EFFECT OF PHASE MISMATCH ON THE ENERGY OF ULTRASHORT PULSES IN A FABRY-PERROT CAVITY

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Analytic expression for the energy of sum frequency ultra short laser pulses in the Fabry-Perrot cavity is obtained in the constant field approximation where complex amplitude of high intensity pump wave remains constant. Effect of phase detuning on the sum frequency ultrashort laser pulse is studied through consideration effects of group velocity delay (GVD), as well as the phase modulation. It is obtained that energy can reach maximum at both positive and negative values of phase mismatch parameter depending on other parameters of problem. Energy maxima correspond to optimum values of reflective index of resonator at various values of phase detuning. In the dependence on the length of nonlinear medium it is observed relative sharp increase in energy when characteristic length of group velocity mismatch is zero. It is observed that energy decreases with increase in phase modulation.

Keywords: constant field approximation, second order dispersion theory, group velocity delay, phase modulated sum frequency laser pulse, Fabry-Perrot resonator.
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1. INTRODUCTION

An interest to the interaction of ultrashort pulses with nonlinear medium is due their prospective in the development of power full sources of light of femtosecond duration [1]. In this paper the frequency conversion of ranning wave was analyzed. The use of Fabry-Perrot cavity [2] also is one of methods to achieve higher efficiencies in the frequency conversion. İn particular the external resonator can be used for this purpose [3]. In general equations describing nonlinear interaction of waves are solved in different approximations in addition to the exact solution. Constant amplitude (field) approximation and constant intensity approximations have found more applications for describing the nonlinear interaction of light waves. Constant intensity approximation [4] has allowed to take into account the reverse reaction of excited or amplified waves onto exciting wave. In [5-9] we have studied processes of frequency conversion by employment of constant intensity approximation. In [10] we have investigated effect of group velocity mismatch and group velocity dispersion on the spectrum and energy of sum frequency ultrshort pulses under phase matching conditions. Goal of present papere is to study effect of phase detuning on energy of sum frequency ultrashort pulses. This is important case since phase-mismatch is more relevant to the real condition of the frequency conversion process.

The purpose of present paper is to study effects of phase mismatch on the energy of sum frequency pulse as well as on the energy depnedencies on the reduced length of medium, reflective index of cavity, characteric lengths of interaction.

2. DISCUSSION AND RESULTS

When the linear losses are negligible in a nonlinear medium the three-wave interaction of waves is described in the second order dispersion theory by the set of coupled equations [1].

$$\left(\frac{\partial(z,t)}{\partial z}+\frac{1}{u_1}\frac{\partial}{\partial t}-i\frac{g_1}{2}\frac{\partial^2}{\partial t^2}\right)A_1=i\gamma_1A_3A_2^*e^{i\Delta z}$$
$$\left(\frac{\partial}{\partial z}+\frac{1}{u_2}\frac{\partial}{\partial t}-i\frac{g_2}{2}\frac{\partial^2}{\partial t^2}\right)A_2=-i\gamma_2A_3A_1^*e^{i\Delta z} \qquad (1)$$
$$\left(\frac{\partial}{\partial z}+\frac{1}{u_3}\frac{\partial}{\partial t}-i\frac{g_3}{2}\frac{\partial^2}{\partial t^2}\right)A_3=-i\gamma_3A_1A_2e^{-i\Delta z}$$

Here A_j (j=1-3)- are the complex amplitudes of a signal, pump and idler waves respectively, u_j –are the group velocities of the interacting waves, $\Delta= k_1-k_2-k_3$ is the phase mismatch between the interacting waves , which plays an important role for the spatial distribution of the electromagnetic field along the sample, $g_j=\partial^2k_j/\partial\omega_j^2$ is the dispersion of group velocities and γ_1, γ_2, γ_3, are the coefficients of nonlinear coupling . Assuming that amplitude of the pump wave is fixed ($A_2=A_{20}=const.$) boundary conditions for the complex amplitudes of interacting waves are presented as follows:

$$A_3^-(l)=R_3A_3^+(l)e^{-i2k_3l};\quad A_3^+(0)=R_{30}A_3^-(0); \qquad (2)$$
$$A_1^+(0)=A_{10}(t;\qquad A_2^+(l)=A_{20}$$

where R_{30} and R_3 are complex coefficients of reflection for the wave of sum frequency from the input and output mirrors respectively when waves are incident

