ON THE THEORY OF INTRACAVITY DISPERSION INTERFEROMETER

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Theory of intracavity dispersion interferometerity is presented in the constant intensity approximation taking into accoun tthe reverse reaction of excited harmonic on the phase of fundamental wave. It was shown that unlike the results of constant field approximation output intensity of second harminc is a function of both intensity of fundamental and harmonic waves as well as the shift in phases of interacting waves. It wasshown that , determination of the dispersion of refractive index is possible due to observed shifts of the locations of exprema in the intensity-phase dependence.

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1. INTRODUCTION

Dispersion interferometry has been proposed as a method for measurement the electron density in tokamak fusion devices [1,2]. It is well known that a conventional dispersion interferometer possesses two nonlinear crystals located before and after the plasma medium whose electron densities are subjected to measurement [3]. Dispersion interferometers are employed for determination the electron concentration of spark plasma produced by laser in the air [4] and diagnosis the micro relief of surfaces of optical elements [5] as well as electron concentration of argon laser discharge [6]. Unlike the classical interferometers the operational principle of those dispersion interferometers is based on the interference of waves with different frequencies after passage identical geometrical paths. Laser beam path is along the plasma. However, since a beam travels a long path, to focus the laser beam on the second nonlinear crystal it meets several difficulties in the heterodyne dispersion interferometer. To avoid such a geometry a new version of heterodyne interferometer i.e. a single crystal dispersion interferometer is developed with the laser wavelength of 1064 nm [7]. The power of second harmonic radiation at the end of second pass depends on the differential phase shift imposed on the two beams during their round trip through plasma. Because the two beams along a common path, the effect of vibrations is cancelled out, leaving the dispersion as only the source of differential phase. A network of laser cavity is shown in Fig.1.

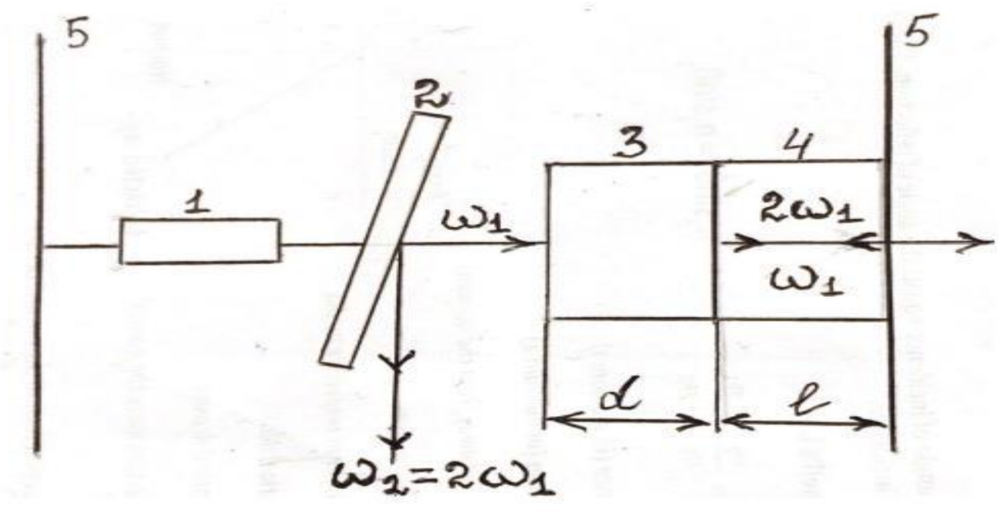

Fig.1. Operational scheeme of inner cavity dispersion interferometer : 1 – laser , 2- optical unit 3– crystal with quadratic nonlinearity , 4– the tested medium, 5- mirrors of cavity.

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2. THEORETICAL APPROACH

Earlier we have employed constant intensity approximation for investigation the dispersion inteferometer with two nonlinear crystals [8-10].

To study intracavity generation of second harmonic we consider the problem in two stages. The set of truncated equations describing nonlinear interaction of both fundamental and harmonic waves propagating in positive and negative directions of z-axis is presented [11]

$$\frac{dA_1^{\pm}}{dz} \pm \delta_1 A_1^{\pm} = \mp i\gamma_1 A_2^{\pm} (A_1^{*})^{\pm} e^{\pm i\Delta z}$$

$$\frac{dA_2^{\pm}}{dz} \pm \delta_2 A_2^{\pm} = \mp i\gamma_2 \left(A_1^{\pm}\right)^2 e^{\mp i\Delta z} \quad (1)$$

