ON THE THEORY OF PARAMETRICAL INTERACTION OF LASER PULSES IN METAMATERIAL

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A theory of parametric interaction of laser pulses in metamaterials has been developed. Analytic expression for the spectral density of a backward signal wave is obtained in the presence of group velocity mismatch and group velocity dispersion. The excited pulse was observed to be splitting out into several peaks at larger nonlinear lengths or when characteristic lengths corresponding to the group velocity mismatch as well as the group velocity dispersion are less than the nonlinear length. It is found that upon parametric interaction between forward (pump and idler) and backward waves, compensation of signal wave losses by the losses of direct waves has allowed the parametric amplification and generation of the backward wave.

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1. INTRODUCTION

Interaction of metamaterials with the electromagnetic waves is distinguished with its specific features [1,2]. In those materials the interacting waves possess negative refractivity at different frequency intervals. Therefore, the energy fluxes of the waves with a positive sign of refractive index will propagate in opposite direction to those with frequencies corresponding to a negative sign of refractive index. The dynamics of three wave interaction in NIM was considered for the case of second harmonic generation in [3]. Results obtained in [4,5] are being used for the developments the metamaterials in the near IR and visible ranges of the spectrum. Earlier we have analyzed the efficiency of energy conversions between two direct waves with respect to the energy of the backward signal wave for the case of signal-wave amplification in metamaterials [6] in the constant intensity approximation (CIA) [7,8], taking into account the reverse reaction of excited wave on the exciting one. By employing the CIA, we have studied the parametric interaction of optical waves in metamaterials under low-frequency pumping in the case of a negative index at a signal wave frequency [9]. The analytic expressions obtained in CIA showed, that the choice of the optimum parameters for the pump intensity, total length of the metamaterial and phase mismatch will facilitate obtaining the regimes of an effective amplification as well as the generation of signal wave.

The characteristic processes observed at parametric interactions of running and counter waves in metamaterials are the transition processes [10]. Authors [11,12,13] were analyzed the transition processes by employment the first order dispersion theory in the medium with quadratic nonlinearity. In case of counter waves the phase, matching condition is executed due to opposite directionality of the Poynting vector to the wave vector. To pump the nonlinear crystal of parametrical amplifier the nanosecond pulses of laser radiation are required. Parametric amplification of light in nonlinear crystals can be used for amplification the radiation being used with the aim of optical stochastic cooling of the relativistic heavy ions [14] Earlier we have employed constant intensity approximation to study the stationary optical parametric amplification [15] in the Fabri-Perrot cavity filled with dissipative dispersive nonlinear medium. Here optimization of various parameters such as the length of the nonlinear medium, wave mismatch, intensities of the pump and idler waves were considered to maximize the signal wave gain.

Upon reduction of the pulse duration the character of interaction of modulated wave significantly depends on the dispersion properties of a medium. The frequency conversion for the ultra-short pulses with running wave was analyzed in [16]. Note that the growing interest to the non-stationary interaction of ultra-short pulses of light in nonlinear medium is related to the development of powerful sources of light pulses of femtosecond duration. Earlier in [17] we were studied influence of group velocity mismatch as well as group velocity dispersion to the generation of sum frequency of ultra- short pulses in an external cavity under the phase matching and absence of linear losses. It was shown that in some cases efficiency of conversion in the existence of GVM and GVD can be significantly higher as compared as to the absence of mismatch and dispersion. Using the Gaussian pulse with quadratic phase modulation as the input pulse led to compression of spectrum with increase in GVM and decrease in GVD. Maximum energy of conversion was obtained not at group phase matching, but at the definite characteristic lengths of group velocity mismatch and group velocity dispersion.

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2. DISCUSSIONS AND RESULTS

The second order dispersion theory is employed to study non-stationary parametric amplification in metamaterials. We consider the nonlinear crystal of length l, and assume that its cross section is much larger than the input laser beam. The beam axis (which we term) is normal to the crystal surface, and this is the direction of the input wave vector. The input surface of the crystal is at $z = l$. We assume for definiteness that for a parametric three-wave interaction in a metamaterial the medium is "left" at the frequency of the signal wave only. Here the pump wave is a long pump pulse with frequency ω_3 an idler wave is at frequency ω_2 and a signal wave is at the difference frequency $\omega_1 = \omega_3 - \omega_2$. The geometry of the problem is so that the pump and idler waves enter the nonlinear medium from the left $(z = 0)$, but the signal wave from the right $(z = l)$ hand side. In such a consideration the wave vectors of all interacting waves in a metamaterial propagate in the positive direction of the z axis. During the wave propagation in a nonlinear medium as a result of the nonlinear interaction the energy exchange occurs between the counter wave packets of two types: direct waves (the idler and pump waves) and a backward wave (the signal wave); this leads to the energy transfer from the pump and idler waves into the signal-wave energy. For the negative values of the dielectric permittivity and magnetic permeability at the signal wave frequency ω_1 and the positive values at the frequencies ω_2, ω_3 the parametrical interaction is described by the system of parametrically coupled equations [1].

