QUANTIZATION OF ELECTRIC CHARGE OF LEPTONS IN WEINBERG-SALAM MODEL WITH RIGHT-HAND NEUTRINO COMPONENT

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Assuming that the neutrino also has a right-hand component, the possibility of obtaining quantization of the electric charge of leptons in the Weinberg-Salam (WS) model is investigated. In this case, the relations following from the conditions of anomaly cancellation are not used. It is shown that the presence of Higgs fields is a necessary condition for quantization of the electric charge of leptons in the WS model.

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Quantization of electric charge in the Standard Model (SM) has been investigated in a number of papers [1-4]. In these papers, quantization of electric charge was obtained using relations following from the conditions of anomaly cancellation.

The authors of [5] took different expressions for the mixing angle of neutral fields in the lepton, quark and Higgs parts of the interaction Lagrangian. From the requirement of equality of these angles, relations are obtained that lead to quantization of the electric charge of particles. We considered the quantization of the electric charge of leptons and quarks in the SM without using relations following from the conditions of anomaly cancellation in [6, 7]. Note that in [6] the neutrino had only a left component. Now we consider the WS model for one family of leptons and assume that the neutrino also has a right component. In this case, we have the following lepton fields

$$\psi_{eL} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \ \psi_{eR} = e_R^-, \ \psi_{\nu R} = \nu_{eR} \quad (1)$$

and the Higgs isodoublet

$$\varphi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix}. \quad (2)$$

The Lagrangian of the interaction of leptons and the Higgs field with gauge fields has the form

$$L = i\overline{\psi}_{eL}\gamma_\mu D_\mu \psi_{eL} + i\overline{\psi}_{eR}\gamma_\mu D_\mu \psi_{eR} + i\overline{\psi}_{\nu R}\gamma_\mu D_\mu \psi_{\nu R} + \left(D_\mu^\varphi \varphi\right)^+ \left(D_\mu^\varphi \varphi\right) \quad (3)$$

where

$$D_\mu = \partial_\mu - ig\left(\vec{t}\vec{b}_\mu\right) - i\frac{g_1}{2}\mathrm{Y}a_\mu, \quad (4)$$

$\psi_{eL}, \psi_{eR}, \psi_{\nu R}$ – are the left and right isomultiplets of the lepton fields; $\vec{b}_\mu$ – Yang - Mills isotriplet, a_μ – Maxwell isosinglet; $\vec{t}$ and Y – is the isospin and hypercharge operator for a given multiplet; g and g_1 – are the interaction constants . In this paper, we do not accept the Gell-Mann-Nishijima relation for weak hypercharges. We accept weak hypercharges as free parameters and fix them from physical requirements. We will consider the hypercharge Y of the lepton fields (1) and the Higgs field (2) to be real and denote them as follows

$$\mathrm{Y}(\psi_{eL}) = y_L, \quad \mathrm{Y}(\psi_{eR}) = y_{eR}, \quad \mathrm{Y}(\psi_{\nu R}) = y_{\nu R}, \quad \mathrm{Y}(\varphi) = y_\varphi . \quad (5)$$

As usual, for isoscalar fields $\vec{t} = 0$, and for isospinor fields ψ_{eL} and $\varphi - \vec{t} = \vec{\tau}/2$ ($\vec{\tau}$ – Pauli matrices). The transformation of the fields b_μ^3 and a_μ into physical fields A_μ and Z_μ will be represented as

$$\begin{aligned} b_\mu^3 &= A_\mu \sin\theta + Z_\mu \cos\theta, \\ a_\mu &= A_\mu \cos\theta - Z_\mu \sin\theta, \end{aligned} \quad (6)$$

where

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$$\sin\theta = \frac{g_1}{\sqrt{g^2 + g_1^2}}, \qquad \cos\theta = \frac{g}{\sqrt{g^2 + g_1^2}}.$$

Let us consider the interaction of leptons with gauge fields. Taking into account (5), (6) and (4) in (3), we obtain

$$L_l = \overline{\psi}_{eL}\left(i\gamma_\mu D_\mu\right)\psi_{eL} + \overline{\psi}_{eR}\left(i\gamma_\mu D_\mu\right)\psi_{eR} + \overline{\psi}_{\nu R}(i\gamma_\mu D_\mu)\psi_{\nu R} = \overline{\psi}_{eL}\gamma_\mu[i\partial_\mu + \frac{1}{2}(g\vec{\tau}\vec{b}_\mu + g_1 y_L a_\mu)]\psi_{eL} + \overline{\psi}_{eR}\gamma_\mu(i\partial_\mu + \frac{1}{2}g_1 y_{eR} a_\mu)\psi_{eR} + \\ + \overline{\psi}_{\nu R}\gamma_\mu(i\partial_\mu + \frac{1}{2}g_1 y_{\nu R} a_\mu)\psi_{\nu R} == L_{KIN} + L_{CC} + L_{NC}. \tag{7}$$

