ANTI-STOKES SCATTERING OF LOW-ENERGY NEUTRINOS AT TRANSVERSELY POLARIZED ULTRA-RELATIVISTIC ELECTRONS IN A MAGNETIC FIELD

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Keywords: relic neutrino, neutrino-electron scattering, magnetic field, low-energy neutrinos, polarization in particle interactions.
PACS: 14.60.Lm., 13.15.+g, 13.88.+e

We investigate the anti-Stokes scattering of sufficiently low-energy neutrinos (including relic neutrinos) at transversely polarized ultra-relativistic electrons in an external constant uniform magnetic field within the framework of the standard Weinberg-Salam model with allowance for the effect of the Z-boson propagator. In the weak field case, the contribution of the field and polarization effects to the cross section of the relic neutrino-electron scattering is negligible and the Z-boson propagator does not contribute to the cross section practically.

1. INTRODUCTION

One of the most important problems of modern nuclear physics, particle physics, astrophysics and cosmology is detection of neutrinos and antineutrinos, especially, relic neutrinos and antineutrinos [1].

The results of the investigations on the neutrino-electron scattering (NES)

$$\nu_i + e^- \rightarrow \nu_i{}' + e^{-\prime} \qquad (1)$$

in an external magnetic field (MF) show that in case of low-energy neutrinos or relic neutrinos (RN) the field effects are essential and neutrinos can be detected [2]. In case of low-energy neutrinos and in the absence of an external MF the cross section for the process (1) depends on the squared energy of the initial neutrino: $\sigma \sim G_F^2 \omega^2/\pi$ [3]. For RN this cross section has extremely small value $\sigma \sim 10^{-63} cm^2$. Behavior of the cross section of the NES in an external MF and various aspects of field and polarization effects arisen in this process were studied by numerous authors [2, 4-12]. The anti-Stokes scattering of low-energy neutrinos on relativistic electrons in a MF was investigated in [8] where the low-energetic (the four-fermion) approximation of the standard Weinberg-Salam electroweak interaction theory was used and some important results were obtained. In [8] the transverse polarization of the spins of the electrons (in both the initial and final states) were taken into account and the asymptotes of the cross section for the process (1) was calculated as a function depending only dynamic (field) parameter

$$\chi = \frac{B}{B_0}\frac{E}{m_e} \qquad (2)$$

that contains the electron energy E and the MF strength B where

$$B_0 = \frac{m_e^2 c^3}{e\hbar} = 4.414 \times 10^{13}\ G \qquad (3)$$

is the Schwinger field strength where $e > 0$ is the elementary electric charge. Hereafter we will use the system of units $c = \hbar = 1$. In this system of units $B_0 = m_e^2/e$. In [8] the authors obtained the formulae for the cross section of the NES in various limiting cases of the parameter χ ($\chi \ll 1$ and $\chi \gg 1$). Detailed analyses of the cross section for the process (1) show that the behavior of cross section in various limiting cases depends not only on the parameter χ but also on the kinematic parameter

$$\kappa = \frac{2\omega E}{m_e^2} \qquad (4)$$

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that contains the neutrino energy besides the electron energy. The role of the kinematic parameter was not taken into account in [8]. At the same time the detailed calculations and analyses show that some coefficients and signs in the expressions for the cross sections of the process (1) are incorrect in the work [8] (one of them was indicated by the authors of [11]). In [11] the cross section for the scattering of a muon neutrino by an electron in a constant external field is calculated within the framework of the standard Weinberg-Salam model with allowance for the effect of the Z-boson propagator. Apart from the work [8] in [11] the authors calculated the dependence of the cross section on the invariant κ and χ parameters. However, in [11] polarizations of the spins of the electrons (in both in the initial and final states) were not taken into account. In [2, 11] it was shown that the influence of the external field on the NES is determined by the new parameter

$$\eta = \frac{\chi}{\kappa}. \tag{5}$$

In [11] the authors investigated the NES in weak ($\eta \ll 1$) and strong ($\eta \gg 1$) field cases but they did not take into account the polarization effects.

