay channels into two leptons: $e^-e^+$- or $\mu^-\mu^+$-pair. This is written as $H_{\rm SM} \to ZZ^* \to 4\ell$, where $\ell$ – one of the leptons is $e^{\mp}$, $\mu^{\mp}$. The decays of the $W$ -boson pair were identified by the channel $H_{\rm SM} \to WW^* \to \ell\nu\ell\nu$, where $\nu$ – is the electron or muon neutrino.

Based on the decay of the Higgs boson into two photons $H_{\rm SM} \to \gamma + \gamma$ its mass is found to be $M_{H_{\rm SM}}(\gamma\gamma) = 126.0 \pm 0.4$ (stat.) $\pm 0.4$ (syst.) GeV [1]. For the decay to four leptons $M_{H_{\rm SM}}(4\ell) = 126.8 \pm 0.2$ (stat.)$\pm$0.7 (syst.) GeV [6]. For a complete picture it is useful to cite the results of the CSM experiment [2], which performed the discovery of a new particle simultaneously with the ATLAS collaboration: $M_{H_{\rm SM}}(\gamma\gamma) = 125.3 \pm 0.4$ (stat.)$\pm$0.5 (syst.) GeV. A mass value of $M_{H_{\rm SM}}(4\ell) = 125.6 \pm 0.4$ (stat.)$\pm$0.2 (system.) GeV was found for the decay channel $H_{\rm SM} \to ZZ^* \to 4\ell$. Consequently, the results of the ATLAS and CSM collaborations match the mass of the Higgs boson.

SM is a successful theory that describes all known elementary particles and strong, electromagnetic, weak interactions between them (the gravitational interaction so far is described by Einstein's general theory of relativity). On the basis of SM one can make accurate calculations and compare them with the corresponding experimental data. The agreement between SM and experiment is strikingly good.

However, SM has its shortcomings. For example, the key point of SM is the Higgs mechanism of electrically weak symmetry, which successfully describes the generation of elementary particle masses. Unfortunately, SM does not give any explanation why there is a Higgs field at all and why it has such property – to form a vacuum condensate.

The second shortcoming of SM is connected with renormalization of the Higgs boson mass. The fact is that for all SM particles the mass renormalization works well. However, in the case of the Higgs boson virtual particles have a strong influence on the mass by trillions of times. Inside SM there is no constraint stopping the Higgs boson mass growth at the expense of virtual particles. This drawback can be eliminated in the following way. If some other particles exist in nature, they in virtual form can compensate their influence on Higgs boson mass. The most important thing here is that in the Minimal Supersymmetric Standard Model (MSSM) such compensation arises by construction of the theory itself. It is such theories that most attract physicists.

According to SM, neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$ are massless particles. However, experiments prove that neutrinos have mass, and in addition, they are very actively mixing with each other, passing from one kind to another. All this suggests that the mass and mixing of neutrinos is not due to the Higgs mechanism, but to a phenomenon of some other nature.



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Again, there are no such phenomena in SM, while there are plenty of such mechanisms in models outside of SM.

The absence of dark matter particles in SM is one of the difficulties of this model. Astro-physicists believe that in the Universe, besides ordinary matter in the form of stars, black hole planets, gas-dust clouds, neutrinos, etc., there are also particles of a completely different nature. We do not see these particles, they are neutral and practically do not interact with ordinary matter and radiation. In the SM there is not a single particle suitable for this role. However, in the MSSM there are such particles as neutralino, sneitrino, gluino, gravitino, which may be candidates for dark matter.

The above facts and a number of other reasons indicate the need to go beyond SM. In this case, the main attention is paid to the MSSM [7-10]. In this model, in contrast to SM, two scalar field doublets with hypercharges –1 and +1 are introduced:

$$\varphi_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad \varphi_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}.$$

To obtain the physical fields of Higgs bosons and determine their masses, the scalar fields $\varphi_1$ and $\varphi_2$ decompose into real and imaginary parts around the vacuum

$$\varphi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \upsilon_1 + H_1^0 + iP_1^0 \\ H_1^- \end{pmatrix},$$

$$\varphi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} H_2^+ \\ \upsilon_2 + H_2^0 + iP_2^0 \end{pmatrix},$$

where $<\varphi_1>= \frac{1}{\sqrt{2}}\upsilon_1$ and $<\varphi_2>= \frac{1}{\sqrt{2}}\upsilon_2$ are the vacuum values of the Higgs boson fields. Mixing the fields $H_1^0$ and $H_2^0$ obtain the CP-even $H$ and $h$ Higgs bosons (mixing angle $\alpha$):

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} H_1^0 \\ H_2^0 \end{pmatrix}.$$

Similarly mix the fields $P_1^0$ and $P_2^0$ ($H_1^\pm$ and $H_2^\pm$) and obtain a Goldston $G^0$-boson and CP odd Higgs boson $A$ (charged Goldston $G^\pm$- and Higgs bosons $H^\pm$) (mixing angle $\beta$):

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} P_1^0 \\ P_2^0 \end{pmatrix},$$

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix}\begin{pmatrix} H_1^\pm \\ H_2^\pm \end{pmatrix}.$$

Thus, there are five Higgs bosons in the MSSM: the CP-even $H$ and $h$-bosons, the CP-odd $A$-boson, and the charged $H^+$- and $H^-$-bosons. The Higgs sector is characterized by the mass parameters $M_H$, $M_h$, $M_A$, $M_{H^\pm}$ and the mixing angles of the scalar fields $\alpha$ and $\beta$. Of these, only two parameters are considered to be free: the mass $M_A$ and the angle $\mathrm{tg}\beta = \upsilon_2/\upsilon_1$. The other parameters are expressed through them:

$$M_{H(h)}^2 = \frac{1}{2}[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2 M_Z^2 \cos^2 2\beta}\,], \quad M_{H^\pm}^2 = M_A^2 + M_Z^2,$$

$$\mathrm{tg}2\alpha = \mathrm{tg}2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad \left(-\frac{\pi}{2} \le \alpha < 0\right),$$

where $M_W$ and $M_Z$ – are the masses of gauge $W^\pm$- and $Z$-bosons.

The supersymmetric (SUSY) partners of gauge $W^\pm$- and Higgs $H^\pm$-bosons are calibrino $\widetilde{W}^\pm$ and higgsino $\widetilde{H}^\pm$. These spinor fields mix and new chargino $\widetilde{\chi}_{1,2}^\pm$ states appear. The neutral counterparts of charginos are called neutralinos and there are four of them $\widetilde{\chi}_j^0$ ($j = 1, 2, 3\,4$). They arise as a result of mixing binos $\widetilde{B}^0$, vino $\widetilde{W}_3^0$ and higgsinos $\widetilde{H}_1^0$, $\widetilde{H}_2^0$. The mass matrix of the neutralino is non-diagonal and depends on the mass parameters wine $M_2$, higgsino $\mu$ and wine $M_1$, as well as on the parameter $\mathrm{tg}\beta$ [7, 8, 11-13]:

$$M_N = \begin{pmatrix} M_1 & 0 & -M_Z\sin\theta_W\cos\beta & M_Z\sin\theta_W\sin\beta \\ 0 & M_2 & M_Z\cos\theta_W\cos\beta & -M_Z\cos\theta_W\sin\beta \\ -M_Z\sin\theta_W\cos\beta & M_Z\cos\theta_W\cos\beta & 0 & -\mu \\ M_Z\sin\theta_W\sin\beta & -M_Z\cos\theta_W\sin\beta & -\mu & 0 \end{pmatrix}.$$

This matrix can be diagonalized by one real matrix $Z$. Expressions of the matrix elements of this matrix $Z_{ij}$ ($i, j = 1, 2, 3\ 4$) and the neutralino mass are given in [11, 13]. For large values of the parameter $|\mu| >> M_{1,2} >> M_Z$, the masses of the neutralino are:

$$m_{\chi_1^0} \approx M_1 - \frac{M_Z^2}{\mu^2}(M_1 + \mu \sin 2\beta)\sin^2\theta_W ,$$

$$m_{\chi_2^0} \approx M_2 - \frac{M_Z^2}{\mu^2}(M_2 + \mu \sin 2\beta)\cos^2\theta_W ,$$

$$m_{\chi_{3/4}^0} \approx |\mu| + \frac{M_Z^2}{2\mu^2}\varepsilon_\mu (1 \mp \sin 2\beta)(\mu \pm M_2 \sin^2\theta_W \mp M_1 \cos^2\theta_W) ,$$

$\theta_W$ – Weinberg angle, $\varepsilon_\mu = \frac{\mu}{|\mu|}$ – sign of the parameter $\mu$. At $|\mu| \to \infty$ two neutralinos correspond to the calibrino state with masses $m_{\chi_1^0} \approx M_1$ and $m_{\chi_2^0} \approx M_2$, and other neutralinos correspond to the higgsino state with masses $m_{\chi_3^0} \approx m_{\chi_4^0} \approx |\mu|$.

Supersymmetric (SUSY) chargino and neutralino particles can be born in the LHC in cascade decays of squarks and gluinos: $\tilde{g} \to q\tilde{q}$, $\tilde{q} \to q\tilde{\chi}_i$. Note that chargino or neutralino pairs can be born in high-energy lepton-antilepton (electron-positron and muon-antimuon) colliders:

$$\ell^- + \ell^+ \to \tilde{\chi}_i^- + \tilde{\chi}_j^+, \quad \ell^- + \ell^+ \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0 .$$

These processes in the case of nonpolarized initial and final particles are considered in [14, 15]. The production of SUSY particles with spin 0 or 1/2 in polarized electron-positron collisions has been studied in [16-18]. In previous papers [19, 20] we have considered the process of chargino pair production in arbitrarily polarized lepton-antilepton interactions. Diagrams with photon and Z-boson exchange, with Higgs boson exchange $H$ ($h$ or $A$), and with scalar neutrino $\tilde{\nu}_L$ exchange have been studied in detail. It is found that in diagrams with photon and Z-boson exchange the lepton and antilepton must have opposite helicities ($\ell_R^-\ell_L^+$ or $\ell_L^-\ell_R^+$); in diagrams with Higgs boson exchange $H$ ($h$ or $A$) the lepton and antilepton must have identical helicities ($\ell_L^-\ell_L^+$ or $\ell_R^-\ell_R^+$); the diagram with sneutrino $\tilde{\nu}_L$ exchange is characterized by the fact that the lepton and antilepton can have only the left helicity ($\ell_L^-\ell_L^+$).

The purpose of the present paper is to study the process of neutralino pair production in arbitrarily polarized lepton-antilepton collisions

$$\ell^- + \ell^+ \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0, \qquad (1)$$

here $\ell^-\ell^+$ – is the lepton-antilepton (electron-positron and or muon-antimuon) pair, $\tilde{\chi}_i^0\tilde{\chi}_j^0$ – neutralino pair. Within the MSSM framework and taking into account the arbitrary polarizations of the lepton-antilepton pair, a general expression for the effective cross section of the process (1) is obtained. The longitudinal and transverse spin asymmetries due to the lepton-antilepton pair polarizations and the degrees of longitudinal and transverse neutralino polarization were determined. In particular, it is shown that the longitudinal spin asymmetry arising from the annihilation of longitudinally polarized leptons with nonpolarized antileptons is equal in magnitude and opposite in sign to the asymmetry arising from the annihilation of longitudinally polarized antileptons with nonpolarized leptons.

## 2. THE AMPLITUDE AND CROSS SECTION OF THE PROCESS $\ell^-\ell^+ \to (Z^*) \to \tilde{\chi}_i^0\tilde{\chi}_j^0$

The annihilation of a lepton-antilepton pair into a neutralino pair is described by the Feynman diagrams in Fig. 1. Diagram a) s-channel diagram with Z-boson exchange, diagram b) also s-channel diagram with Higgs boson exchange $H$ ($h$ or $A$) (this diagram plays an important role in muon-antimuon annihilation). Diagrams c) and d) are t-channel diagrams with an exchange of sleptons $\tilde{\ell}_L$ and $\tilde{\ell}_R$.

