# EFFECTIVE TEMPERATURE OF ELECTRONS IN A SYMMETRY – CENTERED SEMICONDUCTOR QUANTUM WELL

**M.M. BABAYEV**
*Institute of Physics, Ministry of Science and Education of Azerbaijan*
*Baku, Azerbaijan, H. Javid ave. 131, AZ 1073*
m.babayev@physics.science.az

The heating of electrons in semiconductor quantum wells created from crystals with central symmetry under the influence of a strong electric field has been studied. The effective temperature approximation was used, and since the analysis is done for low temperatures of the lattice, electron scattering only from impurity ions and acoustic phonons has been considered. The dependence of the effective temperature of electrons on the intensity of the electric field and the surface density of the electrons has been calculated. The obtained theoretical results have been applied to a $Si/Si_{1-x}Ge_x$ quantum well. It has been determined that the dependence of the effective temperature of the strongly degenerate electron gas on the intensity of the electric field is approximately quadratic.

**Keywords**: quantum well, Pöschl-Teller potential, effective electron temperature.
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## 1. INTRODUCTION

Semiconductor structures with one-dimensional confinement on the nanoscale are the main objects of research in modern nanoelectronics. When a strong electric field is applied, electron and hole heating occurs in these structures. The study of electron heating provides valuable information for understanding the working principles and efficiency of new-generation devices, as well as for investigating electron-phonon interactions and energy relaxation in semiconductors. This phenomenon has been studied both theoretically and experimentally for a long time. Recently, interest in studying this phenomenon has increased significantly [1-5].

In this work, we investigate the heating of electrons in semiconductor quantum wells under the influence of an electric field. We focus on low-temperature regimes where the energy and momentum scattering of electrons are dominated by acoustic phonons and impurity ions. When performing theoretical studies in a quantum well, it is necessary to choose the form of the confining potential. For this purpose, rectangular [6], parabolic [7], or Pöschl-Teller potentials [8-10] are often used. In this work, we will use the modified Pöschl-Teller potential when studying the electron temperature. This potential lies between the rectangular and parabolic potentials and provides a better approximation of the confining potential [8].

## 2. BASIC EQUATIONS OF THE PROBLEM

Let us consider the direction in which the motion of electrons is confined to be the z-axis. Then, the modified Pöschl–Teller potential and the energy of electrons in the potential well are given by [11]:

$$U(z) = \frac{\hbar^2\alpha^2}{m}\tanh^2\alpha z,\ \ \varepsilon_k = \frac{\hbar^2}{2m}(\alpha^2 + k^2) \tag{1}$$

where m is the effective mass of the conducting electrons, α is the parameter of the quantum well, and $\vec{k} = (k_x, k_y)$ is the 2D wave vector of the electrons. We consider the case where the electric field $\vec{E}$ is applied along the x-direction: $E = E_x$. It is clear that in this case, the electric current $j_x$ will also be directed along the x-axis.

We consider the case where the electric field intensity $\vec{E}$ is applied in the plane of the two-dimensional electron gas. Let this direction be denoted by the x-axis: $E = E_x$ . It is evident that the electric current density $j_x$ will also be along the x-axis in this case.

The wave function corresponding to the energy values in equation (1) is given by [11]:

$$\psi_{k_x,k_y}(\vec{r}) = \left[\frac{2\alpha}{L_xL_yL_z}\right]^{\frac{1}{2}} e^{i(k_xx+k_yy)}\ P_1^{-1}(\tanh\alpha z), \tag{2}$$

where $L_z = L$ is the size of the system along the confined direction, while $L_x$ and $L_y$ are the dimensions along the unconfined directions, r = (x, y, z) denotes the position vector of the electron, and $P_1^{-1}(\tanh\alpha z)$ is the associated Legendre function.

The degree of electron heating (effective temperature Te) is found from the balance equation: in a stationary state, the energy received by the electron system from the electric field is equal to the energy it gives to the phonon system [12]:

$$\sigma_2(T_e)E^2 = n\left(\frac{\partial\varepsilon}{\partial t}\right)_1 . \tag{3}$$

131, H.Javid ave, AZ-1073, Baku
Institute of Physics
E-mail: jophphysics@gmail.com

Here $\sigma_2(T_e)$ is the conductivity of the 2D electron gas in a strong electric field, n is the surface density of electrons, and $\left(\frac{\partial\varepsilon}{\partial t}\right)_1$ is the energy lost per second by a single electron due to phonon scattering.

