NONSTATIONARY SUM FREQUENCY GENERATION IN INHOMOGENEOUS OPTICAL FIBER

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An effect of inhomogeneuity of refractive index in optical fiber on the sum frequency generation in analyzed in the constant intensity approximation*(CIA*). It was revealed effect of regular inhomogeneity of medium on the character of nonlinear process at various values of pump intensity as well as medium losses. The influence of inhomogeneity of medium on the duration of sum frequency pulse for the Gaussian shape of pump wave pulse is investigated. It was shown possibility of manipulation of duration of sum frequency output pulse in a medium with regular inhomogeneity. Comparisons of the obtained results with the results of constant field approximation(*CFA*), the accurate calculation as well as the case of homogenous nonlinear medium were carried out.

Keywords: regular inhomogeneuity of refractive index, optical fiber, pulse duration, constant intensity approximation, sum frequency generation.
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1. INTRODUCTION

A study of inhomogeneity of media in nonlinear optics remains urgent since this is the subject of technological problem that reduces efficiency of frequency converters. In reality, nonlinear crystals being the basic element in frequency converters are subjected to high requirements for their homogeneity. When growing and processing nonlinear materials, optical inhomogeneity arises due to inhomogeneity of the composition, the presence of impurities, defects etc. It leads to a non-constant value of refractive index n along the total length of the material. In the sample of optical fiber fabrication process occurs with a refractive index gradient where the core size and refractive index profiles must be kept strictly constant. It should be noticed that, dispersion in a fiber light guide plays the important role in the course of propagation of the short laser pulses. The static inhomogeneity when either refractive index n or the direction of the optical axis changes randomly and the regular inhomogeneity when a change in refractive index can be described analytically are distinguished [1-2]. Above mentioned types of inhomogeneity can be inherent in the nonlinear medium itself or arise as a result of parametric interaction due to effect of laser radiation on the medium parameter. The value of inhomogeneity in n directly affects the efficiency of the converters as well as the parametric amplification threshold. Scientific researches of the world's leading laboratories over past decades confirm the promise of using optical fiber for information transmission. Specifically, optical solitons are used for this purpose; laser pulses that propagate practically without varying the shape of the pulse due to compensation caused by two reverse effects: dispersion broadening and the effect of narrowing the spectrum of laser emission upon propagating along nonlinear medium. Besides, using soliton's it is possible to control the parameters of ultrashort optical pulses [3-4]. Unlike the bulk medium in optical fiber the transvers dimension of laser radiation is preserved along the entire propagation length of optical fiber. For this reason, due to minimum losses and high concentration of a field, a significant increase in conversion efficiency is observed. Both facts also lead to large interaction lengths. Consequently, fiber-optic sensors, devoid of the disadvantages of traditional electrical sensors, are quite successfully used for monitoring oil wells [5-6]. Parametrical three wave interactions in optical waveguides, in particular, the squeezed states of light, wave generation and propagation of optical solitons have been studied in the approximation of undepleted pumping i.e. *CFA* or numerically [3-4]. Note, that to understand the qualitative pattern of the nonlinear interaction of waves the analytical approach for solving the set of reduced wave equations is preferable. A theoretical study of nonlinear processes has showed that the phase relationships between interacting waves play significant role [7-10], which is not taken into account in the *CFA*.

In this paper, non-stationary generation of the sum frequency in a spatially inhomogeneous optical fiber is studied by employment *CIA*. *CIA* [17,19], which has allowed to take into account the depletion of the pump wave and changes in the phases of all interacting waves. In this approximation we have already analyzed the nonlinear processes *SHG* and CARS in an optical fiber, the effects of self-action and cross-interaction [12-13]. The influence of a regular inhomogeneity of the medium on the nature of the nonlinear process is studied for various values of the pump intensity.

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A comparison is made with the case of homogeneous medium. It is considered how the inhomogeneity of the medium affects the duration of the pulse of excited wave.