$$\left(\frac{\partial}{\partial z} + \frac{1}{u_1}\frac{\partial}{\partial t} - i\frac{g_1}{2}\frac{\partial^2}{\partial t^2} + \delta_1\right)A_1 = -i\gamma_1 A_3 A_2^* e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{u_2}\frac{\partial}{\partial t} - i\frac{g_2}{2}\frac{\partial^2}{\partial t^2} + \delta_2\right)A_2 = -i\gamma_2 A_3 A_1^* e^{i\Delta z} \tag{1}$$

$$\left(\frac{\partial}{\partial z} + \frac{1}{u_3}\frac{\partial}{\partial t} - i\frac{g_3}{2}\frac{\partial^2}{\partial t^2} + \delta_1\right)A_3 = -i\gamma_3 A_1 A_2 e^{-i\Delta z}$$

here A_j (j=1-3) are the corresponding complex amplitudes of the signal, idler and pump waves respectively , δ_j are the absorption coefficients of the medium at frequencies ω_j (j=1-3), u_j are the group velocities of the interacting waves, $\Delta = k_1 - k_2 - k_3$ is the phase mismatch between the interacting waves , $g_j = \partial^2 k_j / \partial\omega_j^2$ (the 3-rd term in the Taylor expansion around the central frequency ω_0: $\Delta\omega = \omega - \omega_0$, $k_n(\omega) \cong k_n(\omega_0) + k_n'\Delta\omega + \frac{1}{2}k_n''\Delta\omega^2 + \cdots)$ is the dispersion of group velocities and γ_1 , γ_2 , γ_3, are the coefficients of nonlinear coupling.

Assuming pump wave amplitude to be constant ($A_3 = A_{30} = const.$) and having put substitution $\eta = t - \frac{z}{u_1}$ the set of above equations (1) is reduced to

$$\left(\frac{\partial}{\partial z} - i\frac{g_1}{2}\frac{\partial^2}{\partial \eta^2} + \delta_1\right)A_1(z,\eta) = -i\gamma_1 A_{30} A_2^*(z,\eta) e^{i\Delta z}$$

$$\left(\frac{\partial}{\partial z} + \nu\frac{\partial}{\partial \eta} - i\frac{g_2}{2}\frac{\partial^2}{\partial \eta^2} + \delta_2\right)A_2(z,\eta) = -i\gamma_2 A_{30} A_1^*(z,\eta) e^{i\Delta z} \tag{2}$$

where $\nu = 1/u_2 - 1/u_1$ is a group velocity mismatch?
To analyze the system (2) it is convenient to use the inverse Fourier transformation

$$A_{1,2}(z,\eta) = \int_{-\infty}^{+\infty} A_{1,2}(z,\omega) e^{-i\omega\eta} d\omega \tag{3}$$

Substituting (3) into (2) yields

$$\left(\frac{\partial}{\partial z} + i\frac{g_1}{2}\omega^2 + \delta_1\right)A_1(z,\omega) = -i\gamma_1 A_{30} A_2^*(z,\omega) e^{i(\omega\eta + \Delta z)}$$

$$\left(\frac{\partial}{\partial z} + i\frac{g_2}{2}\omega^2 - i\nu\omega + \delta_2\right)A_2(z,\omega) = -i\gamma_2 A_{30} A_1^*(z,\omega) e^{i(\omega\eta + \Delta z)} \tag{4}$$

Solving this system in the absence of losses ($\delta_i = 0$) gives following expression for the amplitude of a signal wave

$$A_1(\omega, z) = \frac{i\gamma_1 A_{30} A_2}{\lambda - k\tan\lambda l}(\cos\lambda z \cdot \tan\lambda l - \sin\lambda z)e^{-kz} \tag{5}$$

Where $\lambda = {l_{nl}}^{-1} \quad [\left(\frac{1}{4}\frac{l_{nl}}{l_{d.}}(\alpha + 1)\omega^2\tau^2 - \frac{1}{2}\frac{l_{nL}}{l_\nu}\omega\tau + \frac{\Delta}{\Gamma_3}\right)^2 - 1]^{1/2}$,

$$k = l_{nl}{}^{-1}[i(\frac{1}{4}(\alpha-1)\frac{l_{nl}}{l_{d.}}\omega^2\tau^2 - \frac{l_{nl.}}{l_\nu}\omega\tau + \frac{\Delta}{\Gamma_3})], \quad \alpha = \frac{g_2}{g_1}, \ l_d = \frac{\tau^2}{g_1}, \ l_\nu = \frac{\tau}{\nu}$$