The conductivity of the electron gas in a quantum well with a modified Pöschl–Teller potential in weak electric fields (where electron heating does not occur) was calculated in [8]. Using a similar approach, we can calculate the conductivity of the electron gas in a quantum well under a strong electric field. We consider the low-temperature regime (lattice temperature T<10K). In this case, the electrons in the quantum well are in a strongly degenerate state, and for the conductivity of the electron gas in an electric field, we obtain:

$$\sigma_2(T_e) = \frac{e^2 k_0 T_e}{\pi\,\hbar^2}\eta\;\tau_e(\eta), \qquad (4)$$

where $\tau_e(\eta)$ is the electron momentum relaxation time, and

$$\eta = \frac{1}{k_0\,T_e}\left(\zeta - \frac{\hbar^2\alpha^2}{2\,m}\right) \qquad (5)$$

is the reduced chemical potential of the electrons, $k_0$ is the Boltzmann constant, and $\zeta$ is the chemical potential.

Calculations show that the main scattering mechanism for the electron's momentum is scattering from impurity ions. Using the effective temperature approach, the scattering rate $\nu = \tau_e^{-1}(\eta)$ of electrons due to this mechanism in the quantum well is given by:

$$\nu_i = \frac{2\,e^4 m\,n_i}{\chi^{\,2}\hbar^3 n} J_i(\gamma)\ , \qquad (6)$$

where the functionis $J_i(\gamma)$ defined as:

$$J_i(\gamma) = \int_0^1 \frac{dx}{\epsilon(x)\sqrt{1-x^2}}\left(\int_0^\infty e^{-\gamma x t}(1-tanh^2 t)dt\right)^2. \qquad (7)$$

Here, the notations are as follows: $x = \frac{q_2}{2\,k}, \gamma = \frac{2\,k}{\alpha}, t = \frac{\pi\,q_z}{2\,\alpha}$ , where $q_2 = (q_x, q_y)$ and $q_z$ are the components of the phonon wave vector $\vec{q} = (q_x, q_y, q_z)$ parallel and perpendicular to the quantum layer, respectively. $n_i$ is the surface density of impurity ions, $\chi$ is the static dielectric constant, and $\epsilon(x)$ is the dielectric function of the two-dimensional electron gas [8]:

$$\epsilon(x) = 1 + \frac{me^2}{\hbar^2\pi\chi kx}\left\{\frac{1}{2}(\gamma x)^2\Psi^{(1)}\left(-1+\frac{\gamma x}{2}\right) - \frac{4+(\gamma x-1)(\gamma x)^2}{(\gamma x-2)^2}\right\}, \qquad (8)$$

where $\Psi^{(1)}\left(-1+\frac{\gamma x}{2}\right)$ is the trigamma function.

## 3. CALCULATION OF THE ENERGY TRANSFERRED FROM ELECTRONS TO THE CRYSTAL LATTICE

The energy transferred to phonons by a single electron per unit time is given by:

$$\left(\frac{\partial\varepsilon}{\partial t}\right)_1 = \frac{1}{N_0}\sum_{\vec{q}}\left(\frac{\partial N_q}{\partial t}\right)_{ef}\hbar\omega_q\ . \qquad (9)$$

Here, $N_0 = nL_xL_y$ is the total number of electrons in the quantum well, $\hbar\omega_q = \hbar v_0 q$ is the energy of an acoustic phonon, $v_0$ is the speed of sound in the crystal, and $\left(\frac{\partial N_q}{\partial t}\right)_{ef}$ is the rate of change in the number of phonons due to electron-phonon interaction. To compute this quantity, we must calculate the matrix elements of transitions corresponding to phonon emission and absorption by electrons. Since we are considering crystals with a center of symmetry, there is no piezoelectric potential [13], and the scattering of electrons by phonons occurs due to the deformation potential. In this case the electron-phonon interaction is given by:

$$V(\vec{r}) = \frac{i\,E_1}{\sqrt{N}}\sum_{\vec{q}}\frac{q}{\epsilon(q_2)}\left\{b(\vec{q})\,e^{i\,\vec{q}\,\vec{r}} + b^*(\vec{q})e^{-i\,\vec{q}\,\vec{r}}\right\}, \qquad (10)$$

where $N$ is the number of unit cells in the crystal, $E_1$ is the deformation potential constant, and $b(\vec{q})$ are complex normal coordinates [13]. The matrix elements in the quantum well are calculated using the wave function of the unexcited electron-phonon system:

$$\psi_{k_x,k_y,N_q} = \psi_{k_x,k_y}(\vec{r})\,\Pi\varphi_{N_q}(Q_{\vec{q}}) \qquad (11)$$

where $\psi_{k_x,k_y}(\vec{r})$ is the electron wave function, $\Pi\varphi_{N_q}(Q_{\vec{q}})$ is the wave function of the unexcited phonon system [13]. The total energy of the system is:

$$\mathcal{E}_{k,N_q} = \varepsilon_k + \sum_{\vec{q}}\left(N_q + \frac{1}{2}\right)\hbar\,\omega_q\ . \qquad (12)$$