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from the nonlinear medium. Employment the Fourier transformation $A_{1,3}(\omega,t)=\int_{-\infty}^{+\infty}A_{1,3}(z,t)e^{i\omega t}dt$ to the system (1) for the spectral density of sum frequency wave at the output of cavity yields [10]:

$$S_3(\omega,z)=\frac{(1-r_3^2)\gamma_3^2 S_{10}(\omega)I_{20}z^2 sinc^2\mu z}{1-2r_{30}r_3\chi cos\,\Psi+r_3^2\chi^2} \qquad (3)$$

where $\mu^2=\Gamma^2+\frac{\beta^2}{4}$, $\chi^2=cos^2\mu z+\frac{\beta^2}{4\mu^2}sin^2\mu z$,

$\beta=\frac{1}{2}\omega^2 g-\omega\nu-\Delta$, $\quad\Psi=\varphi+arctg(\frac{\beta}{2\mu}tg\mu z)$,

$\varphi=\varphi_r+2k_3 l+\frac{\Delta l}{2}+\frac{\omega^2}{4}(g_1+g_3)-\frac{\omega\nu}{2}l$,

$\varphi_r=\varphi_{10}+\varphi_1$

We consider the idler wave to be Gaussian with quadratic phase modulation in the input of nonlinear medium.

$$A_1(t)=A_{10}\exp[-\left(\frac{1}{2\tau^2}+i\frac{\gamma}{2}\right)t^2] \qquad (4)$$

By employment the inverse Fourier transformation we obtain spectral density of idler wave in the frequency domain

$$A_1(\omega,z)=\frac{A_{10}\tau^2}{2\pi}\frac{1}{\sqrt{1+p}}e^{-\frac{x^2}{1+p}} \qquad (5)$$

where $x=\omega\tau$ and $p=\gamma^2\tau^4$ are the phase and frequency modulation parameters respectively.

According to (3) and (5) substitution of spectral density of idler wave into equation (3) for the energy of the energy sum frequency wave we get

$$W_3(\omega,z)=\int_{-\infty}^{+\infty}S_3(\omega,z)d\omega=K\frac{(1-r_3^2)sinc^2\mu z\times\exp(-\frac{(\omega\tau)^2}{1+(\gamma\tau)^2})}{(1+\mu^2)(1-2r_{30}r_3\chi cos\,\Psi+r_3^2\chi^2)} \qquad (6)$$

where

$$\mu={l_{nl}}^{-1}\left[\frac{1}{4}\frac{l_{nl}}{l_d}(\alpha-1)(\omega\tau)^2-\frac{1}{2}\frac{l_{nl}}{l_\nu}\omega\tau\right]-\frac{\Delta l_{nl}}{2}$$

$$\chi=\left[cos^2\left(\frac{z}{l_{nl}}\sqrt{1+\mu^2}\right)+\frac{\mu^2}{1+\mu^2}sin^2\left(\frac{z}{l_{nl}}\sqrt{1+\mu^2}\right)\right]^{1/2}$$

$$\Psi=\frac{z}{l_{nl}}\left[\frac{1}{4}\frac{l_{nl}}{l_d}(\alpha+1)(\omega\tau)^2-\frac{1}{2}\frac{l_{nl}}{l_\nu}\omega\tau\right]-\frac{\Delta l_{nl}}{2}+arctg\left[\frac{\mu}{\sqrt{1+\mu^2}}tg\left(\frac{z}{l_{nl}}\sqrt{1+\mu^2}\right)\right]$$