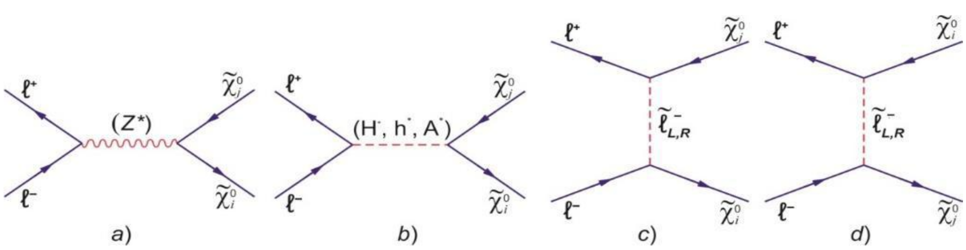


*Fig. 1.* Feynman diagrams for the reaction $\ell^-\ell^+ \to \tilde{\chi}_i^0\tilde{\chi}_j^0$.

The Lagrangians of the Z-boson interactions with a lepton-antilepton pair and neutralino pair are written in the following form:

$$L_{Z\ell\ell} = -\frac{ig}{\cos\theta_W}\bar{\ell}\gamma_\mu(g_L P_L + g_R P_R)\ell Z_\mu\,, \tag{2}$$

$$L_{Z\tilde{\chi}_i^0\tilde{\chi}_j^0} = \frac{ig}{2\cos\theta_W}\tilde{\chi}_i^0\gamma_\mu(g^L_{Z\tilde{\chi}_i^0\tilde{\chi}_j^0}P_L + g^R_{Z\tilde{\chi}_i^0\tilde{\chi}_j^0}P_R)\tilde{\chi}_j^0 Z_\mu\,, \tag{3}$$

here $g = \frac{e}{\sin\theta_W}$ – is the electroweak interaction constant, $g_L$ and $g_R$ ($g^L_{Z\tilde{\chi}_i^0\tilde{\chi}_j^0} \equiv G_L$ and $g^R_{Z\tilde{\chi}_i^0\tilde{\chi}_j^0} \equiv G_R$) – are the left and right interaction constants of the lepton (neutralino) with the Z-boson:

$$g_L = -\frac{1}{2} + \sin^2\theta_W\,, \quad g_R = \sin^2\theta_W\,, \tag{4}$$

$$G_L = -\frac{1}{2\sin\theta_W}[Z_{i3}Z_{j3} - Z_{i4}Z_{j4}],\ \ G_R = \frac{1}{2\sin\theta_W}[Z_{i3}Z_{j3} - Z_{i4}Z_{j4}], \tag{5}$$

$P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$ – kirality matrices.

Diagram (a) of Fig. 1 corresponds the amplitude

$$M^{(Z)}_{i\to f} = \frac{g^2}{2\cos^2\theta_W}\upsilon(p_2,s_2)\gamma_\mu(g_L P_L + g_R P_R)u(p_1,s_1)\cdot D_Z(s)\times$$
$$\times\bar{u}_i(k_1,s)\gamma_\mu(G_L P_L + G_R P_R)\upsilon_j(k_2,s')\,, \tag{6}$$

where $p_1(s_1)$, $p_2(s_2)$, $k_1(s)$ and $k_2(s')$ – are the 4-momentum (polarization vectors) of the lepton, antilepton, and neutralino $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$, $D_Z(s) = (s - M_Z^2 + i\Gamma_Z M_Z)^{-1}$, $s = (p_1 + p_2)^2$ – is the square of the total energy of the lepton-antilepton pair, $\Gamma_Z$ – is the total width of the Z-boson.

We find in the standard way for the modulus of the square of the amplitude (6):

$$\left|M^{(Z)}_{i\to f}\right|^2 = \frac{g^4|D_Z(s)|^2}{4\cos^4\theta_W}L_{\mu\nu}\times\chi_{\mu\nu}\,, \tag{7}$$

where the expressions for the lepton $L_{\mu\nu}$ and neutralino $\chi_{\mu\nu}$ tensors are given in the Appendix.

In the case when the lepton-antilepton pair is polarized arbitrarily and summation is performed on the polarization neutralino states, the expression for the modulus of the amplitude square (6) is obtained:

$$\left|M^{(Z)}_{i\to f}\right|^2 = \frac{g^4|D_Z(s)|^2}{4\cos^4\theta_W}\{(G_L^2 + G_R^2)[(k_1\cdot p_1)(k_2\cdot p_2) + (k_1\cdot p_2)(k_2\cdot p_1) -$$
$$- m_\ell^2((k_1\cdot s_1)(k_2\cdot s_2) + (k_1\cdot s_2)(k_2\cdot s_1))) + 2g_L g_R((p_1\cdot s_2)((k_1\cdot s_1)(k_2\cdot p_2) + (k_2\cdot s_1)(k_1\cdot p_2) -$$
$$-(p_1\cdot p_2)((k_1\cdot s_1)(k_2\cdot s_2) + (k_1\cdot s_2)(k_2\cdot s_1)) + (p_2\cdot s_1)((k_1\cdot s_2)(k_2\cdot p_1) + (k_2\cdot s_2)(k_1\cdot p_1) -$$
$$-(k_1\cdot k_2)(p_1\cdot s_2)) - (s_1\cdot s_2)((k_1\cdot p_1)(k_2\cdot p_2) + (k_1\cdot p_2)(k_2\cdot p_1) - (k_1\cdot k_2)(p_1\cdot p_2)) +$$
$$-(g_L^2 - g_R^2)m_\ell((k_1\cdot p_1)(k_2\cdot s_2) + (k_2\cdot p_1)(k_1\cdot s_2) - (k_1\cdot p_2)(k_2\cdot s_1) - (p_2\cdot k_2)(k_1\cdot s_1)] +$$
$$+2G_L G_R m_{\chi_i}m_{\chi_j}[(g_L^2 + g_R^2)((p_1\cdot p_2) - m_\ell^2(s_1\cdot s_2) + (g_L^2 - g_R^2)((p_1\cdot s_2) - (p_2\cdot s_1))] +$$
$$+(G_L^2 - G_R^2)[(g_L^2 + g_R^2)m_\ell((p_1\cdot k_2)(k_1\cdot s_2) - (p_1\cdot k_1)(k_2\cdot s_2) + (k_2\cdot p_2)(k_1\cdot s_1) - (k_1\cdot p_2)(k_2\cdot s_1)) +$$
$$+(g_L^2 - g_R^2)((k_1\cdot p_2)(k_2\cdot p_1) - (k_1\cdot p_1)(k_2\cdot p_2) + m_\ell^2((k_1\cdot s_1)(k_2\cdot s_2) - (k_1\cdot s_2)(k_2\cdot s_1)) +$$
$$+g_R g_L m_\ell((k_2\cdot p_2)(k_1\cdot s_2) - (k_1\cdot p_2)(k_2\cdot s_2) + (k_1\cdot p_1)(k_2\cdot s_1) - (k_2\cdot p_1)(k_1\cdot s_1))]\}\,, \tag{8}$$

where $m_\ell$ – is the lepton mass.

Using calculations based on (8), for arbitrary polarization of the colliding lepton-antilepton beams in the center-of-mass system, we have the following expression for the differential cross section reaction (1):

$$\frac{d\sigma^{(Z)}}{d\Omega} = \frac{g^4 s|D_Z(s)|^2}{256\pi^2\cos^4\theta_W}\sqrt{\lambda(r_i,r_j)}\{[g_L^2(1-\lambda_1)(1+\lambda_2) + g_R^2(1+\lambda_1)(1-\lambda_2)]\times$$

$$\times[(G_L^2+G_R^2)((1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta)+4G_LG_R\sqrt{r_ir_j}]+g_Lg_R(G_L^2+G_R^2)\eta_1\eta_2\lambda(r_i,r_j)\times$$
$$\times\sin^2\theta\cos(2\varphi-\phi)+[g_L^2(1-\lambda_1)(1+\lambda_2)-g_R^2(1+\lambda_1)(1-\lambda_2)](G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}\cos\theta\}, \quad (9)$$

here $\lambda_1$ and $\lambda_2$ – are the helicities of the lepton and antilepton, $\eta_1$ and $\eta_2$ – are the transverse components of the spin vectors of the lepton-antilepton pair, $\theta$ – the angle of departure of the neutralino $\tilde{\chi}_j^0$ with respect to the lepton momentum direction, $\varphi$ – the azimuthal angle of departure of the neutralino, and $\phi$– the angle between the vectors $\vec{\eta}_1$ and $\vec{\eta}_2$, $r_i=\left(\frac{m_{\tilde{\chi}_i^0}}{\sqrt{s}}\right)^2$, $r_j=\left(\frac{m_{\tilde{\chi}_j^0}}{\sqrt{s}}\right)^2$, $\lambda(r_i,r_j)$ – is the kinematic function of the two-particle phase volume:

$$\lambda(r_i,r_j)=(1-r_i-r_j)^2-4r_ir_j .$$

Let us analyze the differential cross section (9) in various cases of lepton-antilepton pair polarization. It is well known that electrons and positrons moving in storage rings acquire predominantly transverse polarization due to synchrotron radiation. In the case when the lepton-antilepton pair is polarized transversely, the differential cross section of the process (1) has the form:

$$\frac{d\sigma^{(Z)}(\eta_1,\eta_2)}{d\Omega}=\frac{d\sigma_0^{(Z)}(\theta)}{d\Omega}[1+\eta_1\eta_2A_\perp(\theta,\varphi)]. \quad (10)$$

Here

$$\frac{d\sigma_0^{(Z)}(\theta)}{d\Omega}=\frac{g^4s|D_Z(s)|^2}{256\pi^2\cos^4\theta}\sqrt{\lambda(r_i,r_j)}\{(g_L^2+g_R^2)[(G_L^2+G_R^2)((1+r_i-r_j)(1-r_i+r_j)+$$
$$+\lambda(r_i,r_j)\cos^2\theta)+4G_LG_R\sqrt{r_ir_j}]+(g_L^2-g_R^2)(G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}\cos\theta\} \quad (11)$$

– is the differential cross section of the process averaged over the polarization states of the lepton-antilepton pair, a $A_\perp(\theta,\varphi)$ – is the azimuthal angular (or transverse spin) asymmetry determined by the formula (the angle $\phi$ is assumed $\phi=\pi$):

$$A_\perp(\theta,\varphi)=\frac{d\sigma(\theta,2\varphi)/d\Omega-d\sigma(\theta,\pi-2\varphi)/d\Omega}{d\sigma(\theta,2\varphi)/d\Omega+d\sigma(\theta,\pi-2\varphi)/d\Omega}=-g_Lg_R(G_L^2+G_R^2)^2\lambda(r_i,r_j)\sin^2\theta\cos2\varphi\times$$
$$\times\{(g_L^2+g_R^2)[(G_L^2+G_R^2)[(1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta)+4G_LG_R\sqrt{r_ir_j}]+$$
$$+(g_L^2-g_R^2)(G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}\cos\theta\}^{-1}. \quad (12)$$

The differential cross section of the process $\ell^-\ell^+\to(Z^*)\to\tilde{\chi}_i^0\tilde{\chi}_j^0$ in the case of nonpolarized particles (11) is not symmetric when the polar angle is replaced by $\theta\to\pi-\theta$. Hence, the angular distribution neutralino possesses asymmetry. The forward-backward angular asymmetry is defined by formula

$$A_{FB}(\theta)=\frac{d\sigma_0^{(Z)}(\theta)/d(\cos\theta)-d\sigma_0^{(Z)}(\pi-\theta)/d(\cos\theta)}{d\sigma_0^{(Z)}(\theta)/d(\cos\theta)+d\sigma_0^{(Z)}(\pi-\theta)/d(\cos\theta)} \quad (13)$$

and has the following form

$$A_{FB}(\theta)=\frac{g_L^2-g_R^2}{g_L^2+g_R^2}\frac{(G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}\cos\theta}{(G_L^2+G_R^2)[(1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta]+4G_LG_R\sqrt{r_ir_j}}. \quad (14)$$