Furthermore, we assume that the input wave is a Gaussian with a quadratic phase modulation.

$$A_2(t) = A_{20}e^{-\frac{t^2}{2\tau^2} - i\gamma\frac{t^2}{2}}$$

By Fourier transformation for the frequency domain yields

$$A_2(\omega) = \frac{A_{20}}{2\pi}\int_{-\infty}^{+\infty} e^{-\frac{t^2}{2\tau^2} - i\gamma\frac{t^2}{2}} e^{-i\omega t}dt \tag{6}$$

From (6) for the spectral density we obtain

$$S_2(\omega) = \frac{A_{20}\tau^2}{2\pi}\frac{1}{\sqrt{1+p}}e^{-\frac{\mu^2}{1+p}} \tag{7}$$

where $p = \gamma^2\tau^4$ and $\mu = \omega\tau$ are the frequency modulation and phase modulation parameters respectively. Substituting (7) into (5) for spectral density of a signal wave $S_1(\omega, z) = A_1(\omega, z) \cdot A_1^*(\omega, z)$ results

$$S_1(\omega, z) = K\frac{e^{-\frac{\mu^2}{1+p}}(tan\lambda l \cdot cos\lambda z - sin\lambda z)^2}{(\lambda z)^2 + (kz)^2 tan^2\lambda l} \tag{8}$$

where $K = cn\gamma_1^2 I_{30}\, I_{20}\tau^2 z^2 / 16\pi$

As can be seen from (8) the shape of a spectrum of an amplified signal wave is determined not only by the values of z, l_{nl}, l_ν and l_d but also with their quotients z/l_{nl}, l_{nl}/l_ν, l_{nl}/l_d. Effect of phase modulation of idler wave onto the spectral density of amplified signal wave also is demonstrated in fig .1

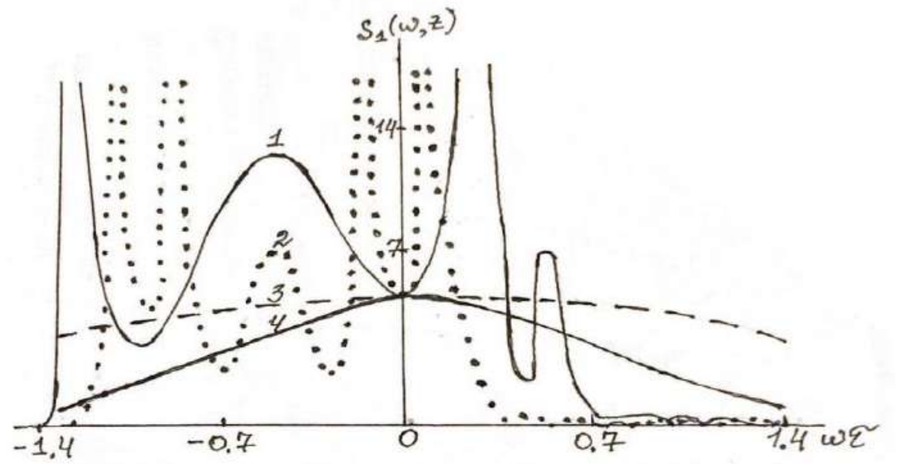

Fig. 1. Dependences of a spectral density $S_1(\omega, z)$ of a signal wave on the phase modulation parameter $\omega\tau$ for $p = 0$ (curve 1), $p = 5$ (curves $1 - 3$) and $z / l_{nl} = 0{,}5$, $\Delta = 0$, $\delta_i = 0$ at different values of ratios of characteristic lengths: $1 - l_{nl}/l_\nu = l_{nl}/l_d = 3$; $2 - \frac{l_{nl}}{l_\nu} = \frac{l_{nl}}{l_d} = 10$; $3 - \frac{l_{nl}}{l_\nu} = \frac{l_{nl}}{l_d} = 0$; $4 - l_{nl}/l_\nu = l_{nl}/l_d = 0$;

As can be seen a spectrum is symmetric when $l_{nl}/l_\nu = 0$ independently on the value of l_{nl}/l_d ratio. Existence of phase modulation leads to increase in the width of spectrum of a signal wave. At larger values of a frequency modulation ($\gamma\tau^2 \gg 1$) the splitting up occurs in the spectral density of amplified pulse (curve 2).

In fig. 2 a spectral density is given as a function of phase modulation at different values of intensity of idler wave. As can be seen at the same values of input intensity (curves 1 and 2) increase in frequency modulation leads to increase in spectral density, however at equal frequency modulations increase in intensity decreases the spectral density of an amplified signal wave (curves 1 and 3).