Here $L_{KIN} = i\overline{\nu}_{eL}\hat{\partial}\nu_{eL} + i\overline{e}_L^-\hat{\partial}e_L^- + i\overline{e}_R^-\hat{\partial}e_R^- + i\overline{\nu}_{eR}\hat{\partial}\nu_{eR}$ – kinetic part of the Lagrangian, $L_{CC} = \frac{g}{2\sqrt{2}}\left(\overline{\nu}_e O_\mu e^- W_\mu^+ + h.c.\right)$ – charged currents interaction Lagrangian (CC), $L_{NC} = \overline{\nu}\gamma_\mu\left(Q_\nu + Q_\nu^{'}\gamma_5\right)\nu A_\mu + \overline{e}\gamma_\mu\left(Q_{oe} + Q_{oe}^{'}\gamma_5\right)eA_\mu + \overline{\nu}\gamma_\mu\left(g_\nu + g_A\gamma_5\right)\nu Z_\mu + \overline{e}\gamma_\mu\left(G_V + G_A\gamma_5\right)eZ_\mu$ – neutral currents interaction Lagrangian (NC).

In the expressions L_{KIN}, L_{CC} and L_{NC} the following notations were introduced

$$O_\mu = \gamma_\mu\left(1 + \gamma_5\right), \quad W_\mu^\pm = \frac{1}{\sqrt{2}}(b_\mu^1 \mp ib_\mu^2), \quad \hat{\partial} = \gamma_\mu\partial_\mu, \ \overline{\psi} = \psi^+\gamma_4.$$

$$Q_\nu = \frac{1}{4}\left[g\sin\theta + g_1\cos\theta\left(y_L + y_{\nu R}\right)\right], \qquad Q_{0e} = \frac{1}{4}\left[-g\sin\theta + g_1\cos\theta\left(y_L + y_{eR}\right)\right],$$
$$Q_\nu^{'} = \frac{1}{4}\left[g\sin\theta + g_1\cos\theta\left(y_L - y_{\nu R}\right)\right], \qquad Q_{0e}^{'} = \frac{1}{4}\left[-g\sin\theta + g_1\cos\theta\left(y_L - y_{eR}\right)\right], \tag{8a}$$

$$g_V = \frac{1}{4}\left[g\cos\theta - g_1\sin\theta\left(y_L + y_{\nu R}\right)\right], \qquad G_V = \frac{1}{4}\left[-g\cos\theta - g_1\sin\theta\left(y_L + y_{eR}\right)\right],$$
$$g_A = \frac{1}{4}\left[g\cos\theta - g_1\sin\theta\left(y_L - y_{\nu R}\right)\right], \qquad G_A = \frac{1}{4}\left[-g\cos\theta - g_1\sin\theta\left(y_L - y_{eR}\right)\right]. \tag{8b}$$

As can be seen from the expressions (8a), in contrast to the case of work [6], the conditions of P-invariance of the electromagnetic interaction and electroneutrality for the neutrino are not equivalent.
Taking into account the covariant derivative D_μ^φ from (4) and moving to the vacuum expectation value of the field (2)

$$\langle\varphi\rangle = \begin{pmatrix} 0 \\ \langle\varphi_0\rangle \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \eta \end{pmatrix},$$

for the part of the Lagrangian responsible for the masses of the vector bosons, we have the following expression

$$\left(D_\mu^\varphi\varphi\right)^+\left(D_\mu^\varphi\varphi\right) = M_W^2 W_\mu^+ W_\mu^- + \frac{1}{2}M_Z^2 Z_\mu^2 + \frac{1}{2}M_A^2 A_\mu^2 + M_Z M_A A_\mu Z_\mu, \tag{9}$$

where

$$M_w = \frac{g\eta}{\sqrt{2}}, \ M_Z = \frac{\eta}{\sqrt{2}}\left(g\cos\theta + g_1 y_\varphi\sin\theta\right), \ M_A = \frac{\eta}{\sqrt{2}}\left(g\sin\theta - g_1 y_\varphi\cos\theta\right) \tag{10}$$

the mass terms of the fields $W_\mu^\pm$, Z_μ and A_μ .