In the presented paper we investigate the anti-Stokes scattering of sufficiently low-energy neutrinos ($\omega \ll m_e$) including RN at transversely polarized ultra-relativistic electrons in an external constant uniform MF with the strength $B \ll B_0$ within the framework of the standard Weinberg-Salam model with allowance for the effect of the Z-boson propagator. Investigation of this process takes an importance for understanding of the cooling of the electron gas.

2. MATRIX ELEMENT OF THE PROCESS

Since we take into account the effect of the Z-boson propagator, muon neutrinos and tau-neutrinos only participate in the process (1). When the momentum transferred is relatively small, $|q^2| \ll m_Z^2$ (m_Z is the Z-boson mass), the results obtained for the cross section of the muon (tau) neutrino-electron scattering will be applicable for all the neutrino flavors including electron neutrinos.

We choose the gauge of a four-potential of the external field as $A^\mu = (0, 0, xB, 0)$. In this gauge the MF is directed along the z-axis. We use the pseudo-Euclidean metric with signature $(+ - - -)$.

Using the standard Feynman rules the matrix element of the considered processes (1) in a MF is written in the form

$$M = \frac{g^2}{8\cos^2\theta_W} \int d^4x d^4x' G^{\alpha\beta}(x - x') \mathrm{N}_\alpha(x') \Lambda_\beta(x) \tag{6}$$

where

$$\mathrm{N}_\alpha(x') = \overline{\psi}_{\nu'}(x') \gamma_\alpha (1 + \gamma^5) \psi_\nu(x') \tag{7}$$

is the neutrino current,

$$\Lambda_\beta(x) = \overline{\psi}_{e'}(x) \gamma_\beta (g_V + g_A \gamma^5) \psi_e(x) \tag{8}$$

is the electron current

$$G^{\alpha\beta}(x - x') = \int G^{\alpha\beta}(q) e^{-iq(x-x')} \frac{d^4q}{(2\pi)^4} \tag{9}$$

is the Z-boson propagator,

$$G^{\alpha\beta}(q) = -\frac{g^{\alpha\beta} - q^\alpha q^\beta / m_Z^2}{q^2 - m_Z^2 + i\varepsilon} \tag{10}$$

$g = e/\sin\theta_W$, θ_W is the Weinberg angle, $\sin^2\theta_W \cong 0.23$, $q = k - k'$, γ^α are the Dirac matrices, $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$, $g_V = -0.5 + 2\sin^2\theta_W$ and $g_A = -0.5$ are for $\nu_\mu e^-(\nu_\tau e^-)$-scattering, $+\psi_\nu(x') = (2\omega V)^{-1/2} u(k) exp(ikx')$ is the wave function of the incident neutrino possessing the four-momentum k and the energy ω, $\overline{\psi}_{\nu'}(x') = \psi_{\nu'}^+(x')\gamma^0$, $\psi_{\nu'}(x') = (2\omega' V)^{-1/2} u(k') exp(ik'x')$ is the wave function of the scattered neutrino possessing the four-momentum k' and energy ω', V is the normalization volume, $u(k)$ and $u(k')$ are the Dirac bispinors of the incident and scattered neutrinos, respectively, $\psi_e(x)$ ($\psi_{e'}(x)$) is the solution of the Dirac equation in a constant homogeneous external MF for the electron in the initial (final) state, $\overline{\psi}_{e'}(x) = \psi_{e'}^+(x)\gamma^0$.