Now assume that the lepton-antilepton pair, as well as the neutralino $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$ polarized longitudinally. In this case, let us represent the differential cross section of the process (1) as follows:

$$\frac{d\sigma^{(Z)}(\lambda_1,\lambda_2;h_1,h_2)}{d\Omega}=\frac{g^4s|D_Z(s)|^2}{1024\pi^2\cos^4\theta_W}\sqrt{\lambda(r_i,r_j)}\{[g_L^2(1-\lambda_1)(1+\lambda_2)+g_R^2(1+\lambda_1)(1-\lambda_2)]\times$$
$$\times[(G_L^2+G_R^2)((1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta-h_1h_2((1+r_i-r_j)(1-r_i+r_j)\cos^2\theta+\lambda(r_i,r_j)))+$$

$$+(G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}(h_2(1+r_i-r_j+(1-r_i+r_j)\cos^2\theta)-h_1(1-r_i+r_j+(1+r_i-r_j)\cos^2\theta))+$$

$$+8G_LG_R\sqrt{r_ir_j}(1-h_1h_2)]+[g_L^2(1-\lambda_1)(1+\lambda_2)-g_R^2(1+\lambda_1)(1-\lambda_2)]\times[(G_L^2+G_R^2)(h_2-h_1)((1+r_i-r_j)\times$$

$$\times(1-r_i+r_j)+\lambda(r_i,r_j)]+2(G_L^2-G_R^2)(1-h_1h_2)\sqrt{\lambda(r_i,r_j)}+8G_LG_R(h_2-h_1)\sqrt{r_i\cdot r_j}]\cos\theta\}, \qquad (15)$$

where $h_1$ and $h_2$ – are the helicities of the neutralino $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$.

As can be seen from the cross section, the lepton and the antilepton must have opposite helicities at annihilation: $\lambda_1=-\lambda_2=\pm1$. If the antilepton is polarized right $\lambda_2=+1(\ell_R^+)$, the lepton must have a left-handed helicity $\lambda_1=-1\ (\ell_L^-)$ and vice versa, if the left-handed antilepton is annihilated ($\lambda_2=-1$; $\ell_L^+$ ), the lepton must have a right-handed helicity: $\lambda_1=+1\ (\ell_R^-)$ (see Fig. 2, which shows the momentum and spin vectors of the lepton-antilepton pair). This is a consequence of the conservation of total momentum in the transition $\ell^-+\ell^+\to Z$. Indeed, consider this process in the center-of-mass system of the lepton-antilepton pair. In this system, the momenta of the lepton and antilepton are equal in magnitude and opposite in direction. In Fig. 2a), the helicity of the lepton is $\lambda_1=-1$, and the helicity of the antilepton is $\lambda_2=+1$. Hence, the projection of the total momentum of the lepton-antilepton pair in the direction of the antilepton momentum will be $+1$ (in units of $\hbar$); the spin of the $Z$-boson also equals $+1$, so the total momentum is conserved in the transition $\ell^-+\ell^+\to Z$.


*Fig. 2*. Impulse and spin $\ell^-\ell^+$-pair directions.

As for the helicities of neutralinos $h_1$ and $h_2$, we note that, according to formula (15), they can be arbitrary independently of each other ($h_1=\pm1$, $h_2=\pm1$). This is due to taking into account the masses $m_{\tilde{\chi}_i^0}$ and $m_{\tilde{\chi}_j^0}$ neutralinos. Suppose that the energy of the counter lepton-antilepton beams is much larger than the masses of the neutralinos ($\sqrt{s}>>m_{\tilde{\chi}_i^0}$, $m_{\tilde{\chi}_j^0}$), then we can neglect the mass terms of the neutralinos. As a result, for the cross section of the process (1) we have the expression:

$$\frac{d\sigma^{(Z)}(\lambda_1,\lambda_2;h_1,h_2)}{d\Omega}=\frac{g^4s|D_Z(s)|^2}{1024\pi^2\cos^4\theta_W}\{[g_L^2(1-\lambda_1)(1+\lambda_2)+g_R^2(1+\lambda_1)(1-\lambda_2)]\times$$

$$\times[G_L^2(1-h_1)(1+h_2)+G_R^2(1+h_1)(1-h_2)](1+\cos^2\theta)+[g_L^2(1-\lambda_1)(1+\lambda_2)-g_R^2(1+\lambda_1)(1-\lambda_2)]\times$$

$$\times[G_L^2(1-h_1)(1+h_2)-G_R^2(1+h_1)(1-h_2)]\cdot2\cos\theta\}. \qquad (16)$$

According to this formula, the neutralinos $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$ must have opposite helicities $h_1=-h_2=\pm1$. At high energies, the process $\ell^-+\ell^+\to\tilde{\chi}_i^0+\tilde{\chi}_j^0$ corresponds to four spiral cross sections:

$$\frac{d\sigma_{LL}^{(Z)}}{d\Omega}\sim[g_LG_L(1+\cos\theta)]^2,$$

$$\frac{d\sigma_{LR}^{(Z)}}{d\Omega}\sim[g_LG_R(1-\cos\theta)]^2,$$

$$\frac{d\sigma_{RR}^{(Z)}}{d\Omega}\sim[g_RG_R(1+\cos\theta)]^2,$$

$$\frac{d\sigma_{RL}^{(Z)}}{d\Omega}\sim[g_RG_L(1-\cos\theta)]^2. \qquad (17)$$

Here, the first and second indices at the cross section show the helicities of the lepton and neutralino $\tilde{\chi}_i^0$, respectively. For example, $\frac{d\sigma_{LL}^{(Z)}}{d\Omega}$ defines the cross section of the spiral process $\ell_L^-+\ell_R^+\to\tilde{\chi}_{iL}^0+\tilde{\chi}_{jR}^0$.

As can be seen from the expression of the spiral sections (17), the sections $\frac{d\sigma_{LL}^{(Z)}}{d\Omega}$ and $\frac{d\sigma_{RR}^{(Z)}}{d\Omega}$ are zero at $\theta=\pi$, and the sections $\frac{d\sigma_{LR}^{(Z)}}{d\Omega}$ and $\frac{d\sigma_{RL}^{(Z)}}{d\Omega}$ are zero at $\theta=0$. This is a consequence of the law of conservation of total momentum (see Fig. 3, where the directions of momenta and spins of initial and final particles are represented).

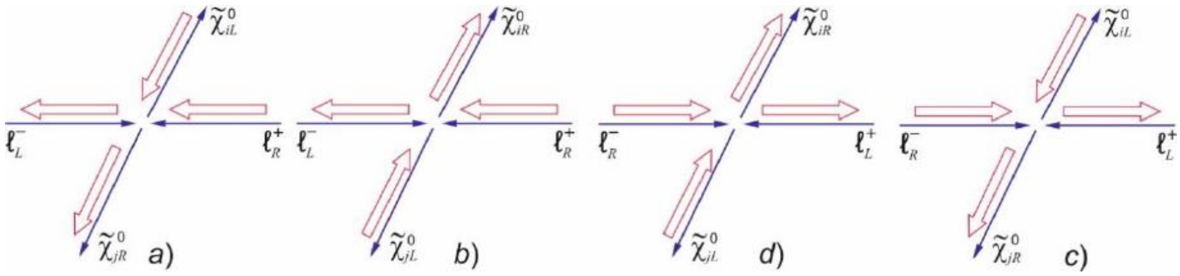


*Fig. 3.* Directions of impulses and spins of particles in the process $\ell^-\ell^+ \to \tilde{\chi}_i^0\tilde{\chi}_j^0$.

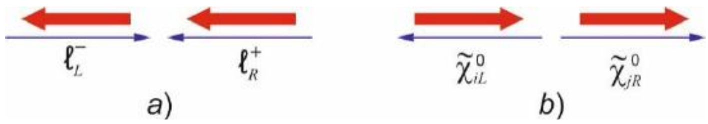


*Fig. 4.* Directions of impulses and spins of the particles at $\theta = \pi$.

Let us consider a spiral process $\ell_L^- + \ell_R^+ \to \tilde{\chi}_{iL}^0 + \tilde{\chi}_{jR}^0$ in the center-of-mass system at $\theta = \pi$. In this case the neutralino $\tilde{\chi}_i^0$ flies out against the momentum of the electron (Fig. 4).

The projection of the total momentum of the initial particles on the direction of the lepton momentum is $-1$. However, the projection of the total momentum of finite particles on the direction of the lepton momentum is equal $+1$ (see Fig. 4b). Thus, the law of conservation of the total momentum is not satisfied. Therefore, the departure of the neutralino $\tilde{\chi}_i^0$ against the lepton momentum is forbidden by the law of conservation of the total momentum. The multiplier $(1+\cos\theta)^2$ in the expression for the corresponding cross section corresponds to this.

Based on the differential effective cross section (15), let us determine the longitudinal spin asymmetry due to lepton (antilepton) polarization:

$$A_1(\theta) = \frac{1}{\lambda_1}\frac{d\sigma^{(Z)}(\lambda_1,0)/d\Omega - d\sigma^{(Z)}(-\lambda_1,0)/d\Omega}{d\sigma^{(Z)}(\lambda_1,0)/d\Omega + d\sigma^{(Z)}(-\lambda_1,0)/d\Omega},$$
$$A_2(\theta) = \frac{1}{\lambda_2}\frac{d\sigma^{(Z)}(0,\lambda_2)/d\Omega - d\sigma^{(Z)}(0,-\lambda_2)/d\Omega}{d\sigma^{(Z)}(0,\lambda_2)/d\Omega + d\sigma^{(Z)}(0,-\lambda_2)/d\Omega}, \qquad (18)$$

where $\frac{d\sigma^{(Z)}(\lambda_1,0)}{d\Omega}\left(\frac{d\sigma^{(Z)}(0,\lambda_2)}{d\Omega}\right)$ – is the differential cross section of the process (1) in the annihilation of the longitudinally polarized lepton and nonpolarized antilepton (nonpolarized lepton and longitudinally polarized antilepton). Given (15) in (18), we have

$$A_2(\theta) = -A_1(\theta) = \{(g_L^2 - g_R^2)[(G_L^2 + G_R^2)[(1+r_i - r_j)(1 - r_i + r_j) + \lambda(r_i, r_j)\cos^2\theta] + 4G_L G_R\sqrt{r_{\chi_i} r_{\chi_j}}\,] +$$
$$+ (g_L^2 - g_R^2)(G_L^2 - G_R^2)\sqrt{\lambda(r_i, r_j)}\cos\theta\} \times \{(g_L^2 + g_R^2)[(G_L^2 + G_R^2)[(1+r_i - r_j)(1 - r_i + r_j) +$$
$$+ \lambda(r_i, r_j)\cos^2\theta] + 4G_L G_R\sqrt{r_{\chi_i} r_{\chi_j}}\,] + (g_L^2 - g_R^2)(G_L^2 - G_R^2)\sqrt{\lambda(r_i, r_j)}\cos\theta\}^{-1}. \qquad (19)$$

Hence, the longitudinal spin asymmetry $A_1(\theta)$, resulting from the annihilation of a polarized lepton with an nonpolarized antilepton, is equal in magnitude but opposite in sign to the longitudinal spin asymmetry $A_2(\theta)$, resulting from the annihilation of an nonpolarized lepton with a polarized antilepton.

Measurement of the transverse spin asymmetry $A_\perp(\theta,\varphi)$, the angular forward-backward asymmetry $A_{FB}(\theta)$, the longitudinal spin asymmetries $A_1(\theta)$ and $A_2(\theta)$ in the experiment allows, in principle, to obtain information about constants of the neutralino with the vector Z-boson $G_L$ and $G_R$.