$$sinc\ x=sinx/x$$

In the fig.1 all dependences are obtained at the constant values of phase modulation parameter $p=5$, reflective index of Fabry-Perrot cavity $r_3=0{,}9$ and reduced lentgth of nonlinear medium $\frac{z}{l_{nl}}=0{,}5$. Characteristic length ratios are varyied for various curves. As can be seen under constant vaules of other parametrs higher value of energy is observed for $\frac{l_{nl}}{l_\nu}=0$ (curve 3) in comparison with $\frac{l_{nl}}{l_d}=0$, (curve 1) and this value corresponds to the positive values of phase detunung .An increse in the characteristic length ration $\frac{l_{nl}}{l_d}$ leads to the shift of energy maximum toward smaller values of phase detunung parameter (comparison of plots 3 and 4). Symmetric behavoir of plots is observed only for $\frac{l_{nl}}{l_d}=0$ only. It is also seen that maxima of energy correspond not only positive values of pahase detuning but also to its negative values.

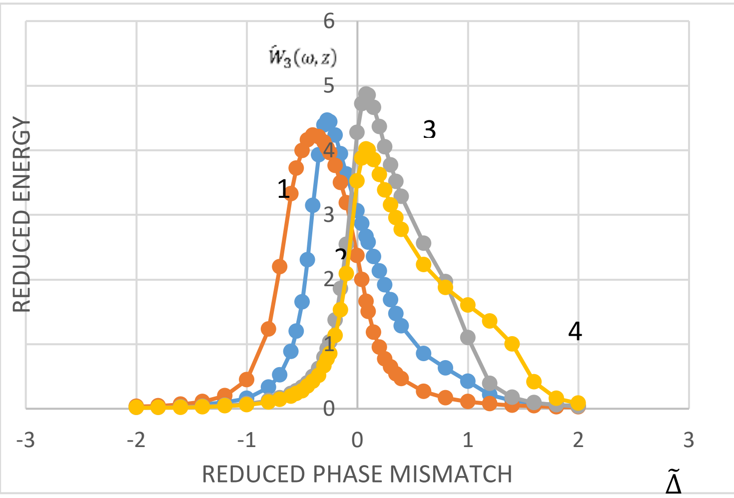

Fig. 1. Dependence of reduced energy of sum frequency ultrashort pulse on the reduced phase mismatch parameter $\tilde{\Delta}=\Delta/2\Gamma$

1. $\frac{z}{l_{nl}} = 0{,}5, \ \frac{l_{nl}}{l_d} = 0, \ \frac{l_{nl}}{l_\nu} = 2, \ p = 5, r_3 = 0{,}9;$
2. $\frac{z}{l_{nl}} = \ 0{,}5 \ , \ \frac{l_{nl}}{l_d} = \frac{l_{nl}}{l_\nu} = 2, p = 5, \ \ r_3 = 0{,}9;$
3. $\frac{z}{l_{nl}} = 0{,}5, \ \frac{l_{nl}}{l_d} = 2, \ \frac{l_{nl}}{l_\nu} = 0, \ p = 5, \ r_3 = 0{,}9;$
4. $\frac{z}{l_{nl}} = 0{,}5, \ \frac{l_{nl}}{l_d} = 3, \ \frac{l_{nl}}{l_\nu} = 0, \ p = 5, \ r_3 = 0{,}9.$

As can be seen from fig.2 an increase in phase mismatch parameter leads to decrease in values of maxima and their shift tovard larger values of reduced length at constant values of characteristic lengths. In spite of increase in phase mismatch relatively other plots its maxium is higher than those at zero value of characteristic length $\frac{l_{nl}}{l_\nu}$(curve 3).