The explicit form of the exact wave function $\psi_e(x)$ for the electron in the initial state is as follows

$$\psi_e(x) = e^{-iEt} \psi_e(\vec{r}) \tag{11}$$

where

$$\psi_e(\vec{r}) = \left(L_y L_z\right)^{-1/2} (eB)^{1/4} e^{i(p_y y + p_z z)} U(\eta), \tag{12}$$

$$U(\eta) = \begin{pmatrix} c_1 u_{n-1}(\eta) \\ i c_2 u_n(\eta) \\ c_3 u_{n-1}(\eta) \\ i c_4 u_n(\eta) \end{pmatrix}, \tag{13}$$

$\eta = (eB)^{1/2}\left(x + \left(p_y/eB\right)\right)$ is the parameter, p_y (p_z) is the y (z)-component of the four-momentum of the electron in the initial state, L_y (L_z) is the normalization length along the y (z)-axis,

$$u_n(\eta) = \left(2^n n!\,\pi^{1/2}\right)^{-1/2} e^{-\eta^2/2} H_n(\eta) \quad (14)$$

are the Hermit functions,

$$H_n(\eta) = (-1)^n e^{\eta^2} \frac{d^n e^{-\eta^2}}{d\eta^n} \quad (15)$$

are the Hermit polynomials [13], $n = 0,1,2,...$ enumerates the Landau energy levels of the electron in the initial state, c_i $(i = 1,2,3,4)$ are the spin coefficients of the electron in the initial state. When the electrons are polarized transversely, the spin coefficients, c_i $(i = 1,2,3,4)$ are given by [13]

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} B_3(A_3 + A_4) \\ B_4(A_4 - A_3) \\ B_3(A_3 - A_4) \\ B_4(A_4 + A_3) \end{pmatrix} \quad (16)$$

where $A_3 = \sqrt{1 + (p_z/E)}$, $A_4 = \zeta\sqrt{1 - (p_z/E)}$, $B_3 = \sqrt{1 + \zeta(m_e/E_\perp)}$, $B_4 = \zeta\sqrt{1 - \zeta(m_e/E_\perp)}$, E - energy of the electron in the initial state, m_e is the electron mass, $E_\perp = \sqrt{E^2 - p_z^2} = m_e\sqrt{1 + 2fn}$, $f = B/B_0$, $\zeta = \pm 1$ is the spin quantum number that determines the projection of the electron spin along (opposite to) the direction of the MF vector $\vec{B}$. The electrons in the final state are described and denoted with the primed quantities that are determined with the same expressions (11)-(16) and above indicated related formulae.

3. CROSS SECTION OF THE PROCESS

We consider that the electrons in the initial and final states are ultra-relativistic ($E^2 \gg m_e^2$, ${E'}^2 \gg m_e^2$) and they possess large transverse momentum ($p_\perp = (2eBn)^{1/2} = m_e(2fn)^{1/2} \gg 1$, $p'_\perp = (2eBn')^{1/2} = m_e(2fn')^{1/2} \gg 1$) in the MF that is not enough strong and satisfies the condition $f \ll 1$. The latter two assumptions mean that the main contribution to the total cross section for the process comes from the electron states occupying high Landau levels ($n, n' \gg 1$). We consider the case when the longitudinal momentum of the electrons in the initial state is zero: $p_z = 0$.

We consider a massless neutrino model that is justified for ultra-relativistic neutrinos ($\omega, \omega' \gg m_\nu$). We also assume that the incident neutrinos fly along the MF direction

$$k^\mu = \omega(1, 0, 0, 1). \quad (17)$$

and their energies are in the range

$$\omega_{min} \ll \omega \ll m_e. \quad (18)$$

where $\omega_{min} = eB/p_\perp$.

The above indicated conditions and restrictions mean that the cross section of the process will depend on two parameters: the field parameter

$$\chi = \frac{e}{m_e^3}\left[-\left(F_{\mu\nu}p^\nu\right)^2\right]^{1/2} = \frac{B}{B_0}\frac{p_\perp}{m_e} \quad (19)$$

and the kinematical parameter

$$\kappa = \frac{2\omega E}{m_e^2} = \frac{2kp}{m_e^2} \quad (20)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the tensor of the external field.