where $A_j^{\pm}$ (j=1, 2) – are complex amplitudes of interacting waves at frequencies ω_j (j=1,2) ("+"- indicate positive direction of Z-axis and "-" refers to negative direction of Z-axis), δ_j – are the absorption coefficients at frequencies ω_j (j=1, 2) , $\Delta = k_2 - 2k_1$ – is the difference in wave numbers for the pump and third harmonic waves, γ_1 and γ_2 nonlinear coupling coefficients given by

$$\gamma_1 = \frac{8\pi\chi_{eff}^2 \omega_1^2 |\varepsilon_1|}{k_1 c^2}, \qquad \gamma_2 = \frac{8\pi\chi_{eff}^2 \omega_2^2 \varepsilon_2}{k_2 c^2}$$

ε_1 and ε_2 dielectric permittivities of medium, χ_{eff}^2 – is the effective dielectric succeptibility of medium. Each of equation in the system (1) describes propagation of waves in both directions along Z-axis. If this circumstance is taken into consideration, we get four equations. However, we do not take into account interaction of counter propagating waves. Differentiating the second equation of (1) yields to following second order differential equation for complex amplitude of second harmonic wave:

$$\frac{d^2 A_2}{dz^2} + (\delta_2 + 2\delta_1 + i\Delta)\frac{dA_2}{dz} + \left[\delta_2(2\delta_1 + i\Delta) + 2\Gamma_1^2\right]A_2 = 0 \quad (2)$$

When waves propagate in the positive direction of Z-axis solution of the equation (2) according to boundary conditions $A_1(z=0) = A_{10}$, $A_2(z=0) = 0$ gives for complex amplitude of second harmonic.

$$A_2(l) = -i\gamma_2 A_{10}^2 \cdot l \cdot \mathrm{sinc}\lambda_1 l \cdot \exp[-(\delta_2 + 2\delta_1 + i\Delta)l/2], \quad (3)$$

where

$$\lambda_1^2 = 2\Gamma_1^2 - \frac{(\delta_2 - 2\delta_1 - i\Delta)^2}{4}, \quad \Gamma_1^2 = \gamma_1\gamma_2 I_{10},$$

$$I_{10} = A_{10} \cdot A_{10}^*, \quad \mathrm{sinc}x = \frac{\sin x}{x}.$$

For the waves, reflected from the surface of second mirror and propagating in opposite direction toward the right-hand side of nonlinear crystal boundary conditions are alter and presented by

$$A_1(z=0) = A_1(l)R_1(\omega_1)\exp[i\varphi_1(2d) + i\varphi_{r,1}],$$

$$A_2(z=0) = A_2(l)R_2(\omega_2)\exp[i\varphi_2(2d) + i\varphi_{r,2}], \quad (4)$$

where где $\varphi_1(2d)$, $\varphi_2(2d)$ - are phase shifts for the pump and harmonic waves in the air gap between nonlinear crystal and second mirror, $i\varphi_{r,1}$ and $i\varphi_{r,2}$ are the phase shifts due to reflections of waves from second mirror, $R_1(\omega_1)$ and $R_2(\omega_2)$ are the coefficients of reflection of pump wave and second harmonic wave respectively. Here we assume, that z=0 again corresponds to the input of second crystal from the right-hand side. Taking into account (3) in the (1) gives expression of the complex amplitude of pump wave:

$$A_1(l) = A_{10}\left(\cos\lambda_1 l + \frac{\delta_2 - 2\delta_1 - i\Delta}{2\lambda_1}\sin\lambda_1 l\right)^{\frac{1}{2}} \cdot \exp\left(-\frac{\delta_2 + 2\delta_1 - i\Delta}{4} l\right). \quad (5)$$

Solving set of equations (1) with boundary conditions (4) results in the complex amplitude of second harmonic wave travelling in the negative direction of the chosen axis:

$$A_{2,out.} = M \times \left[\cot\lambda_2 l + \left(\frac{\lambda_1}{\lambda_2}\cot\lambda_1 l + \frac{\delta_2 - 2\delta_1 - i\Delta}{2\lambda_1}\right)e^{\Psi} + \frac{2\delta_1 - \delta_2 - i\Delta}{2\lambda_2}\right] \quad (6)$$

where $M = \sin\lambda_2 l \times e^{-\frac{\delta_2 - 2\delta_1 - i\Delta}{2\lambda_1} l + i\varphi_2'(2d) + i\varphi_2'}$, $\lambda_2 = [2\Gamma_2^2 - \frac{(\delta_2 - 2\delta_1 - i\Delta)^2}{4}]^{1/2}$, $\Gamma_2^2 = \gamma_1\gamma_2 I_1(l)$,

here $\Psi = \Delta l + 2\varphi_1(2d) - \varphi_2(2d) + 2\varphi_1' - \varphi_2'$, Δl - is the phase shift when waves travel through the first nonlinear crystal. When waves travel from the right toward the left (in the negative direction of the z-axis) the difference in wave numbers equals $\Delta_2 = -\Delta_1 = \Delta$.

