SCATTERING OF LOW-ENERGY NEUTRINOS AT ACCELERATED ELECTRON BEAM PASSING THROUGH SINGLE CRYSTALS

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We investigate the scattering of low-energy neutrinos at the accelerated electron beam passing through the single crystals possessing strong internal electrostatic field in the framework of the Weinberg-Salam electroweak interaction theory. We present the results of our calculations on the average value of the third component of the momentum of the scattered accelerated electrons. These results can be applied in determination of the magnitude of the electrostatic field existing inside the single crystals and in detection of low-energy neutrinos.

Keywords: low-energy neutrinos, single crystals, transversely polarized electrons, effective magnetic field in single crystals
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1. INTRODUCTION

In detection process we measure the observable physical quantities. One of these quantities is the third component of the momentum of the scattered electron when low-energy neutrinos ($\omega \ll m_e$ where ω is the energy of the incident neutrino, m_e is the electron mass) scatter at the accelerated electron beam passing through the single crystals possessing strong internal electrostatic field. The electrons passing through such crystals experience the action of an extended effective magnetic field according to the formula

$$\vec{H}_{ef} = \frac{\left[\vec{E}\vec{\beta}\right]}{\sqrt{1-\beta^2}} \tag{1}$$

where $\vec{\beta} = \beta\vec{n}$, $\beta = v/c$ [1-4]. In this case the third component of the electron situated in an effective magnetic field of the single crystal is conserved. It enables us to measure the third components of the scattered electrons, to register low-energy neutrinos and to obtain invaluable information on the internal electrostatic field of the considered single crystals like a tungsten (W) or a diamond (C).

The main purpose of the presented work is to determine the average third component of the scattered electron when low-energy neutrinos scatter at the accelerated electron beam passing through the single crystals like a tungsten (W) or a diamond (C) that possess strong internal electrostatic field.

We present the results of our calculations of the third component of the scattered electron for the scattering of low-energy neutrinos at transversely polarized accelerated electrons

$$\nu_i + e^- \to \nu_i' + e^{-\prime}, \tag{2}$$

passing through the single crystals possessing strong internal electrostatic field in the framework of the Weinberg-Salam electroweak interaction theory where $\nu_i = \nu_e,\ \nu_\mu,\ \nu_\tau$ are the three flavours of neutrinos in the initial state, $\nu_i' = \nu_e',\ \nu_\mu',\ \nu_\tau'$ are the three flavours of neutrinos in the final state. As we indicated above the electrons passing through such crystals experience the action of an extended effective magnetic field. So, we investigate the process (2) in a constant homogenous magnetic field. We disregard non-homogeneity of the internal electrostatic field (effective magnetic field).

2. METHODS

First of all, we calculate the differential probability of the processes (2) using the Feynman diagram technique [5] and the exact wave function method [6]. The related Feynman diagrams of the processes $\nu_i e^- \to \nu_i' e^{-\prime}$ are given by the Fig.1.

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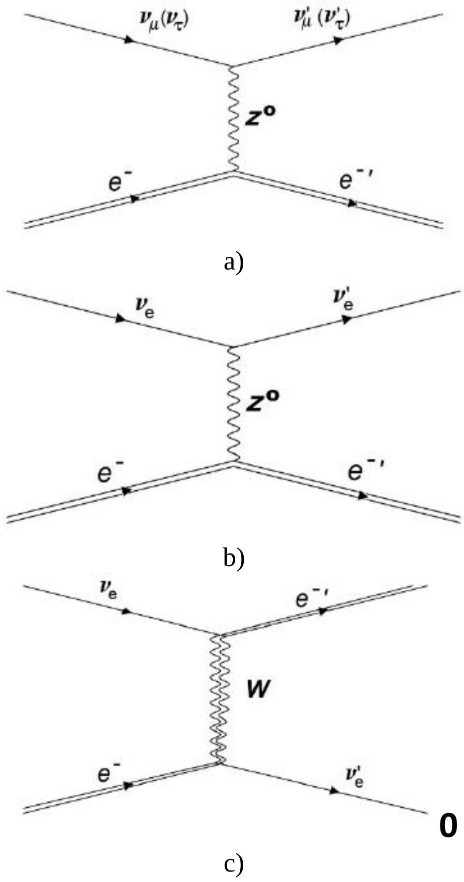