2. THEORY AND DISCUSSIONS

We analyze non-stationary process of sum frequency generation in fiber with optical inhomogeneity. Theoretical analysis will be carried out in the first order dispersion theory, i.e. without taking into account the effect of dispersive spreading of pulses in the course of wave propagation. Then the coupled equations describing three frequency interaction of optical waves by taking into account the medium losses have view [3, 12]:

$$\begin{aligned}
&\frac{\partial A_1}{\partial z}+\frac{1}{u_1}\frac{\partial A_1}{\partial t}+\delta_1 A_1=-i\gamma(|A_1|^2+|A_2|^2+|A_3|^2)A_1-i\beta_1 A_2^* A_3\exp[i\Delta_0 z+i\psi(z)],\\
&\frac{\partial A_2}{\partial z}+\frac{1}{u_2}\frac{\partial A_2}{\partial t}+\delta_2 A_2=-i\gamma(|A_1|^2+|A_2|^2+|A_3|^2)A_2-i\beta_2 A_3 A_1^*\exp[i\Delta_0 z+i\psi(z)],\\
&\frac{\partial A_3}{\partial z}+\frac{1}{u_3}\frac{\partial A_3}{\partial t}+\delta_3 A_3=-i\gamma(|A_1|^2+|A_2|^2+|A_3|^2)A_3-i\beta_3 A_1 A_2\exp[-i\Delta_0 z-i\psi(z)].
\end{aligned}\tag{1}$$

where $A_{1,2,3}$ -are the complex amplitudes of the pump wave (ω_1) and waves at frequencies $\omega_{2,3}$ $\omega_3=\omega_1+\omega_2$), $u_{1,2,3}$ -are the group velocities of the corresponding waves, $\delta_{1,2,3}$ -are the linear losses of the interacting waves, γ is an average value of nonlinear coupling coefficient of the interacting waves where the contribution is imported by the self–phase and cross–phase modulation processes, $\beta_1=\gamma_{SH}^*/2,\ \beta_2=\gamma_{SH},\gamma_{SH}=3\omega_1\varepsilon_o^2\alpha_{SH}f_{112}\chi^{(3)}|E_p|^2|E_{SH}|,$ ε_0 is a dielectric permittivity of free space , α_{SH} is a constant, depending on the microscopic process, f_{112} is an integral of the overlapping, defining through mode distribution for optical fields and average on transverse coordinates x and y [14], E_p is a field of pump wave at frequency ω_p , E_{SH} is a field of a weak seed second harmonic at frequency $2\omega_p$. $\Delta_0=k_3-k_2-k_1-\Delta(z)$, where Δ_0 and $\Delta(z)$ denote the constant and variable parts of the wave parts of the phase detuning between the interacting waves, respectively, and $\psi(z)=\int_0^z\Delta(z')dz'$. For further simplification the above overlap integrals $f_{ijk}\cong f_{112}\cong 1/A_{eff}\quad (i,j,k=1,2,3)$ and the effective area of a fiber core) are taken identical, that is valid for single-mode waveguides [3]. The boundary conditions for the case of two input waves are written as follows

$$A_{1,2}(z=0)=A_{10,20}(t),\ A_3(z=0)=0 \tag{2}$$

We consider a quasistatic approximation when $u_1=u_2=u_3=u$. Then in (1), it is reasonable to substitute variables z and t with local ones z and $\eta=t-z/u$. Then equation (1) can be given by

$$\begin{aligned}
&\frac{\partial A_1}{\partial z}+\delta_1' A_1=-i\beta_1 A_2^* A_3\exp[i\Delta_0 z+i\psi(z)],\\
&\frac{\partial A_2}{\partial z}+\delta_2' A_2=-i\beta_2 A_3 A_1^*\exp[i\Delta_0 z+i\psi(z)],\\
&\frac{\partial A_3}{\partial z}+\delta_3' A_3=-i\beta_3 A_1 A_2\exp[-i\Delta_0 z-i\psi(z)],
\end{aligned}\tag{3}$$

where

$$\delta_j'=\delta_j+i\gamma(|A_1|^2+|A_2|^2+|A_3|^2),\quad j=1\div 3.$$

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Then complex amplitude $A_3(z)$ obeys following second-order differential equation obtained from (3)

$$\frac{d^2A_3}{dz^2}+[\delta_1'+\delta_2'+\delta_3'+i\Delta_0+i\Delta(z)]\frac{dA_3}{dz}+\left[\Gamma_1^2+\Gamma_2^2+(\delta_1'+\delta_2')\delta_3'+i\delta_3'\Delta_0+i\delta_3'\Delta(z)\right]A_3=0, \tag{4}$$

Here $\Gamma_1^2 = |\beta|_2 |\beta_3| I_1(z,\eta)$, $\Gamma_2^2 = |\beta|_1 |\beta_3| I_2(z,\eta)$, $I_{1,2} = A_{1,2} \cdot A_{1,2}^*$. Here $I_{1,2}(z,\eta)$ is a function of two variables. Wherein obtaining a rigorous analytical solution for $A_3(z)$ becomes difficult. However assuming that $I_{1,2}(z,\eta)$ is a function of single variable η, i.e. $I_{1,2}(z,\eta) = I_{1,2}(\eta)$, equation (4) can be solved. This means employment of *CIA*, since no limitations are imposed to the change in the pump wave phase.