Taking into account the fact that the photon mass is zero, from (10) we have

$$g\sin\theta - g_1 y_\varphi\cos\theta = 0 \tag{11}$$

If we take into account (11) in expressions (8a) we have

$$Q_\nu = \frac{g\sin\theta}{4}\left(1+\frac{y_L + y_{\nu R}}{y_\varphi}\right),\quad Q_{oe} = -\frac{g\sin\theta}{4}\left(1-\frac{y_L + y_{eR}}{y_\varphi}\right), \tag{12}$$

$$Q_\nu^{'} = \frac{g\sin\theta}{4}\left(1+\frac{y_L - y_{\nu R}}{y_\varphi}\right),\quad Q_{oe}^{'} = -\frac{g\sin\theta}{4}\left(1-\frac{y_L - y_{eR}}{y_\varphi}\right).$$

Taking into account the P-invariance of the electromagnetic interaction for the neutrino and electron (i.e. $Q_\nu^{'} = 0$, $Q_{oe}^{'} = 0$) , between the hypercharges of the Higgs field and the lepton isomultiplets we obtain the following relations

$$y_L - y_{\nu R} = -y_\varphi, \qquad y_L - y_{eR} = y_\varphi \tag{13}$$

Taking into account (13) in (12) we have

$$Q_\nu = \frac{Q_e}{2}\left(1+\frac{y_L}{y_\varphi}\right),\quad Q_{oe} = -\frac{Q_e}{2}\left(1-\frac{y_L}{y_\varphi}\right), \tag{14}$$

where $Q_e = |e| = g\sin\theta -$ is the modulus of the electron charge.

From expression (14) it is evident that the electric charges of the leptons depend on the hypercharge of the Higgs field and this can be considered as evidence of quantization of the electric charge of the leptons. But expressions (14) do not determine the numerical values of the charge of the leptons.

Taking into account the electroneutrality of the neutrino ($Q_\nu = 0$) from (14) we have

$$y_L = -y_\varphi \tag{15}$$

In the WS model, the masses of the leptons are generated using the Yukawa mass Lagrangian

$$L_{mass}^{l} = f_e\overline{\psi}_{eL}\psi_{eR}\varphi + f_\nu\overline{\psi}_{eL}\psi_{\nu R}\varphi^c + h.c., \tag{16}$$

where $\varphi^c = i\tau_2\varphi^*$.

Taking into account the conservation of the hypercharge from (16) we have

$$y_L = y_{eR} + y_\varphi \ , \qquad y_L = y_{\nu R} - y_\varphi \tag{17}$$

From (13) and (17) it is evident that in this case the P-invariance of the electromagnetic interaction of leptons and the requirement of conservation of hypercharge lead to the same expressions.

If we take into account the P-invariance of the electromagnetic interaction of leptons, the electroneutrality of the neutrino and the masslessness of the photon, expressions (8b) are simplified and take the form

$$g_V = g_A = \frac{g}{4\cos\theta} \ ,$$

$$G_V = \frac{g}{4\cos\theta}\left(-1+4\sin^2\theta\right),$$

$$G_A = -\frac{g}{4\cos\theta} \tag{18}$$

and between the expressions for the mass W and Z-bosons from (10) we obtain the well-known relation

$$M_w = M_z\cos\theta$$

As can be seen from (18), the expressions for the vector and axial constants of interactions with the Z boson for the neutrino (g_V, g_A) and for the electron (G_V, G_A) with the expressions in the WS model and, when obtaining them for the hypercharge of particles, specific numbers were not taken coincide.

If we take into account the relationship $y_L = -y_\varphi$ in (13), we have

$$y_{\nu R} = 0\,,\ y_{eR} = 2y_L = -2y_\varphi$$

From here we can conclude that in the case under consideration, the field A_μ as a photon field will be massless and will have correct electromagnetic interactions with leptons only when

a) the hypercharge of the Higgs boson field and the lepton doublet are equal in magnitude and opposite in sign,

b) the hypercharge of the right-hand component of the neutrino is zero.

Considering the relationships (15) in expressions (14), for the charge of the leptons we have

$$Q_\nu = 0\,,\quad Q_{oe} = -Q_e = -|e|\,.$$

It follows from expressions (15) that when the Higgs field interacts with the lepton field, the lepton isodoublet acquires a hypercharge equal in value to the hypercharge of the Higgs field with the opposite sign. Consequently, inclusion of the Higgs field in the theory leads to the fact that when the Higgs field interacts with lepton isodoublets, the hypercharges of these isodoublets take on quite definite values. The hypercharge of the Higgs field fixes the hypercharges of the lepton isodoublets. Thus, we come to the conclusion that relation (15) leads to quantization of the electric charge of leptons and this indicates the influence of the Higgs field on the quantization of the electric charge.

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