The method for calculating the phonon number variation due to electron-phonon interaction is described in [12, 13]. Calculations carried out in the case of strong degeneracy of the electron gas ($\eta \gg 1$), give the following result for the rate of change of the phonons number:

$$\left(\frac{\partial N_q}{\partial t}\right)_{ef} = \frac{{E_1}^2 m^2 k_0 (T_e - T)}{\pi^2 \hbar^4 \rho v_0 q_2 \epsilon^2(q_2)} \left(\frac{8mk_0 T_e \eta}{\hbar^2} - {q_2}^2\right)^{-\frac{1}{2}} \int_{-\infty}^{\infty} \left(\frac{\pi\, q_z}{2\,\alpha}\right)^2 Csch^2\left(\frac{\pi\, q_z}{2\,\alpha}\right) q\, dq_z\,. \quad (13)$$

Substituting Eq. (13) into Eq. (9) and integrating over all phonons involved in the interaction yields:

$$\left(\frac{\partial \varepsilon}{\partial t}\right)_1 = \frac{4\alpha L {E_1}^2 m^2 k_0 (T_e - T)\, k^3}{\pi^5 \hbar^3\, n\, \rho}\, J_a(\gamma)\ . \quad (14)$$

where:

$$J_a(\gamma) = \int_0^1 dx\, \frac{\sqrt{1+x^2} + x^2 ln\frac{1+\sqrt{1+x^2}}{x}}{\sqrt{1-x^2}\,\epsilon^2(x)} \int_{-\infty}^{\infty} dt\, t^2\ \ Csch^2(t) \sqrt{\left(\frac{2t}{\pi\gamma}\right)^2 + x^2}\ \ . \quad (15)$$

Substituting Eqs. (4) and (14) into the energy balance equation (3) and solving for the effective electron temperature $T_e$ , we obtain the dimensionless temperature ratio $\theta = \frac{T_e}{T}$:

$$\theta = 1 + \left(\frac{E}{E_a}\right)^2, \quad (16)$$

where $E_a$ is the characteristic electric field:

$$E_a = \frac{4\, e\, m^2\, E_1}{\pi^2 \hbar^3 \chi} \left[\frac{(\alpha L) k_0 T\, k_F n_i}{\rho\, n}\, J_i(\gamma)\, J_a(\gamma)\right]^{\frac{1}{2}} \quad (17)$$

From Eqs. (16) and (17), it follows that for relatively high electric field intensities ($E > E_a$), the dependence of the effective temperature of the strongly degenerate electron gas on the field intensity approaches a quadratic form.

## 4. APPLICATION OF THEORETICAL RESULTS TO $Si/Si_{1-x}Ge_x$ QUANTUM WELL

Let us consider the commonly used quantum well width $L = 10^{-8}\, m$, which is frequently employed in experiments. For silicon (Si), the material parameters are taken as [14]: $\rho = 2.33\, \frac{q}{cm^3}$; $v_0 = 9 \cdot 10^5\, \frac{m}{san}$; $\chi = 11.7$; $E_1 = 8$ eV. In silicon crystals, the effective mass of electrons is anisotropic. The isoenergetic surfaces near the minimum of the conduction band are ellipsoids of rotation with the axis of rotation directed along [100]. In Si quantum wells, the confinement direction is typically taken along [100]. In this direction, the effective mass is $m_{\parallel} = 0.92\, m_0$ , while in the direction perpendicular to the layer (in-plane), the effective mass is $m_{\perp} = 0.19 m_0$ , where $m_0$ is the free electron mass [14].

The depth of the quantum well (Δ) depends on the composition (i.e., the value of $x$). For the calculations, we take $\Delta = 0.07\, eV$, which corresponds to a Ge concentration of $x = 0.1$ in the $Si_{1-x}Ge_x$ layer. The value of the parameter α in the modified Pöschl–Teller potential can be determined from the condition that the coefficient in front of the $\tanh^2(\alpha z)$ function is equal to the depth of the quantum well. From Eq. (1), we have:

$$\frac{\hbar^2 \alpha^2}{m_{\parallel}} = \Delta\,. \quad (18)$$

Using $\Delta = 0.07\, eV$, we obtain: $\alpha \cong 9 \times 10^8\, m^{-1}$. This value of $\alpha$ is used in the subsequent calculations.