From the formula of the differential cross section (9), we can obtain expressions for the integral characteristics of the process $\ell^- + \ell^+ \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0$. For this purpose, let us define the following expressions for the cross sections for the production of a neutralino pair:

a) in the case of a transversely polarized lepton-antilepton pair

$$\frac{d\sigma^{(Z)}(\eta_1,\eta_2)}{d\varphi}=\int_0^{\pi}\frac{d\sigma}{d\Omega}d(\cos\theta)=\frac{g^4 s|D_Z(s)|^2}{128\pi^2\cos^4\theta_W}\sqrt{\lambda(r_i,\,r_j)}\times$$

$$\times\left\{(g_L^2+g_R^2)\left[(G_L^2+G_R^2)\left((1+r_i-r_j)(1-r_i+r_j)+\frac{1}{3}\lambda(r_i,\,r_j)\right)+4G_L G_R\sqrt{r_i r_j}\right]-\right.$$

$$\left.-\frac{2}{3}g_L g_R(G_L^2+G_R^2)\eta_1\eta_2\lambda(r_i,\,r_j)\cos 2\varphi\right\}; \qquad (20)$$

b) in the case of longitudinally polarized $\ell^-\ell^+$ -pair

$$\sigma^{(Z)}(\lambda_1,\lambda_2)=2\pi\int_0^{\pi}\frac{d\sigma(\lambda_1,\lambda_2)}{d(\cos\theta)}d(\cos\theta)=$$

$$=\frac{g^4 s|D_Z(s)|^2}{64\pi\cos^4\theta_W}\sqrt{\lambda(r_i,\,r_j)}[g_L^2(1-\lambda_1)(1+\lambda_2)+g_R^2(1+\lambda_1)(1-\lambda_2)]\times$$

$$\times\left\{(G_L^2+G_R^2)\left[(1+r_i-r_j)(1-r_i+r_j)+\frac{1}{3}\lambda(r_i,\,r_j)\right]+4G_L G_R\sqrt{r_i r_j}\right\}. \qquad (21)$$

Let us also determine the cross sections for the neutralino production in the front (F) and back (B) hemispheres in the case of nonpolarized particles:

$$\sigma_F^{(Z)}=2\pi\int_0^{\pi/2}\frac{d\sigma_0(\theta)}{d(\cos\theta)}d(\cos\theta)=\frac{g^4 s|D_Z(s)|^2}{128\pi\cos^4\theta_W}\sqrt{\lambda(r_i,\,r_j)}\times$$

$$\times\left\{(g_L^2+g_R^2)\left[(G_L^2+G_R^2)\left((1+r_i-r_j)(1-r_i+r_j)+\frac{1}{3}\lambda(r_i,\,r_j)\right)+4G_L G_R\sqrt{r_i r_j}\right]+\right.$$

$$\left.+(g_L^2-g_R^2)(G_L^2-G_R^2)\sqrt{\lambda(r_i,\,r_j)}\cdot\frac{1}{2}\right\}, \qquad (22)$$

$$\sigma_B^{(Z)}=2\pi\int_{\pi/2}^{\pi}\frac{d\sigma_0(\theta)}{d(\cos\theta)}d(\cos\theta)=\frac{g^4 s|D_Z(s)|^2}{128\pi\cos^4\theta_W}\sqrt{\lambda(r_i,\,r_j)}\times$$

$$\times\left\{(g_L^2+g_R^2)\left[(G_L^2+G_R^2)\left((1+r_i-r_j)(1-r_i+r_j)+\frac{1}{3}\lambda(r_i,\,r_j)\right)+4G_L G_R\sqrt{r_i r_j}\right]-\right.$$

$$\left.+(g_L^2-g_R^2)(G_L^2-G_R^2)\sqrt{\lambda(r_i,\,r_j)}\cdot\frac{1}{2}\right\}, \qquad (23)$$

From formula (20) of the cross section we define the transverse spin asymmetry $A_\perp(\sqrt{s},\varphi)$, integrated on the polar angle of the neutralino $\theta$:

$$A_\perp(\sqrt{s},\varphi)=-\frac{2g_L g_R}{g_L^2+g_R^2}\frac{(G_L^2+G_R^2)\lambda(r_i,\,r_j)\cos 2\varphi}{(G_L^2+G_R^2)[3(1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,\,r_j)]+12G_L G_R\sqrt{r_i r_j}}. \qquad (24)$$

From the cross section formula (15) for the integral longitudinal spin asymmetry we obtain:

$$A_2=-A_1=\frac{g_L^2-g_R^2}{g_L^2+g_R^2}. \qquad (25)$$

These longitudinal spin asymmetries depend only on the Weinberg parameter $x_W=\sin^2\theta_W$ and at $x_W=0.2315$ $A_2=-A_1=14.7\%$.

For the integral forward-backward asymmetry, we obtain the expression:

$$A_{FB}(\theta)=\frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B}=$$

$$=\frac{g_L^2-g_R^2}{g_L^2+g_R^2}\frac{(G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}}{(G_L^2+G_R^2)[(1+r_i-r_j)(1-r_i+r_j)+\frac{1}{3}\lambda(r_i,r_j)+8G_LG_R\sqrt{r_ir_j}}. \quad (26)$$

Let us estimate the above asymmetries in the processes $e^-+e^+\to\tilde{\chi}_1^0+\tilde{\chi}_2^0$ and $e^-+e^+\to\tilde{\chi}_2^0+\tilde{\chi}_2^0$. For the left and right coupling constants of the neutralino with the Z-boson we obtain expressions:

1) in the process $e^-+e^+\to\tilde{\chi}_1^0+\tilde{\chi}_2^0$:

$$G_R=-G_L=\frac{1}{2\sin\theta_W}[Z_{13}Z_{23}-Z_{14}Z_{24}];$$

2) in the process $e^-+e^+\to\tilde{\chi}_2^0+\tilde{\chi}_2^0$:

$$G_R=-G_L=\frac{1}{2\sin\theta_W}[(Z_{23})^2-(Z_{24})^2],$$

the matrix elements of the and matrix $Z_{13}$, $Z_{23}$, $Z_{14}$ and $Z_{24}$ are given in [11, 13].

Fig. 5 shows the angular dependence of the transverse spin asymmetry $A_\perp(\theta)$ in the reactions $e^-+e^+\to\tilde{\chi}_1^0+\tilde{\chi}_2^0$ (curve 1), $e^-+e^+\to\tilde{\chi}_2^0+\tilde{\chi}_2^0$ (curve 2) at $\varphi=0$, $\sqrt{s}=500$ GeV, $M_2=2M_1=$ 150 GeV, $\mathrm{tg}\beta=3$, $x_W=0.2315$.

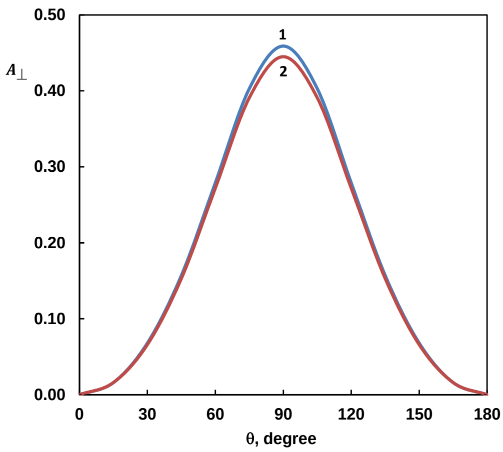


*Fig. 5.* Dependence of the transverse spin asymmetry $A_\perp$ on $\theta$

Fig. 6 illustrates the energy dependence of the transverse spin asymmetry integrated along the polar angle $\theta$ in the reactions $e^-+e^+\to\tilde{\chi}_1^0+\tilde{\chi}_2^0$ (curve 1), $e^-+e^+\to\tilde{\chi}_2^0+\tilde{\chi}_2^0$ (curve 2) at the same values of the parameters as in Fig. 5.

As for the forward-backward angular asymmetry $A_{FB}(\theta)$, as well as the forward-backward integral asymmetry $A_{FB}$, we note that, due to the relation between the neutralino $G_L=-G_R$ bond chiral constants in the reactions considered, they turn to zero $A_{FB}(\theta)=A_{FB}=0$. For this reason, the longitudinal spin asymmetries $A_2(\theta)=-A_1(\theta)$ do not depend on the angle of departure of the neutralino and are only functions of the left and right coupling constants of the lepton with the gauge Z-boson:

$$A_2(\theta)=-A_1(\theta)=\frac{g_L^2-g_R^2}{g_L^2+g_R^2}.$$

The energy dependence of the reaction $e^-+e^+\to\tilde{\chi}_1^0+\tilde{\chi}_2^0$ cross section is presented in Fig. 7 in three cases: 1) when the electron is polarized right: $\lambda_1=+1$; 2) when the electron possesses left-hand helicity: $\lambda_1=-1$; 3) when the electron is non-polarized.

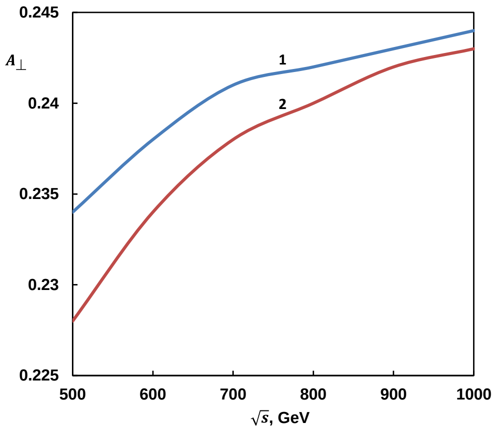


*Fig. 6.* Dependence of transverse spin asymmetry $A_\perp$ on energy $\sqrt{s}$.

## 3. DEGREES OF LONGITUDINAL AND TRANSVERSE POLARIZATION OF THE NEUTRALINO

In the previous section we were interested in the polarization properties of the lepton and antilepton, we determined the transverse and longitudinal spin asymmetries due to the lepton and antilepton polarizations. Note that the study of the degrees of longitudinal and transverse polarizations of the neutralino is also of some interest. They can give valuable information about the interaction constants of the neutralino with the gauge $Z$-boson $G_L$ and $G_R$. In this connection, let us proceed to the study of the polarization characteristics of the neutralino.

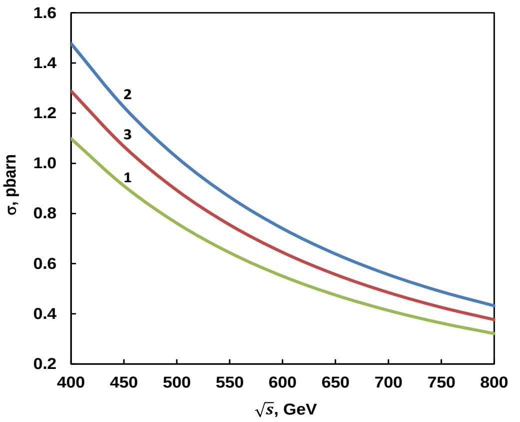


*Fig. 7.* Dependence of the reaction $e^- + e^+ \to \tilde{\chi}_1^0 + \tilde{\chi}_2^0$ cross section on the energy $\sqrt{s}$ at $\lambda_1 = +1$ (curve 1), at $\lambda_1 = -1$ (curve 2) and nonpolarized lepton ( $\lambda_1 = 0$ ) (curve 3).