Having put substitution $A_3 = V \exp[-i\psi(z)]$ in the equation (4) we obtain a differential equation for $V(z)$:

$$\frac{d^2V}{dz^2} + [\delta_1' + \delta_2' + \delta_3' + i\Delta_0 - i\Delta(z)]\frac{dV}{dz} + \left[\Gamma_1^2 + \Gamma_2^2 + \delta_3'(\delta_1' + \delta_2') + \Delta_0\Delta(z) + i\delta_3'\Delta_0 - i(\delta_1' + \delta_2')\Delta(z) - i\frac{d\Delta(z)}{dz}\right]V = 0 \qquad (5)$$

Further consideration will be carried out for the regular linear inhomogeneous medium $\Delta(z) = \alpha \cdot z$, where α -is the constant quantity. As a result from (5) we get ($\Delta_0 = 0$)

$$\frac{d^2V}{dz^2} + [\delta_1' + \delta_2' + \delta_3' - i\Delta(z)]\frac{dV}{dz} + \left[\Gamma_1^2 + \Gamma_2^2 + \delta_3'(\delta_1' + \delta_2') - i(\delta_1' + \delta_2')\Delta(z) - i\frac{d\Delta(z)}{dz}\right]V = 0 \qquad (6)$$

To solve the obtained equation, we put substitution $V(z) = f(\xi) \cdot \exp[(-\delta_1' - \delta_2')z]$, where $\xi = \sqrt{\alpha}(z + i\frac{\delta_3' - \delta_1' - \delta_2'}{\alpha})$. For the function $f(\xi)$ we obtain the Weber's equation [15]:

$$\frac{d^2 f}{d\xi^2} - \xi\frac{df}{d\xi} - lf = 0$$

where $l = (1 + i\frac{\Gamma_1^2 + \Gamma_2^2}{\alpha})$, whose solution is given by

$$f(\xi) = c_1[1 + \frac{l}{2!}\xi^2 + \frac{l(l+2)}{4!}\xi^4] + c_2[\xi + \frac{l+1}{3!}\xi^3 + \frac{(l+1)(l+3)}{5!}\xi^5]$$

Hence, for the complex amplitude of the sum frequency wave we obtain $A_3(z) = V \cdot \exp(-i\psi(z)) = f(\xi) \cdot \exp[-(\delta_1' + \delta_2')z - i\psi(z)]$. After substituting the values of the variables by taking into account the boundary conditions for the relative intensity at sum frequency in the regular inhomogeneous medium we obtain ($\delta_3' = \delta_1' + \delta_2'$):

$$I_3(z)/I_{10}(\eta) = (|\beta_3|z)^2 \cdot I_{20}(\eta) \cdot \exp[(-2(\delta_1 + \delta_2)z)] \times \left\{\left[1 - \frac{1}{3}|\alpha|z^2 + \frac{1}{15}|\alpha|^2 z^4 - \frac{(\Gamma_1^2 + \Gamma_2^2)^2}{120}z^4\right]^2 + \frac{1}{25}\left[1 - \frac{1}{2}|\alpha|z^2\right]^2 \frac{(\Gamma_1^2 + \Gamma_2^2)^2}{4}z^4\right\} \qquad (7)$$

In the absence the inhomogeneity of nonlinear medium i.e. $\Delta(z) = 0$ ($\alpha = 0$), for the homogeneous medium we get

$$I_3(z) = |\beta_3|^2 I_{10}(\eta) I_{20}(\eta) z^2 \cdot \left\{1 - \frac{(\Gamma_1^2 + \Gamma_2^2)^2 z^4}{300}\left[2 - \frac{1}{48}(\Gamma_1^2 + \Gamma_2^2)^2 z^4\right]\right\} \cdot \exp[-2(\delta_1 + \delta_2)z)]$$