In fig.3 the dependences of a spectral density $S_1(\omega, z)$ on the phase modulation parameter $\omega\tau$ are illustrated at different values of l_{nl}/l_ν and l_{nl}/l_d. As can be seen, the shape of a spectrum varies with the change in these ratios, in particular, when $l_{nl}/l_\nu = 0$ (curve 2), the spectrum becomes symmetric relatively negative and positive values of phase modulation parameter.

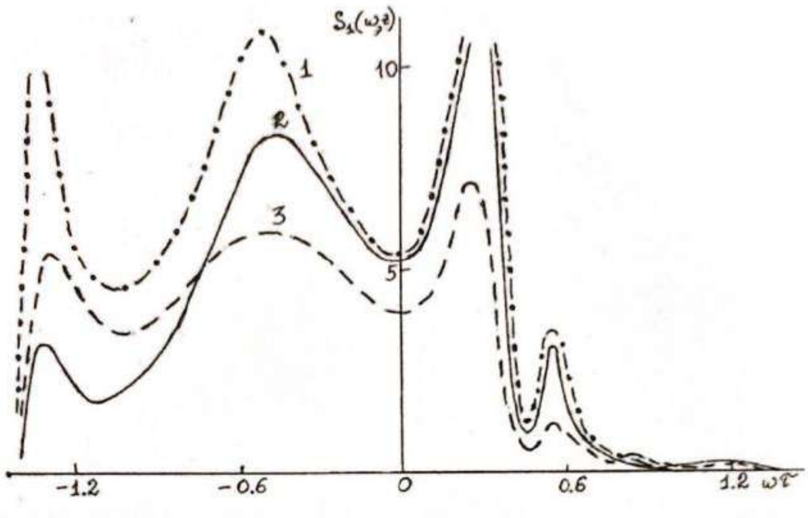

Fig. 2. The reduced spectral density $S_1(\omega, z)$ of a signal wave as a function of phase modulation parameter $\omega\tau$ for z / $l_{nl} = 0{,}7$ (curves 1 and 2) and z/$l_{nl} = 1$ (curve 3), $p = 0$(curve 2), $p = 5$ (curves 1 and 3) and $\Delta = 0$, $\delta_i = 0$

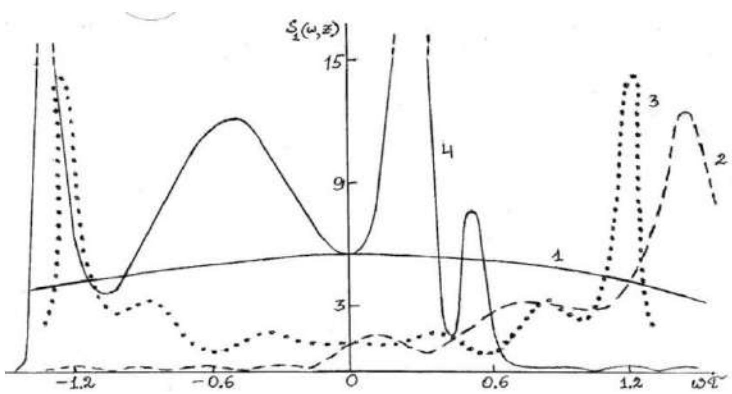

Fig. 3. The reduced spectral density $S_1(\omega, z)$ of a signal wave versus phase modulation parameter $\omega\tau$ for $p = 5$, z / $l_{nl} = 0{,}5$, $\Delta = 0$, $\delta_i = 0$; $1 - l_{nl}/l_\nu = l_{nl}/l_d = 3$; $2 - l_{nl}/l_\nu = 0$, $l_{nl}/l_d = 3$; $3 - l_{nl}/l_\nu = 3$, $l_{nl}/l_d = 0$;

All curves in fig. 1-3 are plotted for the same signs of the coefficients of dispersion of group velocities. Note that when $g_1 = g_2$, amplification of signal wave occurs without dispersion of group velocities. The graphs are plotted for the case when $g_2/g_1 = 3$.

3. CONCLUSIONS

An analytical expression for the spectral density a signal wave in the constant intensity approximation has been derived. In this approximation values of both spectral density and energy of a signal wave are affected by the ratios of characteristic lengths.

When $l_{nl}/l_\nu = 0$, the shape of a dependence for the spectral density becomes symmetric relatively ordinate axis and this has a maximum at positive values of phase modulation when $l_{nl}/l_d = 0$. For the ratios of characteristic lengths differ from zero maxima of spectral density are obtained not at zero $\omega\tau$ but at different values of this parameter. It was found that at the given ratio of l_{nl}/l_d maxima of reduced energy of signal wave displace toward greater values of phase mismatch parameter with increase in the l_{nl}/l_ν.

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