Let us consider the differential section of the process (1) taking into account the longitudinal polarizations of the lepton and neutralino:

$$\frac{d\sigma^{(Z)}(\lambda_1,h)}{d\Omega} = \frac{1}{2}\frac{d\sigma_0^{(Z)}(\lambda_1)}{d\Omega}[1 + h_1 P_{\|}(\sqrt{s},\theta)]\,, \quad (27)$$

$\frac{d\sigma_0^{(Z)}(\lambda_1)}{d\Omega}$ – is the differential cross section of reaction (1) in the annihilation of a polarized lepton and an nonpolarized antilepton:

$$\frac{d\sigma_0^{(Z)}(\lambda_1)}{d\Omega} = \frac{g^4 s|D_Z(s)|^2}{256\pi^2\cos^4\theta_W}\sqrt{\lambda(r_i,r_j)}\{[g_L^2(1-\lambda_1)+g_R^2(1+\lambda_1)]\times$$
$$\times[(G_L^2+G_R^2)((1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta)+4G_LG_R\sqrt{r_ir_j}\,]+$$
$$+[g_L^2(1-\lambda_1)-g_R^2(1+\lambda_1)](G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}\cos\theta\,, \quad (28)$$

a $P_{\|}(\sqrt{s},\theta)$ – is the degree of longitudinal polarization of the neutralino:

$$P_{\|}(\sqrt{s},\theta) = \{[g_L^2(1-\lambda_1)+g_R^2(1+\lambda_1)](G_R^2-G_L^2)\sqrt{\lambda(r_i,r_j)}[1-r_i+r_j+(1+r_i-r_j)\cos^2\theta -$$
$$-2[g_L^2(1-\lambda_1)-g_R^2(1+\lambda_1)][(G_L^2+G_R^2)(1-r_i-r_j)+4G_LG_R\sqrt{r_ir_j}\cos\theta\}\times$$
$$\times\{[g_L^2(1-\lambda_1)+g_R^2(1+\lambda_1)](G_L^2+G_R^2)((1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta)+8G_LG_R\sqrt{r_ir_j}\,]+$$
$$+2[g_L^2(1-\lambda_1)-g_R^2(1+\lambda_1)](G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}\}^{-1}. \quad (29)$$

If the neutralino is polarized transversely in the plane of production, then the differential cross section of the reaction $e^- + e^+ \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0$ will take the form (the lepton is polarized longitudinally):

$$\frac{d\sigma^{(Z)}(\lambda_1,\eta)}{d\Omega} = \frac{1}{2}\frac{d\sigma_0^{(Z)}(\lambda_1)}{d\Omega}[1+\eta P_{\perp}(\sqrt{s},\theta)]\,, \quad (30)$$

where $\eta$ – is the transverse component of the neutralino spin vector, $\frac{d\sigma_0^{(Z)}(\lambda_1)}{d\Omega}$ – is the differential cross section for the annihilation of a longitudinally polarized lepton and an nonpolarized antilepton (formula (28)), and $P_{\perp}(\sqrt{s},\theta)$ – the degree of transverse polarization of the neutralino is defined by the expression:

$$P_{\perp}(\sqrt{s},\theta) = \sqrt{\lambda(r_i,r_j)}\sin 2\theta\{[g_L^2(1-\lambda_1)+g_R^2(1+\lambda_1)](G_L^2-G_R^2)\sqrt{r_j}\,+$$
$$+[g_L^2(1-\lambda_1)-g_R^2(1+\lambda_1)][-(G_L^2+G_R^2)\sqrt{r_i}+8G_LG_R\sqrt{r_j}\,]\}\cdot\{[g_L^2(1-\lambda_1)+g_R^2(1+\lambda_1)]\times$$
$$\times[(G_L^2+G_R^2)[(1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta]+8G_LG_R\sqrt{r_ir_j}\,]+$$
$$+2[g_L^2(1-\lambda_1)-g_R^2(1+\lambda_1)](G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}\cos\theta\}^{-1}. \quad (31)$$

Figure 8 illustrates the dependence of the degree of longitudinal polarization of the neutralino in the process $e^- + e^+ \to \tilde{\chi}_1^0 + \tilde{\chi}_2^0$ on the angle $\theta$ at $\sqrt{s}$ = 500 GeV and $\lambda_1 = +1$ (curve 1), $\lambda_1 = -1$ (curve 2) and at nonpolarized electron (curve 3). It follows from the figure that at $\lambda_1 = +1$ ( $\lambda_1 = -1$ ) the degree of longitudinal polarization of the neutralino is minimal (maximum), with an increase in the angle $\theta$ it increases (decreases) and vanishes at an angle of $\theta$ =90°. With a further increase in the angle $\theta$, the degree of longitudinal polarization of the neutralino changes sign and increases (decreases).

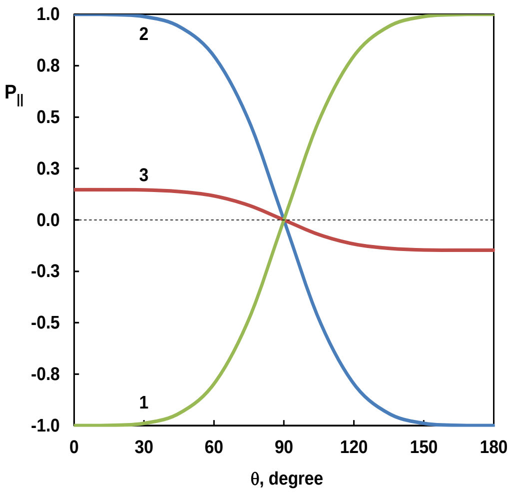


*Fig. 8.* Degree of longitudinal polarization of the neutralino in the reaction $e^-e^+ \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ as a function of the polar angle $\theta$ at $\lambda_1 = +1$ (curve 1), $\lambda_1 = -1$ (curve 2), at nonpolarized $e^-e^+$-pair (curve 3).

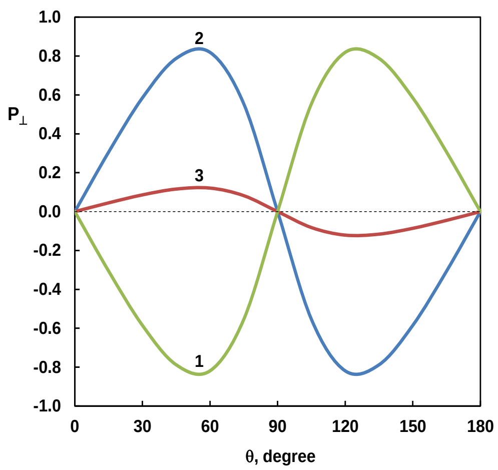


*Fig. 9.* Dependence of the transverse spin asymmetry $P_\perp(\sqrt{s},\,\theta)$ on the angle $\theta$ in the reaction $e^-e^+ \to \tilde{\chi}_2^0\tilde{\chi}_2^0$.

In the case of nonpolarized initial particles, the degree of longitudinal polarization of neutralino at the beginning of the angular spectrum is positive and gradually decreases with increasing angle.

The angular dependence of the degree of transverse polarization of the neutralino $P_\perp(\sqrt{s},\,\theta)$ in the process $e^- + e^+ \to \tilde{\chi}_2^0 + \tilde{\chi}_2^0$ is shown in Fig. 9 for $\lambda_1 = +1$ (curve 1), $\lambda_1 = -1$ (curve 2) and at nonpolarized electron (curve 3). As you can see, the degree of transverse polarization is maximum or minimum near the angle 60° or 150°, vanishes at $\theta$ = 0°; 90° and 180°.

## 4. AMPLITUDE AND CROSS SECTION OF THE PROCESS $\ell^-\ell^+ \to (\Phi^*) \to \tilde{\chi}_2^0\tilde{\chi}_2^0$

We now turn to the study of the effective cross section of the process corresponding to the diagram b) of Fig. 1 with the Higgs boson exchange $\Phi^* = H^*(h^*;\,A^*)$. The Lagrangians of the interaction of the Higgs boson $\Phi$ with a lepton pair and a neutralino pair are written in the following form [7, 20]:

$$L_{\Phi\ell\ell} = -g_{\Phi\ell\ell}\bar{\ell}\gamma_\mu(a + b\gamma_5)\ell\cdot\Phi,$$
$$L_{\Phi\chi_i^0\chi_j^0} = g\cdot\tilde{\chi}_i^0\gamma_\mu(G_L P_L + G_R P_R)\tilde{\chi}_j^0\cdot\Phi, \qquad (32)$$

where in the case of CP-even Higgs bosons $H$ and $h$ $a$ =1 and $b$ =0 and

$$g_{H\ell\ell} = i\frac{m_\ell}{\upsilon}\cdot\frac{\cos\alpha}{\cos\beta}, \quad g_{h\ell\ell} = -i\frac{m_\ell}{\upsilon}\cdot\frac{\sin\alpha}{\cos\beta},$$

and in the case of CP-odd $A$-bosons $a$ =0 and $b$ =1 and

$$a = \frac{m_\ell}{\upsilon}\mathrm{tg}\beta,$$

$\upsilon = 246$ GeV is the vacuum value of the Higgs boson field, $G_L$ and $G_R$ – are the left and right interaction constants of the Higgs boson with the neutralino pair

$$G_L = \frac{1}{2\sin\theta_W}(Z_{j2} - \mathrm{tg}\theta_W Z_{j1})(e_k Z_{i3} + d_k Z_{i4}) + i \leftrightarrow j,$$
$$G_R = \frac{1}{2\sin\theta_W}(Z_{j2} - \mathrm{tg}\theta_W Z_{j1})(e_k Z_{i3} + d_k Z_{i4})\varepsilon_k + i \leftrightarrow j, \qquad (33)$$

$\varepsilon_1 = \varepsilon_2 = -\varepsilon_3 = 1$, the coefficients of $e_k$ and $d_k$ are equal:

$$e_1 = +\cos\alpha, \quad e_2 = -\sin\alpha, \quad e_3 = -\sin\beta,$$
$$d_1 = -\sin\alpha, \quad d_2 = -\cos\alpha, \quad d_3 = +\cos\beta.$$

Based on the Lagrangians (32), let us write down the amplitude of the corresponding diagram b) of Fig. 1:

$$M^{(\Phi)}_{i\to f} = g_{\Phi\ell\ell} g D_{\Phi}(s)\bar{\upsilon}_{\ell}(p_2,s_2)(a+b\gamma_5)u_{\ell}(p_1,s_1)g[\bar{u}_i(k_1,s)(G_L P_L + G_R P_R)\upsilon_j(k_2,s'). \quad (34)$$

Here $D_{\Phi}(s)=(s-M^2_{\Phi}+i\Gamma_{\Phi}M_{\Phi})^{-1}$, $M_{\Phi}$ and $\Gamma_{\Phi}$ – are the mass and total width of the $\Phi$-boson.

The square of the modulus of the matrix element (34) with simultaneous accounting of the polarizations of all particles involved has the form:

$$\left|M^{(\Phi)}_{i\to f}\right|^2 = g_{\Phi\ell\ell} g^2 \left|D_{\Phi}(s)\right|^2 L\times\chi\,, \quad (35)$$

where $L$ and $\chi$ – are the scalars functions of the lepton-antilepton pair and the neutralino pair:

$$L=[|a|^2+|b|^2][(p_1\cdot p_2)+m^2_{\ell}(s_1\cdot s_2)]+[|a|^2-|b|^2][-m^2_{\ell}-(s_1\cdot s_2)(p_1\cdot p_2)+(p_1\cdot s_2)(p_2\cdot s_1)]-$$
$$-2\,\mathrm{Re}(ab^*)m_{\ell}[(p_1\cdot s_2)+(p_2\cdot s_1)]+2\,\mathrm{Im}(ab^*)(p_1p_2s_1s_2)_{\varepsilon}\}; \quad (36)$$

$$\chi=\frac{1}{2}(G^2_L+G^2_R)[(k_1\cdot k_2)+m_{\chi_1}m_{\chi_2}(s\cdot s')]+\frac{1}{2}(G^2_L-G^2_R)[m_{\chi_i}(k_2\cdot s)+m_{\chi_j}(k_1\cdot s')]+$$
$$+G_LG_R[-m_{\chi_i}m_{\chi_j}-(k_1\cdot k_2)(s\cdot s')+(k_1\cdot s')(k_2\cdot s)]\,. \quad (37)$$

The effective cross section of the process in the case of arbitrary polarizations of the initial and longitudinal polarizations of the final particles can be represented as (in the center-of-mass system):

$$\sigma^{(\Phi)}=\frac{g^2_{\Phi\ell\ell}g^2 s}{128}|D_{\Phi}(s)|^2\sqrt{\lambda(r_i,\,r_j)}\cdot\{[|a|^2+|b|^2][1-(\vec{n}\vec{\xi}_1)(\vec{n}\vec{\xi}_2)]+[|a|^2-|b|^2]\times$$
$$\times[(\vec{\xi}_1\vec{\xi}_2)-(\vec{n}\vec{\xi}_1)(\vec{n}\vec{\xi}_2)]+2\,\mathrm{Re}(ab^*)[(\vec{n}\vec{\xi}_2)-(\vec{n}\vec{\xi}_1)]-2\,\mathrm{Im}(ab^*)(\vec{n}[\vec{\xi}_1\vec{\xi}_2])\}\{[(G^2_L+G^2_R)\times$$
$$\times(1-r_i-r_j)-4G_LG_R\sqrt{r_i\cdot r_j}\,](1+h_1h_2)+(G^2_L-G^2_R)\sqrt{\lambda(r_i,r_j)}(h_1+h_2)\}\,, \quad (38)$$

where $\vec{n}$ – a unit vector in the lepton momentum direction; $\vec{\xi}_1$ and $\vec{\xi}_2$ – unit vectors directed along the lepton and antilepton spins in their rest systems, respectively.