Standard calculations of the cross section of the process gives the following general formula

$$\sigma = \frac{G_F^2 m_e^2}{4\pi^{3/2}} \int_0^\infty \left[A\Phi_1(z) - B\left(\frac{\chi}{u}\right)^{2/3} \Phi'(z) - C\left(\frac{\chi}{u}\right)^{1/3} \Phi(z)\right] N(u,\kappa) \frac{u\,du}{(1+u)^4} \quad (21)$$

where

$$A = \frac{\kappa}{2u}\left[g_L^2(1+u)^2 + g_R^2 + 2g_L g_R \zeta\zeta'(1+u)\right] - g_L g_R(1+\zeta\zeta')(1+u), \quad (22)$$

$$B = g_L^2(1+u)^2 + g_R^2 + 2g_L g_R \zeta\zeta'(1+u), \quad (23)$$

$$C = g_L^2 \zeta'(1+u)^2 - g_R^2\zeta + g_L g_R(\zeta - \zeta')(1+u), \quad (24)$$

$$N(u,\kappa) = \left[\kappa\left(\frac{m_e}{m_Z}\right)^2 \frac{u}{1+u} + 1\right]^{-2}, \quad (25)$$

u is the invariant spectral variable

$$u = \frac{\chi}{\chi'} - 1 = \frac{p_\perp}{p'_\perp} - 1 \simeq \frac{\omega'}{E - \omega'}, \quad (26)$$

the field parameter χ' belongs to the electrons in the final state,

$$\Phi(z) = \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} dt exp\left[i\left(zt + \frac{t^3}{3}\right)\right] \quad (27)$$

is the Airy function depending on the argument

$$z = \left(\frac{u}{\chi}\right)^{2/3}\left(1 - \frac{\kappa}{u}\right), \tag{28}$$

$\Phi'(z) = d\Phi(z)/dz$ and $\Phi_1(z) = \int_z^{\infty}\Phi(y)dy$.

After the averaging over the initial electron polarization states and summation over the final electron polarization states, in particular case, we obtain the corresponding formula of the work [11] from the formula (21) of this work. In [8] and [7] the process (1) was investigated in the low-energetic (the four-fermion) approximation of the standard Weinberg-Salam electroweak interaction theory. Therefore, the results of the works [8] and [7] are derived from our formula (21) as a particular case if we disregard the propagator effect in the formulae (21) and (25), i.e. in the limiting case $\kappa(m_e/m_Z)^2 = 2\omega E/m_Z^2 \ll 1$ the multiplier $N(u,\kappa)$ in the integrand in (21) is replaced with $N(u,\kappa) \cong 1$.

4. CONCLUSION

The behavior of the cross section (21) depends on the behavior of the Airy function $\Phi(z)$, its derivative $\Phi'(z)$ and the function $\Phi_1(z)$. At the same time the behavior of the functions $\Phi(z)$, $\Phi'(z)$ and $\Phi_1(z)$ depends on the behavior of the argument

$$z = \frac{1}{\eta^{2/3}}\frac{w-1}{w^{1/3}} \tag{29}$$

where $w = u/\kappa$.

The factor $1/\eta^{2/3}$ takes a leading role in behavior of the argument z. When $\eta \ll 1$, for the relic neutrino energy $\omega \sim 10^{-4}\ eV$ and for the realistic values of the electron energy E and the magnetic field strength B that can be achieved in the laboratory conditions the parameter $\chi \ll 1$ and the factor $N(u,\kappa)$ is replaced with $N(u,\kappa) \cong 1$ because of $\kappa(m_e/m_Z)^2 = 2\omega E/m_Z^2 \ll 1$. So, in the weak field case ($\eta \ll 1$) the contribution of the field ($\sigma \sim \chi^2$) and polarization ($\sigma \sim \zeta'\chi$) effects to the cross section of the relic neutrino-electron scattering in a MF is negligible and the Z-boson propagator does not contribute to the cross section practically.

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Received: 11.11.2021