Hence in *CFA*, when $\beta_1 = 0$ ($\Gamma_1 = 0$) and δ_1=0, we get

$I_3^{CFA}(z) = |\beta_3|^2 I_{10}(\eta) \cdot I_{20}(\eta) z^2 (1 - \frac{\Gamma_2^4 z^4}{6}) \cdot \exp[-2\delta_2 z)]$, and for the stationary case

$$I_3^{CFA}(z) = \left|\beta_3\right|^2 I_{10} \cdot I_{20}\, z^2 (1 - \frac{\Gamma_2^4 z^4}{180}) \cdot \exp\left[-2\delta_2 z)\right] \text{ [16].}$$

Below we analyze expression (7), obtained in the CIA, for the cases of a weak and strong inhomogeneous medium with a pulsed character of the pump wave and the wave at frequency of ω_2 . Assume that the ultrashort pump wave with duration of τ_1 and a long pulse wave at frequency of ω_2 with duration τ_2 at the left entrance to the optical fiber have the form of a Gaussian beam $A_{10,20}(\eta) = A_{10,20} \exp(-\eta^2 / 2\tau_{1,2}^2)$, provided that $\tau_1 << \tau_2$ Wherein relationships for the $\Gamma_{1,2} z$ in the *CIA* have the view

$\Gamma_1 z = \sqrt{|\beta_2||\beta_3| I_{10}(\eta)} z = \sqrt{|\beta_2||\beta_3| I_{10}} \cdot z \exp(-\eta^2 / 2\tau_1^2) = \Gamma_1(0) z$, where $\Gamma_1(0) = \sqrt{|\beta_2||\beta_3| I_{10}}$,

$I_{10} = A_{10} \cdot A_{10}^*$ and $\Gamma_2 z = \sqrt{|\beta_1||\beta_3| I_{20}(\eta)} z = \sqrt{|\beta_1||\beta_3| I_{20}} \cdot z \exp(-\eta^2 / 2\tau_2^2) = \Gamma_2(0) z$, here $\Gamma_2(0) = \sqrt{|\beta_1||\beta_3| I_{20}}$.

In the case of a weak inhomogeneous medium $|\alpha| z^2 < 1$ and small interaction lengths $\Gamma z < 1$, from (7) we get

$$I_3(z) \approx \left[|\beta_3| z\right]^2 I_{10}(\eta) I_{20}(\eta) \cdot \left[1 - \frac{2}{3}|\alpha| z^2 - \frac{(\Gamma_1^2 + \Gamma_2^2)^2}{60} z^4\right] \cdot \exp[-2(\delta_2 + \delta_1) z], \qquad (8)$$

Hence from (8) in the *CFA* we obtain

$$I_3(z) \approx \left[|\beta_3| z\right]^2 I_{10}(\eta) I_{20}(\eta) \left(1 - \frac{2}{3}|\alpha| z^2 - \frac{\Gamma_1^4}{60} z^4\right) \cdot \exp(-2\delta_2 z) \approx \left[|\beta_3| z\right]^2 I_{10}(\eta) I_{20}(\eta)$$

Taking into account the pulsed character of pump wave, we rewrite (8) in the form

$I_3(z) / I_{10} \approx I_2 \exp\left\{(-\eta^2 / \tau_1^2) \cdot \left[1 - \frac{1}{30}\left(\Gamma_1^2(0) + \Gamma_2^2(0)\right)\Gamma_1^2(0) z^4\right]\right\}$. It is seen that an increase in the input pulse duration is determined by the following relation

$\tau_{3in\hom} = \tau_1 / \sqrt{1 - \frac{1}{30}\left(\Gamma_1^2(0) + \Gamma_2^2(0)\right)\Gamma_1^2(0) z^4}$. In the CFA, this effect of increasing the pulse duration is weak $\tau_{3in\hom} = \tau_1 / \sqrt{1 - \frac{1}{30}\Gamma_1^4(0) z^4}$ since Γ_2 .