The interaction constant of the $\Phi$-boson with a lepton pair is proportional to the lepton mass $m_{\ell}$, therefore the study of the process of production of the neutralino pair in muon-antimuon collisions is of particular interest. Therefore, let us consider the process $\mu^-+\mu^+\to(\Phi^*)\to\tilde{\chi}^0_i+\tilde{\chi}^0_j$ in the case of a longitudinally polarized muon-antimuon pair: in which $(\vec{n}\vec{\xi}_1)=\lambda_1$, $(\vec{n}\vec{\xi}_2)=-\lambda_2$, $(\vec{\xi}_1\vec{\xi}_2)=-\lambda_1\lambda_2$, where $\lambda_1$ and $\lambda_2$ – are the helicities of the muon and antimuon:

$$\sigma^{(\Phi)}(\lambda_1,\lambda_2,h_1,h_2)=\frac{g^2_{\Phi\ell\ell}g^2 s}{128}\sqrt{\lambda(r_i,\,r_j)}\cdot\{[|a|^2+|b|^2](1+\lambda_1\lambda_2)-2\,\mathrm{Re}(ab^*)(\lambda_1+\lambda_2)\}\times$$
$$\times\{[(G^2_L+G^2_R)(1-r_i-r_j)-4G_LG_R\sqrt{r_i,r_j}\,](1+h_1h_2)+(G^2_L-G^2_R)\sqrt{\lambda(r_i,r_j)}(h_1+h_2)\}\,. \quad (39)$$

It follows from this formula for the effective cross section that the muon and antimuon as well as the neutralino $\tilde{\chi}^0_i$ and $\tilde{\chi}^0_j$ must have the same helicities: $\lambda_1=\lambda_2=\pm1$, $h_1=h_2=\pm1$. This is a consequence of the conservation of the total angular momentum in the transitions $\ell^-+\ell^+\to\Phi$ and $\Phi\to\tilde{\chi}^0_i+\tilde{\chi}^0_j$. Diagram b) of Fig. 1 corresponds to four spiral sections:

1) all particles are left-polarized: ($\lambda_1=\lambda_2=h_1=h_2=-1$):

$$\sigma^{(\Phi)}_{LL}\sim|a+b|^2\{(G^2_L+G^2_R)(1-r_i-r_j)-4G_LG_R\sqrt{r_ir_j}-(G^2_L-G^2_R)\sqrt{\lambda(r_i,r_j)}\}\,,$$

2) all particles are right-handedly polarized: ($\lambda_1=\lambda_2=h_1=h_2=+1$):

$$\sigma^{(\Phi)}_{RR}\sim|a-b|^2\{(G^2_L+G^2_R)(1-r_i-r_j)-4G_LG_R\sqrt{r_ir_j}+(G^2_L-G^2_R)\sqrt{\lambda(r_i,r_j)}\}\,;$$

3) initial particles are left-polarized and final particles are right-polarized: ($\lambda_1=\lambda_2=-1,\ h_1=h_2=+1$):

$$\sigma^{(\Phi)}_{LR}\sim|a+b|^2\{(G^2_L+G^2_R)(1-r_i-r_j)-4G_LG_R\sqrt{r_ir_j}-(G^2_L-G^2_R)\sqrt{\lambda(r_i,r_j)}\}\,;$$

4) initial particles are right-polarized and final particles are left-polarized ($\lambda_1=\lambda_2=+1,\ h_1=h_2=-1$):

$$\sigma^{(\Phi)}_{RL}\sim|a-b|^2\{(G^2_L+G^2_R)(1-r_i-r_j)-4G_LG_R\sqrt{r_ir_j}-(G^2_L-G^2_R)\sqrt{\lambda(r_i,r_j)}\}$$

In these cases directions of impulses and spins of particles are shown in Fig. 10.

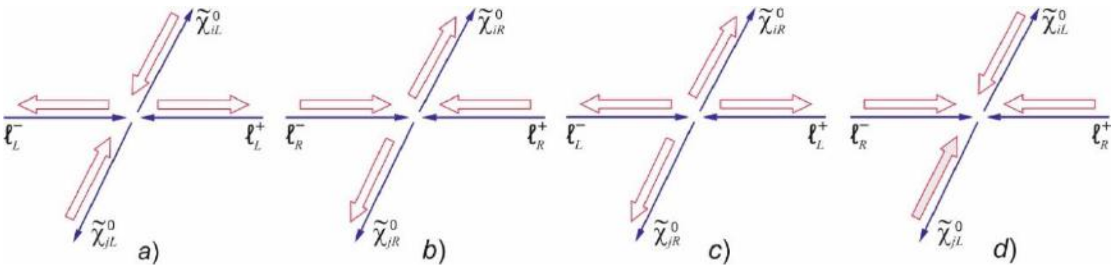

*Fig. 10.* Directions of impulses and spins of particles in the process $\ell^- \ell^+ \to (\Phi^*) \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$.

As can be seen from Fig. 10, the directions of spins of the lepton and antilepton, as well as neutralino $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$ are directed opposite to each other, therefore, their total momentum is zero, the spin of the intermediate Higgs boson $\Phi$ is also zero, so in transitions $\ell^- + \ell^+ \to \Phi$ and $\Phi \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0$ the law of conservation of total momentum is satisfied.

It follows from the above reasoning that we can distinguish contributions to the cross section of diagrams a) and b) in Fig. 1 from the spirals of the lepton-antilepton pair. The contribution of the diagram with vector $Z$-boson exchange differs from zero if the lepton and antilepton have opposite helicities $\lambda_1 = -\lambda_2 = \pm 1$. However, the contribution of the $\Phi$-boson exchange diagram is zero in this case. If the lepton and the antilepton have the same helicity $\lambda_1 = \lambda_2 = \pm 1$, then the contribution of diagram a) is zero, and the contribution of diagram b) is different from zero.

On the basis of the effective cross section formula (39), let us determine the longitudinal spin asymmetries due to the polarizations of the lepton and antilepton:

$$A_1 = \frac{1}{\lambda_1} \frac{\sigma^{(\Phi)}(\lambda_1, 0) - \sigma^{(\Phi)}(-\lambda_1, 0)}{\sigma(\lambda_1, 0) + \sigma(-\lambda_1, 0)} = -\frac{2\,\mathrm{Re}(ab^*)}{|a|^2 + |b|^2},$$
$$A_2 = \frac{1}{\lambda_2} \frac{\sigma^{(\Phi)}(0, \lambda_2) - \sigma^{(\Phi)}(0, -\lambda_2)}{\sigma^{(\Phi)}(0, \lambda_2) + \sigma^{(\Phi)}(0, -\lambda_2)} = -\frac{2\,\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}, \tag{40}$$

here $\sigma^{(\Phi)}(\lambda_1, 0)$ ($\sigma^{(\Phi)}(0, \lambda_2)$) – is the annihilation cross section of the polarized lepton and nonpolarized antilepton (nonpolarized lepton and polarized antilepton). From formulas (40) it follows that the longitudinal spin asymmetry arising from the interaction of a polarized lepton with nonpolarized antileptons is equal to the longitudinal spin asymmetry arising from the interaction of polarized antileptons with nonpolarized leptons.

Experimental study of these asymmetries

$$A_1 = A_2 = -\frac{2\,\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}$$

in the process $\mu^- + \mu^+ \to (\Phi^*) \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0$ can provide valuable information about the nature of the $\Phi$-boson. If the $\Phi$-boson is a purely CP-even particle (like the $H(h)$ Higgs boson) or a CP odd particle (like the $A$-boson), the experiments will not reveal longitudinal spin asymmetry.

In the case where the lepton-antilepton pair is polarized transversely, the effective cross section (38) will take the form:

$$\sigma^{(\Phi)} = \frac{g_{\Phi\ell\ell}^2 g^2 s}{32} |D_\Phi(s)|^2 \sqrt{\lambda(r_i, r_j)} \cdot [|a|^2 + |b|^2 + (|a|^2 - |b|^2)\eta_1\eta_2 \cos\phi - 2\,\mathrm{Im}(ab^*)\eta_1\eta_2 \sin\phi] \times$$
$$\times [(G_L^2 + G_R^2)(1 - r_i - r_j) - 4G_L G_R \sqrt{r_i \cdot r_j}\,], \tag{41}$$

where $\phi -$ the angle between the transverse spin vectors $\vec{\eta}_1$ and $\vec{\eta}_2$. This section leads to the following transverse spin asymmetries due to the lepton-antilepton pair polarizations:

$$A_3 = \frac{1}{\eta_1\eta_2} \frac{\sigma^{(\Phi)}(\phi = 0) - \sigma^{(\Phi)}(\phi = \pi)}{\sigma^{(\Phi)}(\phi = 0) + \sigma^{(\Phi)}(\phi = \pi)} = \frac{|a|^2 - |b|^2}{|a|^2 + |b|^2}, \tag{42}$$

$$A_4 = \frac{1}{\eta_1\eta_2}\frac{\sigma^{(\Phi)}(\phi=-\pi/2)-\sigma^{(\Phi)}(\phi=\pi/2)}{\sigma^{(\Phi)}(\phi=-\pi/2)+\sigma^{(\Phi)}(\phi=\pi/2)} = \frac{2\,\mathrm{Im}(ab^*)}{|a|^2+|b|^2}. \quad (43)$$

The study of these transverse spin asymmetries is also a source of information about the nature of the $\Phi$-boson. If the $\Phi$-boson is CP-even then the asymmetry is $A_3=+1$, and if it is CP-odd then this asymmetry is $A_3=-1$. The difference from zero of the transverse spin asymmetry $A_4$ also indicates a violation of the CP-accountability in the process $\mu^- + \mu^+ \to (\Phi^*) \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0$.