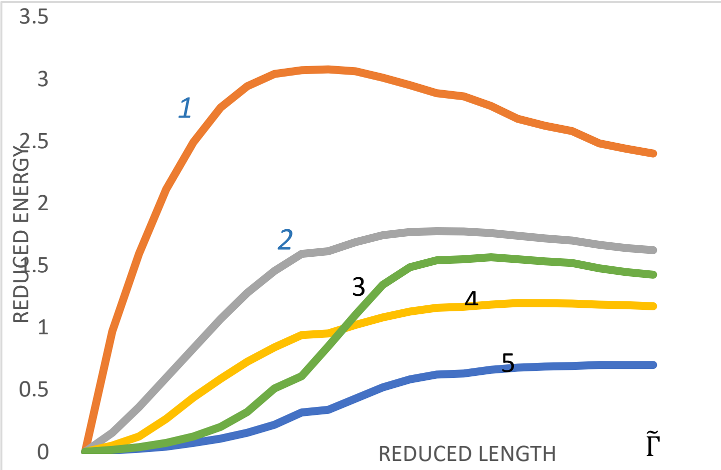

Fig. 2. Dependence of reduced energy $\widetilde{W}_3$ of sum frequency ultrashort pulse on the reduced lentgh $\tilde{\Gamma} = \frac{z}{l_{nl}}$ of nonlinear medium at different values of reduced phase mismatch $\tilde{\Delta} = \Delta/2\Gamma$.

1. $\tilde{\Delta} = 0 \ , \ \frac{l_{nl}}{l_d} = \frac{l_{nl}}{l_\nu} = 2, \ p = 5, \ \ \ r_3 = 0{,}9;$
2. $\tilde{\Delta} = 0{,}3 \ , \ \frac{l_{nl}}{l_d} = \frac{l_{nl}}{l_\nu} = 2, p = 5, \ \ r_3 = 0{,}9;$
3. $\tilde{\Delta} = 1, \ \frac{l_{nl}}{l_d} = 2, \ \ \frac{l_{nl}}{l_\nu} = 0, \ p = 5, \ r_3 = 0{,}9;$
4. $\tilde{\Delta} = 0{,}5 \ , \frac{l_{nl}}{l_d} = \frac{l_{nl}}{l_\nu} = 2, \ p = 5, \ \ \ r_3 = 0{,}9;$
5. $\tilde{\Delta} = 1 \ , \ \frac{l_{nl}}{l_d} = \frac{l_{nl}}{l_\nu} = 2, \ p = 5, \ \ \ \ r_3 = 0{,}9$

Fig.3 demonstrates that maxima of reduced energy are obtained at optimum values of reflective index for various values of other parameters. Maximum of energy in case of negative value of phase mismatch is higher than that of case with its zero value due to due to increase in the value of characteristic lengths (compare curves 2 and 3).