In a strong inhomogeneous medium $|\alpha| z^2 > 1$ and $\Gamma z < 1$, from (7) we obtain

$$\begin{gathered} I_3(z) \approx \left[|\beta_3| z\right]^2 \cdot I_{10}(\eta) I_{20}(\eta) \exp\left[\left(-2(\delta_1 + \delta_2) z\right)\right] \times \\ \left(1 - \frac{2}{3}|\alpha| z^2 + \frac{11}{45}|\alpha|^2 z^4 - \frac{2}{45}|\alpha|^3 z^6 - \frac{1}{45 \cdot 4}|\alpha| z^2 \left(\Gamma_1^2 + \Gamma_2^2\right) z^4\right) \cdot \end{gathered} \qquad (9)$$

Hence in the *CFA* we obtain

$$I_3(z) / I_{10} \approx \left[|\beta_2| I_{20}(\eta) z\right]^2 \cdot \left(1 - \frac{2}{3}|\alpha| z^2 + \frac{11}{45}\alpha^2 z^4 - \frac{2}{45}|\alpha|^3 z^6 - \frac{1}{45 \cdot 4}|\alpha| z^2 \Gamma_1^4 z^4\right).$$

In a homogeneous environment $I_2(z) / I_{10} \approx \left[|\beta_2| I_{10}(\eta) z\right]^2 \cdot \exp\left[\left(-2(\delta_1 + \delta_2) z\right)\right]$

For the homogeneous medium $I_2(z)/I_{10} \approx \left[\beta_2 \left|I_{10}(\eta)z\right]^2 \cdot \exp[(-2(\delta_1+\delta_2)z)]\right.$.

Analysis of (9) for the duration of the sum frequency pulse with $|\alpha|z^2 < 1.5$ yields

$$I_3(z)/I_{10} \approx I_{20}\exp\left[(-\eta^2/\tau_1^2)\cdot\left(1-\frac{1}{45}|\alpha|z^2\left(\Gamma_1^2(0)+\Gamma_2^2(0)\right)\Gamma_1^2(0)z^4\right)\right]$$

Hence duration of output pulse of radiation at sum frequency ω_3 in a strong inhomogeneous medium is determined by $\tau_{3in\,\hom} = \tau_1 / \sqrt{1-\frac{1}{45}|\alpha|z^2\left(\Gamma_1^2(0)+\Gamma_2^2(0)\right)\Gamma_1^2(0)z^4}$, and in the homogeneous medium $\tau_{3\hom} = \tau_1$. This fact of increasing the duration of pulse becomes significantly weak in *CFA* $\tau_{3in\,\hom} = \tau_1 / \sqrt{1-\frac{1}{45}|\alpha|z^2\Gamma_1^4(0)z^4}$ As can be seen from (7) intensity of the wave of sum frequency depends on the relationship between parameters Γ_1^2 and Γ_2^2 whose ratio equals $\Gamma_1^2/\Gamma_2^2 = \beta_2 I_{10}/\beta_1 I_{20} = n_1\omega_2 I_{10}/n_2\omega_1 I_{20}$. For the intensity of pump wave $I_{10} >> I_{20}$ and frequency conversion upward $\omega_1 >> \omega_2$ the ratio $\Gamma_1^2/\Gamma_2^2 \sim 1$, however when $\omega_1 << \omega_2$ the ratio $\Gamma_1^2/\Gamma_2^2 >> 1$ and one can neglect Γ_2^2 in this case. Consider second case to evaluate the pulse duration at sum frequency for various inhomogeneous media. Let $\Gamma_1(0)z = 0.9$ and we neglect parameter Γ_2.

As expected, with increase in the degree of inhomogeneity of the optical system, the quality (*Q*-factor) of the system, by analogy with the *Q*-factor of an optical resonator, decreases, that leads to broadening of the sum frequency pump excited in the given system. Really in the case of strong inhomogeneity the term of regular inhomogeneity included into exponential dependence (9) affects onto duration of pulse increasing it. If in a weak inhomogeneous medium $|\alpha|z^2 = 0.3, \tau_{2in\,\hom} = 1.0041\cdot\tau_1$, but in a strong inhomogeneous medium if $|\alpha|z^2 = 1.5$ we obtain $\tau_{2in\,\hom} = 1.01378\cdot\tau_1$.