From the effective cross section formula (39), let us determine the degree of longitudinal polarization of the neutralino by the standard formula :

$$P = \frac{\sigma^{(\Phi)}(h_1=1)-\sigma^{(\Phi)}(h_1=-1)}{\sigma^{(\Phi)}(h_1=1)+\sigma^{(\Phi)}(h_1=-1)} = \frac{(G_L^2-G_R^2)\sqrt{\lambda(r_i,r_j)}}{(G_L^2+G_R^2)(1-r_i-r_j)-4G_LG_R\sqrt{r_i\cdot r_j}}. \quad (44)$$

In the case when the neutralino $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$ are polarized transversely, the differential cross section of the process $\ell^- + \ell^+ \to (\Phi^*) \to \tilde{\chi}_i^0 + \tilde{\chi}_j^0$ has the following form:

$$\frac{d\sigma^{(\Phi)}}{d\Omega} = \frac{1}{4}\frac{d\sigma_0^{(\Phi)}}{d\Omega}(1+\eta\eta' A_\perp), \quad (45)$$

Where

$$\frac{d\sigma_0^{(\Phi)}}{d\Omega} = \frac{g_{\Phi\ell\ell}^2 g^2\cdot s|D_\Phi(s)|}{512}\sqrt{\lambda(r_i,r_j)}\cdot[|a|^2+|b|^2][(G_L^2+G_R^2)(1-r_i-r_j)-4G_LG_R\sqrt{r_i\cdot r_j}] \quad (46)$$

– is the differential cross section of the process, $\eta$ and $\eta'$ – are the transverse components of the spin vectors of the neutralino $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$, $A_\perp$ – is the degree of transverse polarization of the neutralino:

$$A_\perp = \frac{2\cos\varphi\cdot[G_LG_R(1-r_i-r_j)-(G_L^2+G_R^2)\sqrt{r_i\cdot r_j}]}{(G_L^2+G_R^2)(1-r_{\chi_i}-r_{\chi_j})-4G_LG_R\sqrt{r_i\cdot r_j}}. \quad (47)$$

Let us estimate the degree of longitudinal ($P$) and transverse ($A_\perp$) polarization in the process $\mu^- + \mu^+ \to (H^*) \to \tilde{\chi}_1^0 + \tilde{\chi}_2^0$. According to (33), the left and right Higgs boson $H$ coupling constants of the neutralino pair $\tilde{\chi}_i^0$ and $\tilde{\chi}_j^0$ are equal to each other $G_L = G_R$. As a consequence, the degree of longitudinal polarization is zero, while the degree of transverse polarization of the neutralino is equal to the cosine of the angle $\varphi$:

$$A_\perp = \cos\varphi.$$

The degree of transverse polarization is maximum at angle $\varphi = 0$ ($A_\perp$ =100%) and turns to zero at $\varphi = \frac{\pi}{2}$, then $A_\perp$ changes sign and decreases with increasing angle $\varphi$ and reaches a minimum at $\varphi=\pi$: $A_\perp$ =–100%.

Figure 11 illustrates the energy dependence of the cross section of the process
$\mu^- + \mu^+ \to (H^*) \to \tilde{\chi}_1^0 + \tilde{\chi}_2^0$ at $\mathrm{tg}\beta = 3$, $M_A$ =500 GeV, and $\Gamma_H$ =4 GeV.

It can be seen that the cross section is maximum when the energy of the muon-antimuon pair is equal to the Higgs boson mass: $\sqrt{s} = M_H$ =500 GeV.

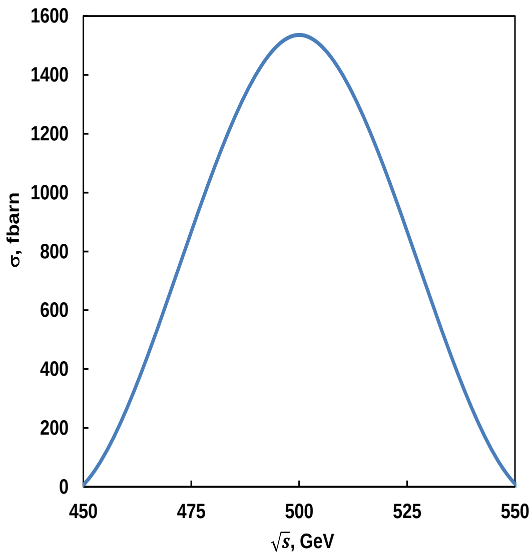


*Fig. 11.* Energy dependence of the cross section of the pro cess $\mu^-\mu^+ \to (H^*) \to \tilde{\chi}_i^0\tilde{\chi}_j^0$.

## 5. AMPLITUDE AND CROSS SECTION OF THE REACTION $\ell^-\ell^+ \to (\tilde{\ell}_L, \tilde{\ell}_R) \to \tilde{\chi}_i^0\tilde{\chi}_j^0$

We turn to the study of diagrams c) and d) of Fig. 1 with the exchange of scalar $\tilde{\ell}_L$ and $\tilde{\ell}_R$ leptons The Lagrangian of the interaction of lepton $\ell$, neutralino $\tilde{\chi}_i^0$ and scalar lepton $\tilde{\ell}_L$ ($\tilde{\ell}_R$) is written as follows:

$$L_{\ell\tilde{\ell}\tilde{\chi}_i^0} = gf_{\ell i}^L(\bar{\ell}P_R\tilde{\chi}_i^0)\tilde{\ell}_R + gf_{\ell i}^R(\bar{\ell}P_L\tilde{\chi}_i^0)\tilde{\ell}_R + \text{e.c.} \quad (48)$$

Based on this Lagrangian, it is easy to write the amplitudes of the t- and u-channel diagrams c) and d) of Fig. 1:

$$M_c = ig^2\{D_{\tilde{\ell}_L}(t)f_{\ell i}^L f_{\ell j}^L(\bar{u}_i(k_1)P_L u_\ell(p_1))(\bar{\upsilon}_\ell(p_2)P_R\upsilon_j(k_2)) +$$
$$+D_{\tilde{\ell}_R}(t)f_{\ell i}^R f_{\ell j}^R(\bar{u}_i(k_1)P_R u_\ell(p_1))(\bar{\upsilon}_\ell(p_2)P_L\upsilon_j(k_2))\}, \quad (49)$$

$$M_d = -ig^2\{D_{\tilde{\ell}_L}(u)f_{\ell i}^L f_{\ell j}^L(\bar{u}_j(k_2)P_L u_\ell(p_1))(\bar{\upsilon}_\ell(p_2)P_R\upsilon_i(k_1)) +$$
$$+D_{\tilde{\ell}_R}(u)f_{\ell i}^L f_{\ell j}^L(\bar{u}_j(k_2)P_R u_\ell(p_1))(\bar{\upsilon}_\ell(p_2)P_L\upsilon_i(k_2))\}. \quad (50)$$

Here $t=(p_1-k_1)^2$ and $u=(p_1-k_2)^2$ – are the kinematic variables $D_{\tilde{\ell}_{L,R}}(x)=(x-m^2_{\tilde{\ell}_{L,R}}(x))^{-1}$, $f_{\ell i}^L$ and $f_{\ell j}^R$ – are the left and right interaction constants of the lepton, neutralino and scalar lepton [7, 15]:

$$f_{\ell i}^L = -\sqrt{2}\left[\frac{1}{\cos\theta_W}(T_3(\ell)-e_\ell\sin^2\theta_W)Z_{i2}+e_\ell\sin\theta_W Z_{i1}\right],$$
$$f_{\ell i}^R = -\sqrt{2}e_\ell\sin\theta_W(\mathrm{tg}\theta_W Z_{i2}^* - Z_{i1}^*), \quad (51)$$

$e_\ell$ and $T_3(\ell)$ – and is the electric charge and the third projection of the weak lepton isospin $\ell$.

Now let us find the square of the matrix element $\overline{|M_c+M_d|^2}$, summed over the spin states of the neutralino (lepton and antilepton are longitudinally polarized):

$$\overline{|M_c+M_d|^2} = g^4\{(f_{\ell i}^L f_{\ell j}^L)^2[|D_{\tilde{\ell}_L}(t)|^2(p_1\cdot k_1)(p_2\cdot k_2)+|D_{\tilde{\ell}_L}(u)|^2(p_1\cdot k_2)(p_2\cdot k_1)-$$
$$-2\,\mathrm{Re}(D_{\tilde{\ell}_L}(t)D^*_{\tilde{\ell}_L}(u))\eta_i\eta_j m_{\chi_i}m_{\chi_j}s](1-\lambda_1)(1-\lambda_2)+(f_{\ell i}^R f_{\ell j}^R)^2|D_{\tilde{\ell}_R}(t)|^2(p_1\cdot k_1)(p_2\cdot k_2)+$$
$$+|D_{\tilde{\ell}_R}(u)|^2(p_1\cdot k_2)(p_2\cdot k_1)-2\,\mathrm{Re}(D_{\tilde{\ell}_R}(t)D^*_{\tilde{\ell}_R}(u))\eta_i\eta_j m_{\chi_i}m_{\chi_j}s](1+\lambda_1)(1-\lambda_2)\}. \quad (52)$$

Here $\eta_i$, $\eta_j=\pm1$ are the sign factors appearing from the operator products in the S-matrix in connection with Vick's theorems [15].

Having the square of the matrix element, it is easy to calculate in a standard way the differential effective cross section of this process in the center-of-mass system we used the following relations:

$$(p_1\cdot k_1)=\frac{s}{4}[1+r_i-r_j-\sqrt{\lambda(r_i,r_j)}\cos\theta],\ (p_2\cdot k_2)=\frac{s}{4}[1-r_i+r_j-\sqrt{\lambda(r_i,r_j)}\cos\theta],$$
$$(p_1\cdot k_2)=\frac{s}{4}[1-r_i+r_j+\sqrt{\lambda(r_i,r_j)}\cos\theta],\ (p_2\cdot k_1)=\frac{s}{4}[1+r_i-r_j+\sqrt{\lambda(r_i,r_j)}\cos\theta],$$
$$\frac{d\sigma^{(\tilde{\ell})}}{d(\cos\theta)}=\frac{g^4 s}{2^9\pi}\sqrt{\lambda(r_i,r_j)}\{(f_{\ell i}^L f_{\ell j}^L)^2[|D_{\tilde{\ell}_L}(t)|^2((1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta-$$

$$-2\sqrt{\lambda(r_i,r_j)}\cos\theta)+\left|D_{\tilde{\ell}_L}(u)\right|^2((1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta+$$
$$+2\sqrt{\lambda(r_i,r_j)}\cos\theta)-8\,\mathrm{Re}(D_{\tilde{\ell}_L}(t)D^*_{\tilde{\ell}_L}(u))\eta_i\eta_j\sqrt{r_ir_j}\,](1-\lambda_1)(1+\lambda_2)+$$
$$+(f^R_{\ell i}f^R_{\ell j})^2[\left|D_{\tilde{\ell}_R}(t)\right|^2((1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta-2\sqrt{\lambda(r_i,r_j)}\cos\theta)+$$
$$+\left|D_{\tilde{\ell}_R}(u)\right|^2((1+r_i-r_j)(1-r_i+r_j)+\lambda(r_i,r_j)\cos^2\theta+2\sqrt{\lambda(r_i,r_j)}\cos\theta)-$$
$$-8\,\mathrm{Re}(D_{\tilde{\ell}_R}(t)D^*_{\tilde{\ell}_R}(u))\eta_i\eta_j\sqrt{r_ir_j}\,](1+\lambda_1)(1-\lambda_2)\,. \qquad (53)$$

It follows from this expression that, in the annihilation process, the lepton and the antilepton must have opposite helicities, either $\lambda_1=-\lambda_2=+1$, or $\lambda_1=-\lambda_2=-1$.