Fig 1. The Feynman diagrams describing the processes $\nu_i e^- \to \nu_i' e^{-\prime}$

Fig.1, a corresponds to the processes proceeding at the expense of a purely neutral weak current ($\nu_\mu e^- \to \nu_\mu{}' e^{-\prime}$ or $\nu_\tau e^- \to \nu_\tau{}' e^{-\prime}$ processes). Fig. 1, b and fig. 1, c correspond to the process proceeding at the expense of both a neutral weak current and charged weak current ($\nu_e e^- \to \nu_e{}' e^{-\prime}$ process), respectively.

3. DIFFERENTIAL PROBABILITY OF THE PROCESS AND THE AVERAGE THIRD COMPONENT OF THE SCATTERED ELECTRON

At first, we calculate the differential probability of the elastic scattering process describing by the reaction $\nu_\mu e^- \to \nu_\mu{}' e^{-\prime}$ (or $\nu_\tau e^- \to \nu_\tau{}' e^{-\prime}$) when a Z-boson propagator only contributes.

We assume that electrons in the initial and final states are ultra-relativistic

$$\varepsilon^2 \gg m_e^2, \qquad \varepsilon'^2 \gg m_e^2 \tag{3}$$

where $\varepsilon = \gamma m_e$, $\gamma = \sqrt{1 + 2fn + (p_z/m_e)^2}$, p_z and n are the energy, relativistic factor, z-component of the momentum and the number of the Landau energy level belonging to the electron in the initial state, respectively. The primed quantities $\varepsilon' = \gamma' m_e$, $\gamma' = \sqrt{1 + 2fn' + (p_z'/m_e)^2}$, p_z' and n' belong to the electron in the final state. f is the dimensionless field parameter characterizing the external magnetic field $f = B/B_0$ where B is the magnitude of the magnetic field vector $\vec{B}$ that is directed along the z -axis and $B_0 = m_e^2/e \cong 4.414 \times 10^{13} G$ is the Schwinger field strength in the system of units $c = \hbar = 1$. B is assumed to be $B \ll B_0$ (or $f \ll 1$). We also assume that electrons in the initial and final states possess large transverse momenta

$$p_\perp = (2eBn)^{1/2} = m_e(2fn)^{1/2} \gg m_e, \tag{4}$$

$$p_\perp' = (2eBn')^{1/2} = m_e(2fn')^{1/2} \gg m_e. \tag{5}$$

The assumptions $\varepsilon^2 \gg m_e^2$, $\varepsilon'^2 \gg m_e^2$, $p_\perp \gg m_e$, $p_\perp' \gg m_e$ and $f \ll 1$ mean that the main contribution to the differential probability of the process comes from the electron states occupying high Landau levels ($n,\ n' \gg 1$). In this case motion of the electrons in the initial and final states are semiclassical.

We consider the case when the longitudinal momentum of the electrons in the initial state is zero: $p_z = 0$.

Let the incident low-energy massless neutrino fly along the z-axis (along the magnetic field direction) and its energy is in the range $\omega_{min} \ll \omega \ll m_e$, (or $f/\sqrt{\gamma^2 - 1} \ll \omega/m_e \ll 1$) where $\omega_{min} = eB/p_\perp$.