From equation (6) for the output intensity of second harmonic wave we obtain

$$I_{2,out} = I_{10}\frac{\gamma_2}{\gamma_1\rho_1}\Gamma_1^2(sin^2x_1 + sh^2y_1)\cdot e^{[-2(\delta_2+2\delta_1)l]} \times$$

$$\times [A^2 + B^2 + S^2 + D^2 + 2(AS + BD)cos\Psi + 2(AD - BS)sin\Psi] \quad (7)$$

where

$$A = Msinx_2chy_2 - Ncosx_2shy_2,$$

$$B = Hsinx_2chy_2 + Fcosx_2shy_2,$$

$$S = cosx_2shy_2 - \left(\frac{\Delta}{2\sqrt{\rho_2}}sin\frac{\xi_2}{2} - \frac{2\delta_1 - \delta_2}{2\sqrt{\rho_2}}cos\frac{\xi_2}{2}\right)sinx_2chy_2 +$$

$$+\left(\frac{\Delta}{2\sqrt{\rho_2}}cos\frac{\xi_2}{2} - \frac{2\delta_1 - \delta_2}{2\sqrt{\rho_2}}sin\frac{\xi_2}{2}\right)shy_2cosx_2$$

$$D = -sinx_2shy_2 - \left(\frac{\Delta}{2\sqrt{\rho_2}}cos\frac{\xi_2}{2} - \frac{2\delta_1 - \delta_2}{2\sqrt{\rho_2}}sin\frac{\xi_2}{2}\right)sinx_2chy_2 +$$

$$-\left(\frac{\Delta}{2\sqrt{\rho_2}}sin\frac{\xi_2}{2} - \frac{2\delta_1 - \delta_2}{2\sqrt{\rho_2}}cos\frac{\xi_2}{2}\right)shy_2cosx_2$$

$$F = \sqrt{\frac{\rho_1}{\rho_2}}(B_1cos\theta - B_2sin\theta) - \frac{\Delta}{2\sqrt{\rho_2}}sin\frac{\xi_2}{2} - \frac{2\delta_1 - \delta_2}{2\sqrt{\rho_2}}cos\frac{\xi_2}{2}$$

$$H = \sqrt{\frac{\rho_1}{\rho_2}}(B_1sin\theta - B_2cos\theta) - \frac{\Delta}{2\sqrt{\rho_2}}cos\frac{\xi_2}{2} + \frac{2\delta_1 - \delta_2}{2\sqrt{\rho_2}}sin\frac{\xi_2}{2}$$

$$B_1 = \frac{tgx_1/ch^2y_1}{tg^2x_1 + th^2y_1}, \quad B_2 = \frac{thy_1/cos^2x_1}{tg^2x_1 + th^2y_1}$$

$$x_{1,2} = \sqrt{\rho_{1,2}}lcos\frac{\xi_{1,2}}{2}, \quad y_{1,2} = \sqrt{\rho_{1,2}}lsin\frac{\xi_{1,2}}{2},$$

$$\rho_{1,2}^2 = \left[2\Gamma_{1,2}^2 + \frac{\Delta_{1,2}^2}{4} - \frac{(\delta_2 - 2\delta_1)^2}{4}\right]^2 + \frac{\Delta_{1,2}^2}{4}(\delta_2 - 2\delta_1)^2, \quad \Psi = 2n\pi,$$

$$\xi_{1,2} = arctg\frac{\Delta_{1,2}/2(\delta_2 - 2\delta_1)}{2\Gamma_{1,2}^2 + \Delta_{1,2}^2/4 - (\delta_2 - 2\delta_1)^2/4}, \quad \theta = \frac{\xi_1 - \xi_2}{2}$$

3. CONCLUSIONS

On the basis of studies above one can conclude , that output intensity of second harmonic wave is a function of generelized phase shift between interacting waves and oscillations take place due to trigonometric functions. Since both fundamental and harmonic waves are two times travelling through investigated medium a certain diplacement can be observed in the dependence of output intensity versus phase shift. The dispersion of refractive index can be determined due to shift in the positions of extrema points in the intensity-phase shift dependence.

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