For a degenerate electron gas, the coefficients $J_i(\gamma)$ and $J_a(\gamma)$ are only weakly dependent on the lattice temperature. Thus, as it follows from equation (17), the characteristic electric field $E_a$ increases, and the dimensionless temperature of electrons $\theta$ decreases with the growth of the lattice temperature.

Fig.1 shows the dependence of the dimensionless electron temperature $\theta$ on the electric field strength $E$ for various values of the lattice temperature: $T = 4, 6, 8, 10\, K$. For small lattice temperatures and relatively high electric field strengths, the electron temperature becomes an order of magnitude higher than the lattice temperature.

The degree of electron heating also depends on the electron surface density $n$. Note that in quantum wells, as the surface density of impurity ions increases, the electron surface density increases as well. For 3D degenerate n-type semiconductors, the electron concentration is approximately equal to the ion concentration, whereas in 2D systems, typically $n \gg n_i$ (by a factor of 10–100) [15], although the ratio $n/n_i$ changes only slightly. Taking this ratio to be constant, we can analyze the dependence of the characteristic field $E_a$ on $n$.

For a degenerate electron gas, the Fermi wave number depends only on the surface density: $k_F = \sqrt{2\pi n}$ . Additionally, the product $J_i(\gamma)\, J_a(\gamma)$ increases slightly with increasing $n$. As a result, the characteristic electric field $E_a$ increases with increasing $n$, meaning that the degree of electron heating decreases as the electron surface density increases.

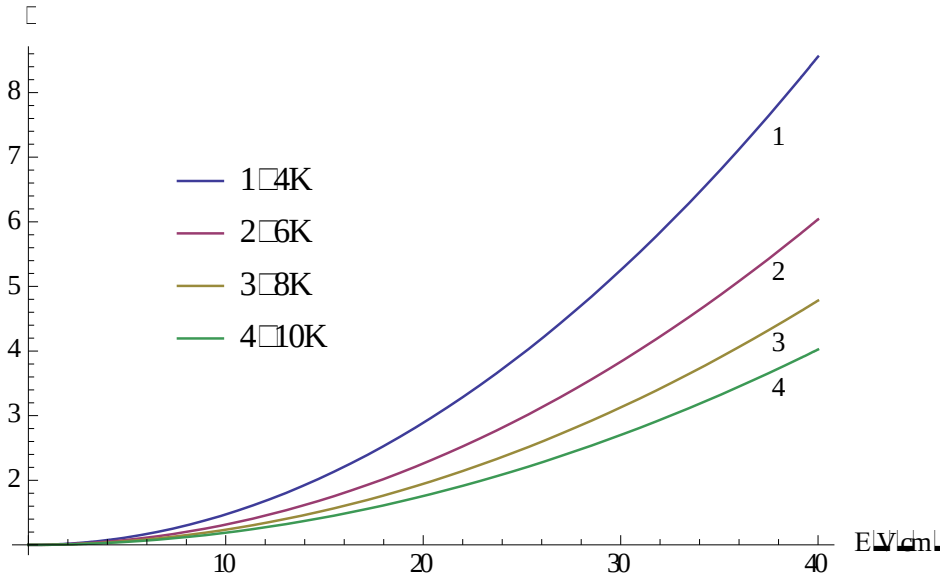


*Fig.1.* Dependence of the dimensionless electron temperature $\theta$ on electric field strength $E$ at different lattice temperatures.

Using the above parameters, at a lattice temperature $T = 4K$ and a surface electron density $= 2 \times 10^{15}\ m^{-2}$ , $\frac{n}{n_i} = 50$, for the characteristic field we obtain: $E_a$=*14.5V/cm*. Let's estimate the characteristic field at different values of the surface density of electrons at the given lattice temperature ($T = 4K$): for $n = 10^{15}\ m^{-2}$ : $E_a = 9.8\ V/cm$ ; for $n = 3 \times 10^{15}\ m^{-2}$: $E_a = 17.8\ V/cm$; for $n = 4 \times 10^{15}\ m^{-2}$ : $E_a = 20.3\ V/cm$.

## 5. CONCLUSIONS

At electric field intensities exceeding the characteristic field $E_a$, the dependence of the effective temperature of a strongly degenerate 2D electron gas in a quantum well on the field intensity is close to quadratic. The value of the characteristic field increases with both the lattice temperature and the electron surface density. Consequently, at lower lattice temperatures and electron surface densities, the degree of electron heating under strong electric fields increases.


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