The above indicated conditions and restrictions mean that the differential probability of the process will depend on two parameters: the field parameter

$$\chi = \frac{e}{m_e^3}\left[-\left(F_{\mu\nu}p^\nu\right)^2\right]^{1/2} = \frac{B}{B_0}\frac{p_\perp}{m_e} = [f^2(\gamma^2 - 1)]^{1/2} \tag{6}$$

and the kinematical parameter

$$\kappa = \frac{2\omega E}{m_e^2} = \frac{2kp}{m_e^2} \tag{7}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the tensor of the external field.

We obtain the following general formula for the differential probability of the process $\nu_\mu e^- \to \nu_\mu{}' e^{-\prime}$ (or $\nu_\tau e^- \to \nu_\tau{}' e^{-\prime}$):

$$dw = \frac{G_F^2 m_e^2}{4\pi^{3/2}}\frac{1}{V}\left[A\Phi_1(z) - B\left(\frac{\chi}{u}\right)^{2/3}\Phi'(z) - C\left(\frac{\chi}{u}\right)^{1/3}\Phi(z)\right]N(u,\kappa)\frac{u\,du}{(1+u)^4} \tag{8}$$

where

$$A = \frac{\kappa}{2u}\left[g_L^2(1+u)^2 + g_R^2 + 2g_L g_R \zeta\zeta'(1+u)\right] - g_L g_R(1 + \zeta\zeta')(1+u), \tag{9}$$

$$B = g_L^2(1+u)^2 + g_R^2 + 2g_L g_R \zeta\zeta'(1+u), \quad (10)$$

$$C = g_L^2\zeta'(1+u)^2 - g_R^2\zeta + g_L g_R(\zeta - \zeta')(1+u), \quad (11)$$

$$N(u,\kappa) = \left[\kappa\left(\frac{m_e}{m_Z}\right)^2 \frac{u}{1+u} + 1\right]^{-2}, \quad (12)$$

u is the invariant spectral variable

$$u = \frac{\chi}{\chi'} - 1 = \frac{p_\perp}{p'_\perp} - 1 \simeq \frac{\omega'}{E-\omega'}, \quad (13)$$

the field parameter χ' belongs to the electrons in the final state,

$$\Phi(z) = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{+\infty} dt exp\left[i\left(zt + \frac{t^3}{3}\right)\right] \quad (14)$$

is the Airy function depending on the argument

$$z = \left(\frac{u}{\chi}\right)^{2/3}\left(1 - \frac{\kappa}{u}\right), \quad (15)$$

$\Phi'(z) = d\Phi(z)/dz$, $\Phi_1(z) = \int_z^\infty \Phi(y)dy$,

$g_L = -0.5 + \sin^2\theta_W$, $g_R = \sin^2\theta_W$.

The analyses of the general formula obtained for the differential probability and the argument z show that the influence of the external magnetic field on the low-energy neutrino-electron scattering is determined by the parameter

$$\eta = \frac{\chi}{\kappa} = \frac{1}{2}\frac{B}{B_0}\frac{m_e}{\omega}. \quad (16)$$

When the parameter $\eta \gtrsim 1$, the field effects become essentially. To achieve a higher η the energy of the incident neutrino ω is to be as low as possible and the magnitude of the magnetic field vector is to be as high as possible but much less than B_0 . For this purpose, relic neutrinos are the most suitable neutrinos due to their extremely low energy. To achieve $\eta \gtrsim 1$ for relic neutrinos B is to satisfy the condition

$$2.897 \times 10^4\ G \lesssim B \ll 4.414 \times 10^{13} G. \quad (17)$$

Not exceeding the unitarity limit that is $\sqrt{s} \approx 600\ GeV$ [7] for the neutrino-electron scattering we obtain the following values for the kinematical parameter κ and for the multiplier $\kappa(m_e/m_Z)^2$ participating in the formula for the differential probability, respectively:

$$\kappa = \frac{2\omega\varepsilon}{m_e^2} \lesssim 10^{-4}, \kappa\left(\frac{m_e}{m_Z}\right)^2 \lesssim 10^{-14}. \quad (18)$$

In this case is the multiplier $N(u,\kappa)$ is replaced with $N(u,\kappa) \cong 1$ and the corresponding formula will describe not only the $\nu_\mu e^- \to \nu_\mu{}' e^{-\prime}$ and $\nu_\tau e^- \to \nu_\tau{}' e^{-\prime}$ processes but also the $\nu_e e^- \to \nu_e{}' e^{-\prime}$ process. However, it should be taken into account that $g_L = 0.5 + \sin^2\theta_W$ and $g_R = \sin^2\theta_W$ for the $\nu_e e^- \to \nu_e{}' e^{-\prime}$ process.