Let us carry out numerical analysis of equation (7) obtained in *CIA*. Fig.1 shows, how the pulse duration varies depending on problem parameters according to analysis of equations (8) and (9). Really, in the case of strong inhomogeneity the term of regular inhomogeneity included into exponential dependence (9) affects onto duration of pulse increasing it (compare solid and marked curves). Variation of intensities of

frequency ω_3 in a strong inhomogeneous medium is determined by

$\tau_{3in\,\hom} = \tau_1 / \sqrt{1-\frac{1}{45}|\alpha|z^2\left(\Gamma_1^2(0)+\Gamma_2^2(0)\right)\Gamma_1^2(0)z^4}$

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As a result of the nonlinear interaction of optical waves upon generation of the wave at sum frequency occurs broadening of the wave at sum frequency in the regular inhomogeneous medium as compared with the case of homogeneous medium. It has allowed to manipulate the duration of the output harmonic pulse of sum frequency wave in a regular inhomogeneous medium by changing intensities of both pump wave and wave with frequency ω_2 at the input.

Thus, the possibility of controlling the parameter of the output pulse by a nonlinear optical method is shown. In a regular inhomogeneous medium, one can manipulate the parameter of the output radiation by adjusting the input pump intensity.

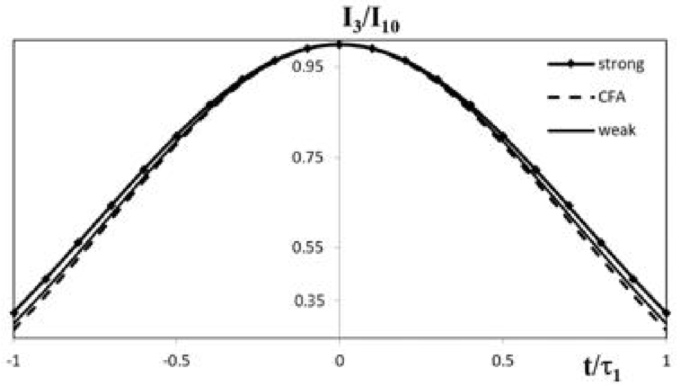

Fig.1. Dependences of relative intensity of sum frequency wave I_3 / I_{10} versus time t / τ_1 at $\delta_3 = 0$, $\Gamma_1 z$ =0.9, $\Gamma_2 = 0.8\Gamma_1$. The dashed curve is the result of calculation for uniform medium ($\alpha = 0$), so and in *CFA* ($\Gamma_2 z$ =0). Solid curve corresponds to the weak inhomogeneous medium, solid line with dots corresponds to the strong inhomogeneous medium at $|\alpha| z^2$ =1.5 calculated in *CIA*.

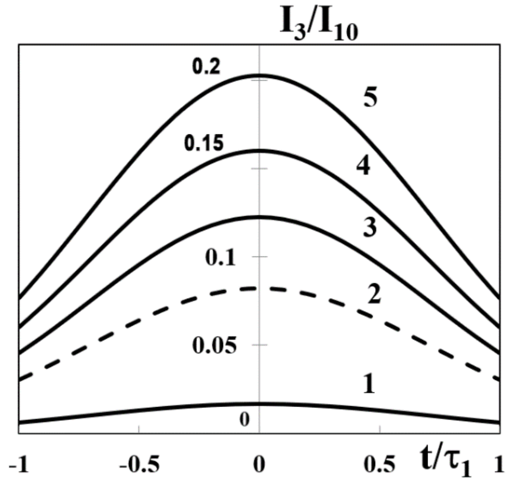

Fig. 2. Dependences of relative intensity of sum frequency wave I_3 / I_{10} versus time t / τ_1 at $\Gamma_2 = 0.5\Gamma_1$, $|\alpha| z^2$ =1.5 for $\Gamma_1 z$ =0.9 (curve 4), 0.8 (curve 3), 0.7 (curves 1 and 2), $\delta_3 z$ =0 (curves 2-5) and 0.5 (curve (1) Calculations were carried out in *CIA* (curves 1, 3-5) and in *CFA* (dashed curve 2).

3. CONCLUSIONS

Thus, in the constant intensity approximation, we have carried out investigation of sum frequency generation in an optical fiber with a linear regular inhomogeneity of the refractive index. An analytical expression is obtained for the intensity of sum frequency wave by taking into account the effects of self-action and cross-actions of interacting three waves. The analytical and numerical results are compared with the results in the *CFA*. A broadening of a pulse wave at sum frequency in a regular inhomogeneous medium as compared to the case of a homogeneous medium.

The possibility of manipulating the duration of the output pulse of generated wave in a regular inhomogeneous medium due to variations of input values of intensities both pump wave and wave with frequency ω_2 is shown.

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