Integrating by the angles of departure of the neutralino , we finally obtain

$$\sigma^{(\tilde{\ell})}_{tot}=\frac{1}{2}\sigma_{\tilde{\ell}}(2-\delta_{ij})\,, \qquad (54)$$

where

$$\sigma_{\tilde{\ell}}=\frac{g^4\sqrt{\lambda(r_i,r_j)}}{32\pi s}\times$$
$$\times\left\{(f^L_{\ell i}f^L_{\ell j})^2\left[2-\frac{r_{\tilde{\ell}_L}}{r_ir_j+r_{\tilde{\ell}_L}(1+r_{\tilde{\ell}_L}-r_i-r_j)}-\frac{1}{\sqrt{\lambda(r_i,r_j)}}\left(r_i+r_j-2r_{\tilde{\ell}_L}-\frac{2\eta_i\eta_j\sqrt{r_ir_j}}{1+2r_{\tilde{\ell}_L}-r_i-r_j}\right)L(\tilde{\ell}_L)\right]+\right.$$
$$\left.+(f^R_{\ell i}f^R_{\ell j})^2\left[2-\frac{4r_{\tilde{\ell}_R}}{1+2r_{\tilde{\ell}_R}-r_i-r_j}+\frac{1}{\sqrt{\lambda(r_i,r_j)}}\left(r_i+r_j-2r_{\tilde{\ell}_R}-\frac{2\eta_i\eta_j\sqrt{r_ir_j}}{1+2r_{\tilde{\ell}_R}-r_i-r_j}\right)L(\tilde{\ell}_R)\right]\right\}, \qquad (55)$$
$$L(\tilde{\ell}_{L,R})=\ln\left|\frac{1+2r_{\tilde{\ell}_{L,R}}-r_i-r_j+\sqrt{\lambda(r_i,r_j)}}{1+2r_{\tilde{\ell}_{L,R}}-r_i-r_j-\sqrt{\lambda(r_i,r_j)}}\right|,\ r_{\tilde{\ell}_L}=\left(\frac{m_{\tilde{\ell}_L}}{\sqrt{s}}\right)^2,\ r_{\tilde{\ell}_R}=\left(\frac{m_{\tilde{\ell}_R}}{\sqrt{s}}\right)^2,$$

$\delta_{ij}=0$ at the production of different neutralinos ($i\neq j$) and $\delta_{ij}=1$ at the production of identical neutralinos ($i=j=1,\ 2,\ 3,\ 4$), $m_{\tilde{\ell}_L}$ and $m_{\tilde{\ell}_R}$ – are the masses scalar lepton $\tilde{\ell}_L$ and $\tilde{\ell}_R$.

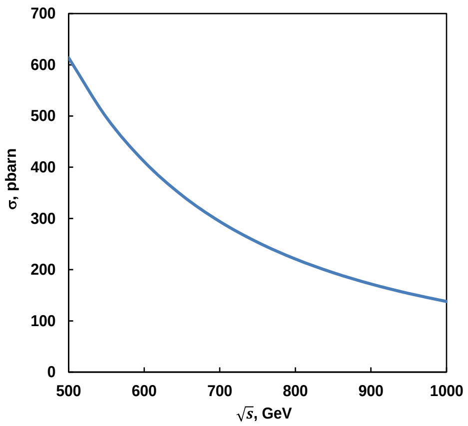


*Fig. 12.* Energy dependence of the cross section of the reaction $e^-e^+\to(\tilde{\ell}_L;\,\tilde{\ell}_R)\to\tilde{\chi}^0_2\tilde{\chi}^0_2$.

Figure 12 illustrates the dependence of the effective cross section of the process $e^-+e^+\to(\tilde{\ell}_L;\,\tilde{\ell}_R)\to\tilde{\chi}^0_2+\tilde{\chi}^0_2$ on the energy $\sqrt{s}$ of the electron-positron beams at parameter values $m_{\tilde{\ell}_L}$ = $m_{\tilde{\ell}_R}$=40 GeV, $x_W=0.2315$, $M_2=2M_1=150$ GeV, $\mu=200$ GeV, $\mathrm{tg}\beta=3$. As can be seen from the figure, the cross section of the process $e^-+e^+\to(\tilde{e}_L;\,\tilde{e}_R)\to\tilde{\chi}^0_2+\tilde{\chi}^0_2$ decreases with increasing energy of the electron-positron beams.

## CONCLUSION

Thus, we have discussed the process of neutralino pair production in arbitrarily polarized lepton-antilepton (electron-positron or muon-antimuon) collisions $\ell^-+\ell^+\to\tilde{\chi}^0_i+\tilde{\chi}^0_j$. Diagrams with exchanges of neutral Z-bosons, scalar $H$ and $h$, pseudoscalar $A$-bosons, and scalar $\tilde{\ell}_L$ and $\tilde{\ell}_R$ leptons have been studied in detail. Expressions for the differential and integral cross sections of the process are obtained, and the longitudinal and transverse spin asymmetries due to lepton-antilepton pair polarizations, the forward-backward angular asymmetry, and the degrees of longitudinal and transverse neutralino polarization are determined. The angular and energy dependences of these characteristics and the total cross section of the reaction are studied in detail. The results of the calculations are illustrated by graphs.

## APPENDIX

Here we give the expressions for the lepton tensor $L_{\mu\nu}$ and the neutralino tensor $\chi_{\mu\nu}$:

$$L_{\mu\nu} = \frac{1}{2}(g_L^2 + g_R^2)[p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu} - (p_1 \cdot p_2)g_{\mu\nu} - m_\ell^2(s_{1\mu}s_{2\nu} + s_{2\mu}s_{1\nu} - (s_1 \cdot s_2)g_{\mu\nu}] +$$

$$+\frac{1}{2}(g_L^2 - g_R^2)m_\ell[p_{1\mu}s_{2\nu} + s_{2\mu}p_{1\nu} - (p_1 \cdot s_2)g_{\mu\nu} - s_{1\mu}p_{2\nu} - p_{2\mu}s_{1\nu} + (p_2 \cdot s_1)g_{\mu\nu}] +$$

$$+ g_L g_R\{(p_1 \cdot s_2)[s_{1\mu}p_{2\nu} + p_{2\mu}s_{1\nu} - (p_2 \cdot s_1)g_{\mu\nu}] - (p_1 \cdot p_2)[s_{1\mu}s_{2\nu} + s_{2\mu}s_{1\nu} - (s_1 \cdot s_2)g_{\mu\nu}] +$$

$$+ (p_2 \cdot s_1)[s_{2\mu}p_{1\nu} + p_{1\mu}s_{2\nu}] - (s_1 \cdot s_2)[p_{1\mu}p_{2\nu} + p_{2\mu}p_{1\nu}]\} + \frac{1}{2}(g_L^2 + g_R^2)(-m_\ell)[(\mu\nu p_1 s_2)_\varepsilon +$$

$$+ (\mu\nu p_2 s_1)_\varepsilon] + \frac{1}{2}(g_L^2 - g_R^2)i[-(\mu\nu p_1 p_2)_\varepsilon + m_\ell^2(\mu\nu s_1 s_2)_\varepsilon] + g_L g_R i m_\ell[(\mu\nu p_1 s_1)_\varepsilon - (\mu\nu p_2 s_2)_\varepsilon];$$

$$\chi_{\mu\nu} = \frac{1}{2}(G_L^2 + G_R^2)[k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu} - (k_1 \cdot k_2)g_{\mu\nu} - m_{\chi_i}m_{\chi_j}(s_\mu s'_\nu + s_\nu s'_\mu - (s \cdot s')g_{\mu\nu}] +$$

$$+\frac{1}{2}(G_L^2 - G_R^2)[m_{\chi_j}(k_{1\mu}s'_\nu + k_{1\nu}s'_\mu - (k_1 \cdot s')g_{\mu\nu} - m_{\chi_i}(s_\mu k_{2\nu} + s_\nu k_{2\mu} - (k_2 \cdot s)g_{\mu\nu})] +$$

$$+ G_L G_R[-m_{\chi_i}m_{\chi_j}g_{\mu\nu} - (k_1 \cdot k_2)(s_\mu s'_\nu + s_\nu s'_\mu - (s \cdot s')g_{\mu\nu}) - (s \cdot s')(k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu}) +$$

$$+ (k_1 \cdot s')[s_\mu k_{2\nu} + s_\nu k_{2\mu} - (k_2 \cdot s)g_{\mu\nu}] + (s \cdot k_2)[k_{1\mu}s'_\nu + k_{1\nu}s'_\mu] + \frac{1}{2}(g_L^2 + g_R^2)i[m_{\chi_j}(\mu\nu k_1 s')_\varepsilon +$$

$$m_{\chi_i}(\mu\nu k_2 s)_\varepsilon] + \frac{1}{2}(g_L^2 - g_R^2)i[(\mu\nu k_1 k_2)_\varepsilon - m_{\chi_i}m_{\chi_j}(\mu\nu s s')_\varepsilon] + g_L g_R i[m_{\chi_i}(\mu\nu k_2 s')_\varepsilon + (\mu\nu k_1 s)_\varepsilon],$$

the notation is introduced here $(\mu\nu ab)_\varepsilon = \varepsilon_{\mu\nu\rho\sigma}a_\rho b_\sigma$.

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[1] ATLAS Collaboration. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector of the LHC. Phys. Letters, 2012, B 716, p. 1-29.

[2] CMS Collaboration. Observation of a new boson at mass of 125 GeV with the CMS experiment at the LHC. Phys. Letters, 2012, B 716, p. 30-60.

[3] *V.A. Rubakov*. UFN, 2012, V.182, No 10, p.1017-1025 (in Russian).

[4] *A.V. Lanev*. CMS Collaboration results: Higgs boson and search for new physics. UFN, 2014, V. 184, No 9, p. 996-1004 (In Russian).

[5] *D.I. Kazakov*. The Higgs boson is found: what is next? UFN, 2014, V. 184, No 9, p. 1004-1017 (In Russian).

[6] ATLAS Collaboration. Measurements of Higgs boson production and couplings in diboson final states with the ATLAS detector at the LHC. Phys. Letters, 2013, B 726, p. 88-119.

[7] *A. Djouadi*. The Anatomy of Electro-Weak Symmetry Breaking. Tome II: The Higgs boson in the Minimal Supersymmetric Standard Model. arXiv: hep-ph/0503172v2, 2003; DOI: 10.1016/j.physrep.2007.10.004.

[8] *J.F. Gunion, H.E. Haber*. Higgs bosons in Supersymmetric. Phys. Rev., 2003, D 67, p. 0750.19.

[9] *D.I. Kazakov*. Supersymmetry on the Run: LHC and Dark Matter. Nucl. Phys. B Proc. Suppl., 2010, Vol. 203-204, p. 118-154.

[10] *R.K. Barman et al*. Current status of MSSM Higgs sector with LHC 13 TeV data. Eur. Phys. J. Plus, 2019, Vol. 134: 150, No 4 arXiv: 1608.02573v3, [hep-ph]

[11] *M.M. El Kheishen, A.A. Shafik, A.A. Aboshousha*. Phys. Rev., 1992, Vol. 45, No. 11, pp. 4345-4348. DOI:https://doi.org/10.1103/PhysRevD.45.4345

[12] *A. Djouadi, P. Janot, J. Kalinowski, P.M. Zerwas*. SUSY Decays of Higgs Particles. CERN PPE/96-34, 1996, 13p.

[13] *A. Djouadi, J. Kalinowski, P. Ohmann, P.M. Zerwas*. Heavy SUSY Higgs bosons at $e^-e^+$ linear colliders. Z. Phys., 1997, C 74, p. 93-111.

[14] *A. Bartl, H. Fraas, W. Majerotto*. Signatures of chargino production in $e^+e^-$collisions. Z. Phys., 1986, B 30, p. 441-449.

[15] *A. Bartl, H. Fraas, W. Majerotto*. Production and decay of neutralinos in $e^+e^-$-collisions. J. Nucl. Phys., 1986, B 278, p. 1-25

[16] *E.Ch. Christova, N.P. Nedelcheva*. Neutralino production on polarized $e^+e^-$-collisions. Preprint, Dubna, 1988, E2-88-607, 14p.

[17] *S.M. Bilenky, N.P. Nedelcheva*. Possible test for supersymmetry in $e^-e^+$-collisions with polarized beams. Preprint, Dubna, 1986, E2-88-494, 10p.

[18] *P. Cheapetta, J. Soffer, P. Taxil, F.M. Renard, P. Sorba*. Supersymmetry and polarization in $e^-$

$e^+$-collisions (I). Nucl. Phys., 1984, Vol. B259, p. 365-396.
[19] *S.K. Abdullayev, M.Sh. Gojayev, A.K. Gulayeva.* The production of a chargino pair in polarized lepton-antilepton collisions (I). AJP, Fizika, 2020, V. XXVI, No 3, p. 20-30.
[20] *S.K. Abdullayev, M.Sh. Gojayev, A.K. Gulayeva.* The production of a chargino pair in polarized lepton-antilepton collisions (II). AJP, Fizika, 2020, V. XXVI, No 4, p. 19-27.

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