4. RESULTS

Using the general formula (8) for the differential probability and the formula for the average third component of the scattered electron

$$\langle p'_z\rangle = \frac{\int_0^\infty p'_z dw}{\int_0^\infty dw} \quad (19)$$

we obtain the asymptotic formula for the average third component of the scattered electron in the limiting case $\chi \gg 1 > \kappa$ $(\eta \gg 1)$:

$$< p'_z > = \frac{m_e}{4\Gamma\left(\frac{2}{3}\right)} \frac{c_1(3\chi)^{1/3} - \frac{2}{3}\Gamma\left(\frac{2}{3}\right)c_2}{c_3} \quad (20)$$

where

$$c_1 = g_L^2 + \frac{1}{6}g_R^2 + \frac{2}{3}g_L g_R\zeta\zeta', \quad (21)$$

$$c_2 = 2g_L^2\zeta' - \frac{5}{27}g_R^2\zeta + \frac{4}{9}g_L g_R(\zeta - \zeta'), \quad (22)$$

$$c_3 = g_L^2 + \frac{5}{27}g_R^2 + \frac{2}{3}g_L g_R\zeta\zeta'. \quad (23)$$

In particular case, when $\chi \gg 10^2$, we obtain from the formula (20) the following simple asymptotic formula for the average value of the third component of the scattered electron

$$< p'_z > = \frac{m_e}{4\Gamma\left(\frac{2}{3}\right)} \frac{g_L^2 + \frac{1}{6}g_R^2 + \frac{2}{3}g_L g_R\zeta\zeta'}{g_L^2 + \frac{5}{27}g_R^2 + \frac{2}{3}g_L g_R\zeta\zeta'}(3\chi)^{1/3} \quad (24)$$

which is known [8].

5. DISCUSSION

The analysis of the formulae (20) and (24) shows that the average value of the third component of the scattered electron is determined by the field parameter, the spin quantum number ζ (ζ') of the electron in the initial (final) state, the structural constants g_L and g_R of the electroweak interactions. The formula (20) shows that the average value of the third component of the scattered electron is the function depending on the field parameter χ. Since the field parameter χ contains B (the magnitude of the magnetic field vector $\vec{B}$), B is determined by H_{ef} and H_{ef} is determined by $E = |\vec{E}|$, we can come to the conclusion that the average value of the third component of the scattered electron contains the information on the magnitude of the electrostatic field existing inside the single crystals.

The z-component (the third component) of the momentum in an external field is a conserved physical quantity. At the same time $< p'_z >$ is an observable physical quantity. It means that $< p'_z >$ can be measured in the experiment.

The measurement of $< p_z' >$ has a great importance in detection of low-energy neutrinos.

6. CONCLUSIONS

Thus, we have investigated the scattering of low-energy neutrinos at the accelerated electron beam passing through the single crystals possessing strong internal electrostatic field in the framework of the Weinberg-Salam electroweak interaction theory. We have obtained the analytical formula for the average value of the third component of the momentum of the scattered accelerated electrons.

Since the average value of the third component of the scattered electron contains the information on the magnitude of the electrostatic field existing inside the single crystals, this result enables the experimentalists to determine the magnitude of the indicated electrostatic field.

Determination of the average value of the third component of the momentum of the scattered accelerated electrons has a great importance in detection of low-energy neutrinos.

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