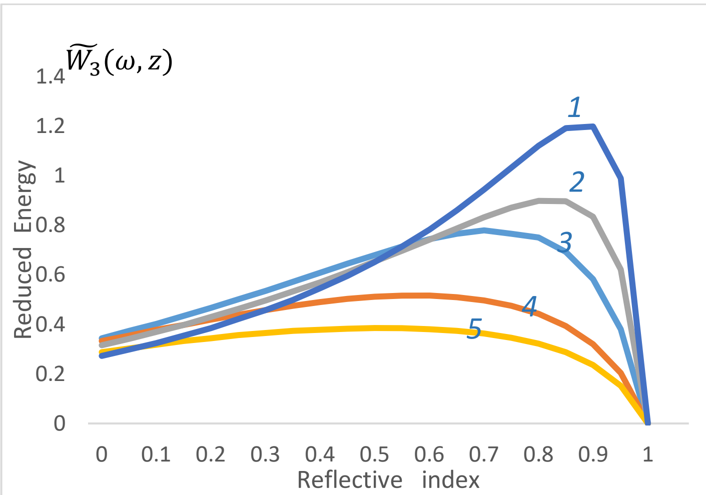

Fig. 3. Dependence of reduced energy $\widetilde{W}_3 \ (\omega, z)$ of sum frequency ultrashort pulse on the reflective coefficient r_3 of Fabri-Perrot cavity at different values of of reduced phase mismatch $\tilde{\Delta} = \Delta/2\Gamma$ and modulation parameter p : $\frac{z}{l_{nl}} = 0{,}5, \ \frac{l_{nl}}{l_d} = \frac{l_{nl}}{l_\nu} = 2, \ p = 0$ (curves 1, 5) ; 5 (curves 2, 3,4); $\frac{z}{l_{nl}} = 0{,}5, \ \frac{l_{nl}}{l_d} = \frac{l_{nl}}{l_\nu} = 3,$ (curves 4, 5) : 1. $\tilde{\Delta} = 0{,}3$; 2. $\tilde{\Delta} = -0{,}3$; 3. $\tilde{\Delta} = 0$; 3. 4. $\tilde{\Delta} = 0{,}3$; p=5, 5. $\tilde{\Delta} = 0{,}3$; p=0

In fig.4 the plots of sum frequency energy on the characteristic lengths are presented. As can be seen these dependences demonstrate maxima, decrease and the saturation behavoir .

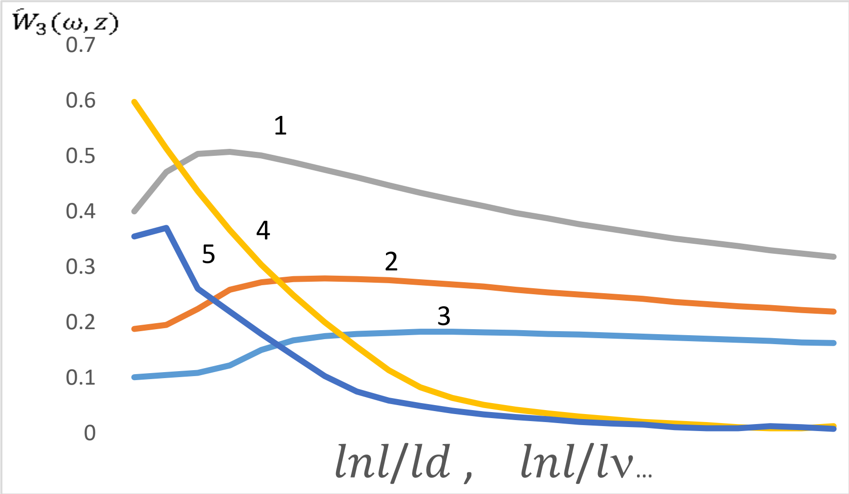

Fig. 4. Dependence of reduced energy $\widetilde{W}_3\,(\omega, z)$ of sum frequency ultrashort pulse on the characteristic length ratio $\frac{l_{nl}}{l_d}$ (curves 1,2,3) and $\frac{l_{nl}}{l_\nu}$ (curves 4,5) at different values of reduced phase mismatch $\tilde{\Delta} = \Delta/2\Gamma$ when $p = 0$, $r_3 = 0{,}9$ and $\frac{z}{l_{nl}} = 0{,}5$, and $\frac{l_{nl}}{l_\nu} = 3$ (plots 1,2,3), $\frac{l_{nl}}{l_d} = 3$ (curves 4,5):
1- $\tilde{\Delta} = -0{,}3$; 2- $\tilde{\Delta} = 0$; 3-$\tilde{\Delta} = 0{,}3$; 4- $\tilde{\Delta} = 0$; 5-$\tilde{\Delta} = 0{,}3$

3. CONCLUSIONS

On the basis of above stated one can conclude that energy of sum frequency decreases with increase in phase modulation. Energy possess maximum depending on the phase detunung parameter. Maxima are obtained for both positive and negative values of phase detuning. An increse in the characteristic length ration $\frac{l_{nl}}{l_d}$ leads to the shift of energy maximum toward smaller values of phase detunung parameter. Dependence of energy on the reflective index also has pronounced maximum. An increase in characteristic length leads to decrease in the energy of sum frequency. At definite values of problem parameters maximum corresponding to negative values of phase detuning is higher than the value for positive value of phase mismatch. Dependence of of sum frequency energy versus reduced length of nonlinear medium has pronounced maxiumum corresponding to optimum value of medium length.

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